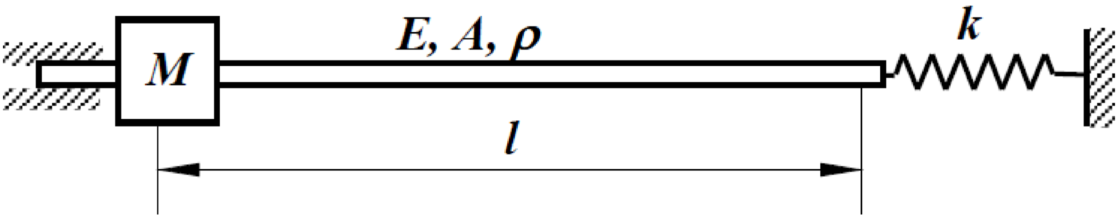


$f(t) := 1$ $\qquad \omega := \frac{\beta \cdot c}{l}$

$u(x,t) := \left(A \cdot \cos\left(\frac{\omega \cdot x}{c}\right) + B \cdot \sin\left(\frac{\omega \cdot x}{c}\right)\right) \cdot f(t)$ Soluzione dell'eq. di D'Alembert

$\frac{\partial}{\partial x} u(x,t) \text{ collect}, \left(\frac{\omega}{c}\right) \rightarrow \left(B \cdot \cos\left(\frac{\beta}{l} \cdot x\right) - A \cdot \sin\left(\frac{\beta}{l} \cdot x\right)\right) \cdot \frac{\beta}{l}$

$N(x,t) := E \cdot A_s \cdot \frac{\partial}{\partial x} u(x,t)$

$u_{pp}(x,t) := -\omega^2 \cdot u(x,t)$

Condizioni al contorno

$M \cdot u_{pp}(0,t) - N(0,t) \text{ collect}, A, B \rightarrow \left(-\frac{M \cdot \beta^2 \cdot c^2}{l^2}\right) \cdot A + \left(-\frac{A_s \cdot E \cdot \beta}{l}\right) \cdot B$

$k \cdot u(l,t) + N(l,t) \text{ collect}, A, B \rightarrow \left(k \cdot \cos(\beta) - \frac{A_s \cdot E \cdot \beta \cdot \sin(\beta)}{l}\right) \cdot A + \left(k \cdot \sin(\beta) + \frac{A_s \cdot E \cdot \beta \cdot \cos(\beta)}{l}\right) \cdot B$

Dati numerici

$E := 206000 \cdot 10^6 = 2.06 \times 10^{11}$

$l := 1.4$

$\rho := 7800$

$d := 12 \cdot 10^{-3} = 0.012$

$A_s := \frac{\pi \cdot d^2}{4} = 1.130973 \times 10^{-4}$

$M := 5$

$m := \rho \cdot A_s \cdot l = 1.235023$ Massa della barra

$\mu := \frac{m}{M} = 0.247005$

$k := 3 \cdot 10^7$

$c := \sqrt{\frac{E}{\rho}} = 5139.091$

$a := \frac{E \cdot A_s}{l} = 1.664147 \times 10^7$

$f_{11}(\beta) := \left(-\frac{M \cdot \beta^2 \cdot c^2}{l^2} \right)$

$f_{12}(\beta) := \left(-\frac{A_s \cdot E \cdot \beta}{l} \right)$

$f_{21}(\beta) := \left(k \cdot cos(\beta) - \frac{A_s \cdot E \cdot \beta \cdot sin(\beta)}{l} \right)$

$f_{22}(\beta) := \left(k \cdot sin(\beta) + \frac{A_s \cdot E \cdot \beta \cdot cos(\beta)}{l} \right)$

$\Delta(\beta) := \begin{pmatrix} f_{11}(\beta) & f_{12}(\beta) \\ f_{21}(\beta) & f_{22}(\beta) \end{pmatrix}$

