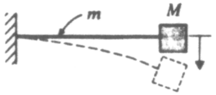
Equivalent Masses, Springs and Dampers

Equivalent masses

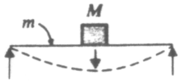
Mass (M) attached at end of spring of mass m

$$m_{eq} = M + \frac{m}{3}$$



Cantilever beam of mass m carrying an end mass M

$$m_{eq} = M + \frac{33}{140}m$$



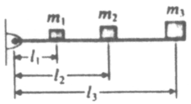
Simply supported beam of mass m carrying a mass M at the middle

$$m_{eq} = M + \frac{17}{35}m$$

Coupled translational and rotational masses

$$m_{eq} = m + \frac{J_0}{R^2}$$

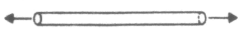
$$J_{eq} = J_0 + mR^2$$



Masses on a hinged bar

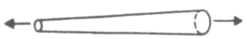
$$m_{eq_1} = m_1 + \left(\frac{l_2}{l_1}\right)^2 m_2 + \left(\frac{l_3}{l_1}\right)^2 m_3$$

Equivalent springs



Rod under axial load (l = length, A = cross sectional area)

$$k_{eq} = \frac{EA}{l}$$



Tapered rod under axial load (D, d = end diameters)

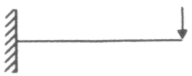
$$k_{eq} = \frac{\pi EDd}{4l}$$

Helical spring under axial load (d = wire diameter, D = mean coil diameter, n = number of active turns)

$$k_{eq} = \frac{Gd^4}{8nD^3}$$

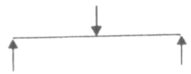
Fixed-fixed beam with load at the middle

$$k_{eq} = \frac{192EI}{l^3}$$



Cantilever beam with end load

$$k_{eq} = \frac{3EI}{l^3}$$

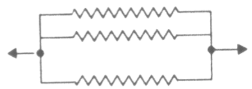


Simply supported beam with load at the middle

$$k_{eq} = \frac{48EI}{l^3}$$

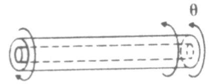
Springs in series

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots + \frac{1}{k_n}$$



Springs in parallel

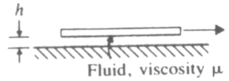
$$k_{eq} = k_1 + k_2 + \cdots + k_n$$



Hollow shaft under torsion (l = length, D = outer diameter, d = inner diameter)

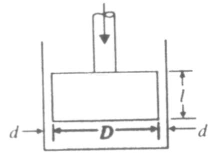
$$k_{eq} = \frac{\pi G}{32l}(D^4 - d^4)$$

Equivalent viscous dampers



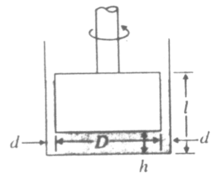
Relative motion between parallel surfaces (A = area of smaller plate)

$$c_{eq} = \frac{\mu A}{h}$$



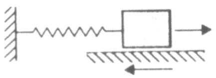
Dashpot (axial motion of a piston in a cylinder)

$$c_{eq} = \mu\frac{3\pi D^3 l}{4d^3}\left(1 + \frac{2d}{D}\right)$$



Torsional damper

$$c_{eq} = \frac{\pi\mu D^2(l-h)}{2d} + \frac{\pi\mu D^3}{32h}$$



Dry friction (Coulomb damping) (fN = friction force, ω = frequency, X = amplitude of vibration)

$$c_{eq} = \frac{4fN}{\pi\omega X}$$