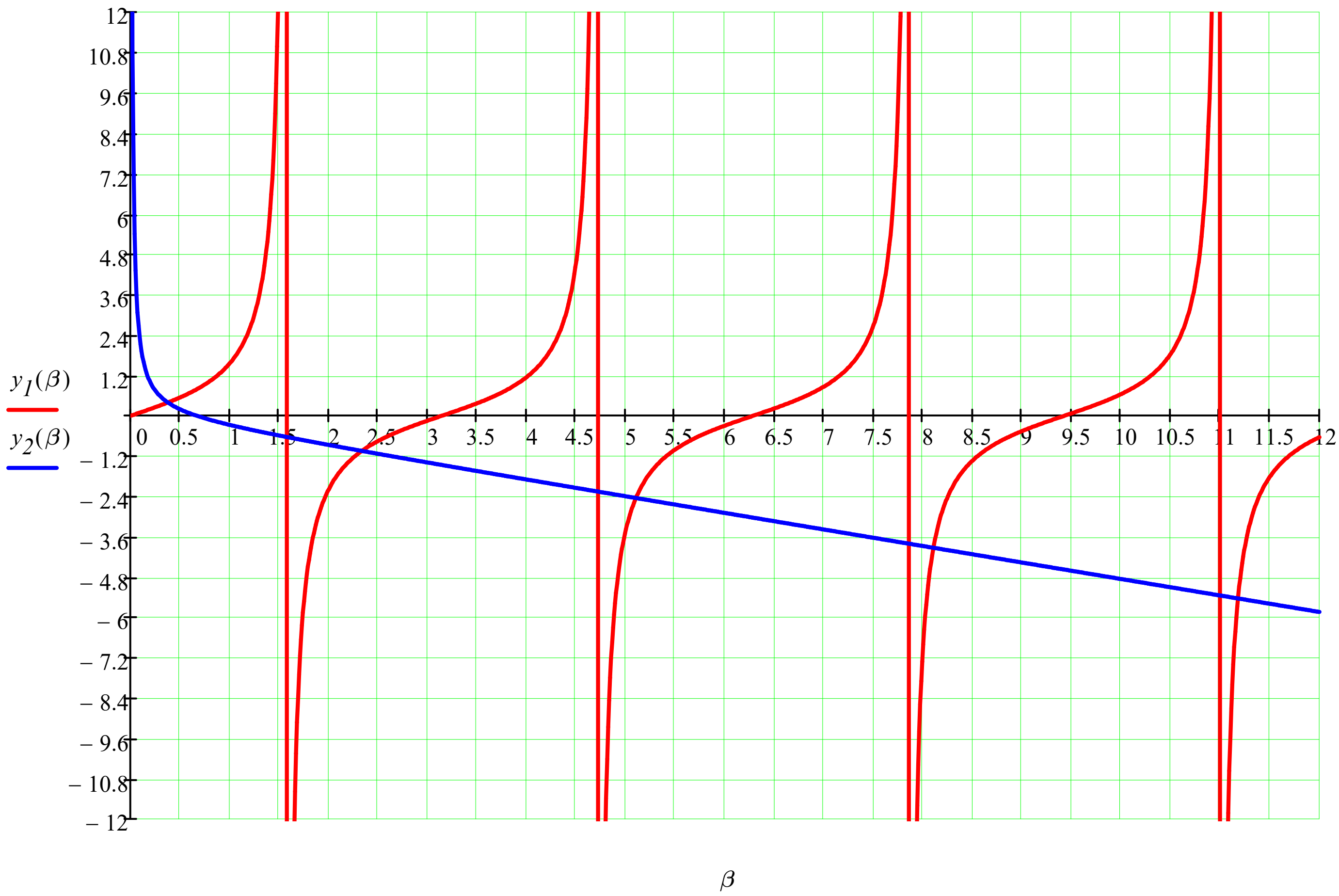
Equazione delle frequenze

$$tan(\beta) = \frac{\frac{k \cdot \mu}{\beta} - a \cdot \beta}{k + \mu \cdot a}$$

$y_1(\beta) := tan(\beta)$

$y_2(\beta) := \frac{\frac{k \cdot \mu}{\beta} - a \cdot \beta}{k + \mu \cdot a}$

$\beta := 0, 0.01 .. 12$ $\qquad$ $FS := 12$



$F(\beta) := y_1(\beta) - y_2(\beta)$

$\beta := 0.5$

$\beta_1 := root(F(\beta), \beta) = 0.375856$

$\beta := 2.5$

$\beta_2 := root(F(\beta), \beta) = 2.333921$

$\beta := 5.2$

$\beta_3 := root(F(\beta), \beta) = 5.100515$

$\beta := 8.2$

$\beta_4 := root(F(\beta), \beta) = 8.103356$

$\beta := 11$

$\beta_5 := root(F(\beta), \beta) = 11.177573$

rad/s Hz

$\beta_1 = 0.375856$ $\omega_1 := \frac{c}{l}\cdot\beta_1 = 1379.685$ $f_1 := \frac{\omega_1}{2\cdot\pi} = 219.584$

$\beta_2 = 2.333921$ $\omega_2 := \frac{c}{l}\cdot\beta_2 = 8567.31$ $f_2 := \frac{\omega_2}{2\cdot\pi} = 1363.53$

$\beta_3 = 5.100515$ $\omega_3 := \frac{c}{l}\cdot\beta_3 = 18722.864$ $f_3 := \frac{\omega_3}{2\cdot\pi} = 2979.836$

$\beta_4 = 8.103356$ $\omega_4 := \frac{c}{l}\cdot\beta_4 = 29745.631$ $f_4 := \frac{\omega_4}{2\cdot\pi} = 4734.164$

$\beta_5 = 11.177573$ $\omega_5 := \frac{c}{l}\cdot\beta_5 = 41030.402$ $f_5 := \frac{\omega_5}{2\cdot\pi} = 6530.191$

Calcolo diretto del determinante in forma numerica, senza sviluppi di calcolo simbolico

$det(\beta) := |\Delta(\beta)|$

$\beta := 0.5$ $root(det(\beta),\beta) = 0.375856$

$\beta := 2.5$ $root(det(\beta),\beta) = 2.333921$

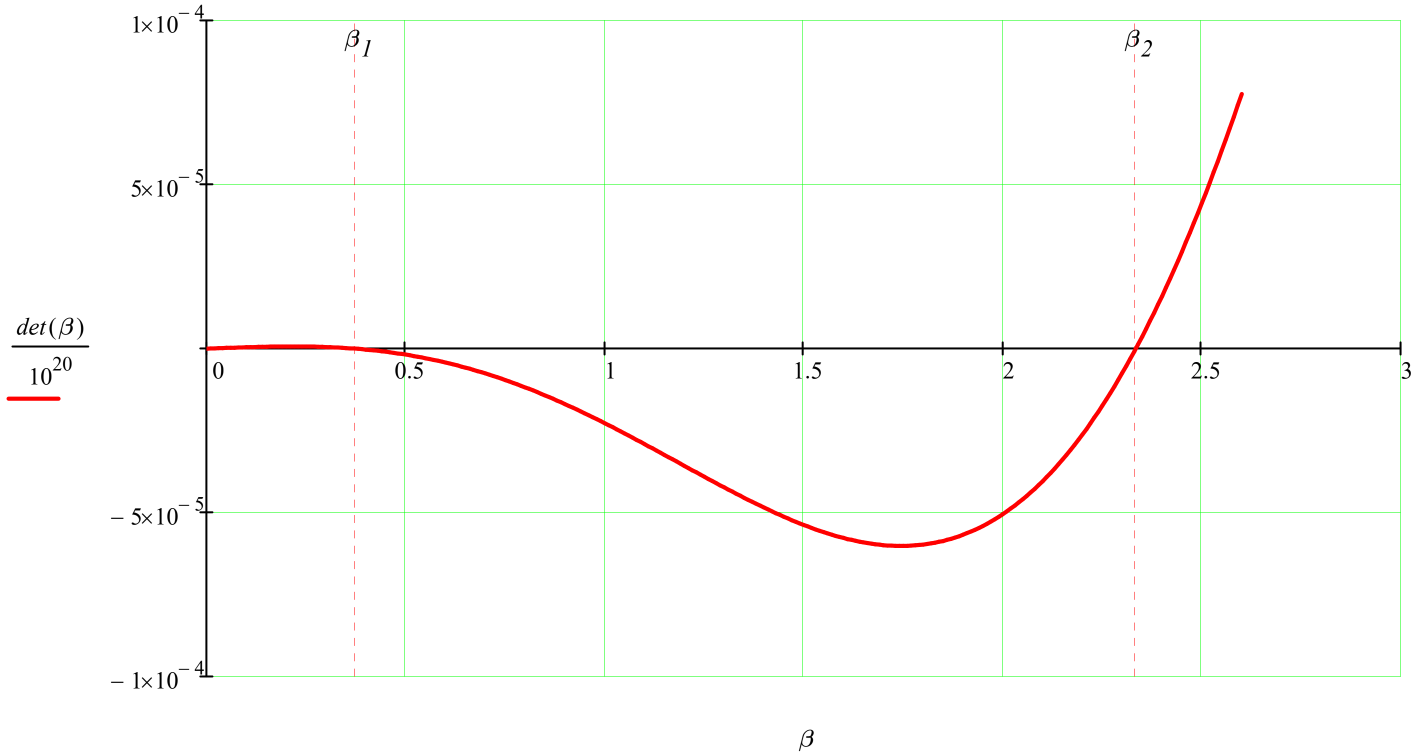
$\beta := 5.2$ $root(det(\beta),\beta) = 5.100515$

$\beta := 8.2$ $root(det(\beta),\beta) = 8.103356$

$\beta := 11$ $root(det(\beta),\beta) = 11.177573$

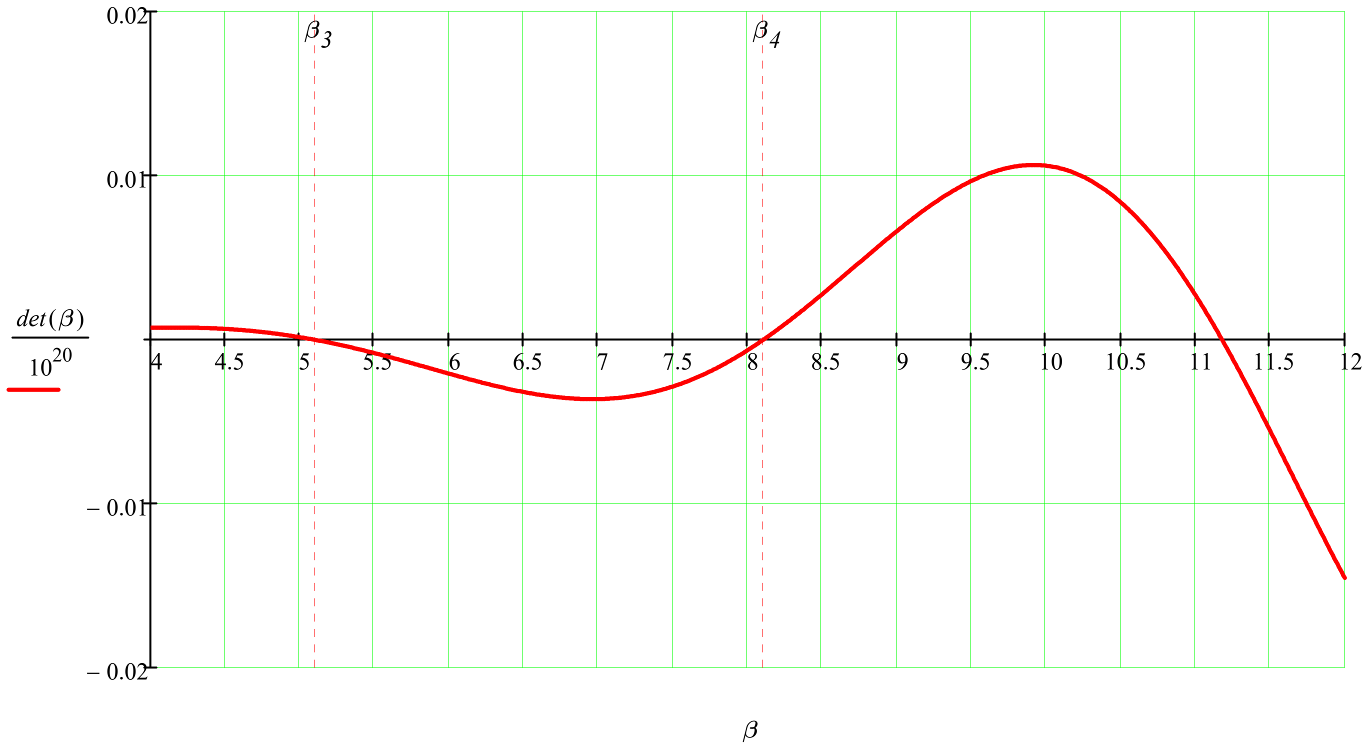
$\beta_1 = 0.375856$ $\beta_2 = 2.333921$ $\beta_3 = 5.100515$ $\beta_4 = 8.103356$ $\beta_5 = 11.177573$

$\beta := 0, 0.01 .. 2.6$



1×10^−4
5×10^−5
−5×10^−5
−1×10^−4
β1
β2
det(β)/10^20
0
0.5
1
1.5
2
2.5
3
β

$\beta := 4, 4.01 .. 12$



0.02
0.01
−0.01
−0.02
β3
β4
det(β)/10^20
4
4.5
5
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
10.5
11
11.5
12
β