

$$f(t) := 1 \quad \omega := \frac{\beta \cdot c}{l}$$

$$u(x, t) := \left( A \cdot \cos\left(\frac{\omega \cdot x}{c}\right) + B \cdot \sin\left(\frac{\omega \cdot x}{c}\right) \right) \cdot f(t)$$

Soluzione dell'eq. di D'Alembert

$$\frac{\partial}{\partial x} u(x, t) \text{ collect, } \left(\frac{\omega}{c}\right) \rightarrow \left( B \cdot \cos\left(\frac{\beta}{l} \cdot x\right) - A \cdot \sin\left(\frac{\beta}{l} \cdot x\right) \right) \cdot \frac{\beta}{l}$$

$$N(x, t) := E \cdot A_s \cdot \frac{\partial}{\partial x} u(x, t)$$

$$u_{pp}(x, t) := -\omega^2 \cdot u(x, t)$$

Condizioni al contorno

$$M \cdot u_{pp}(0, t) - N(0, t) \text{ collect, } A, B \rightarrow \left( -\frac{M \cdot \beta^2 \cdot c^2}{l^2} \right) \cdot A + \left( -\frac{A_s \cdot E \cdot \beta}{l} \right) \cdot B$$

$$k \cdot u(l, t) + N(l, t) \text{ collect, } A, B \rightarrow \left( k \cdot \cos(\beta) - \frac{A_s \cdot E \cdot \beta \cdot \sin(\beta)}{l} \right) \cdot A + \left( k \cdot \sin(\beta) + \frac{A_s \cdot E \cdot \beta \cdot \cos(\beta)}{l} \right) \cdot B$$

Dati numerici

$$E := 206000 \cdot 10^6 = 2.06 \times 10^{11}$$

$$\underset{\text{mm}}{l} := 1.4$$

$$\rho := 7800$$

$$d := 12 \cdot 10^{-3} = 0.012$$

$$A_s := \frac{\pi \cdot d^2}{4} = 1.130973 \times 10^{-4}$$

$$M := 5$$

$$\underset{\text{kg}}{m} := \rho \cdot A_s \cdot l = 1.235023$$

Massa della barra

$$\mu := \frac{m}{M} = 0.247005$$

$$k := 3 \cdot 10^7$$

$$\textcolor{green}{c} := \sqrt{\frac{E}{\rho}} = 5139.091$$

$$a := \frac{E \cdot A_s}{l} = 1.664147 \times 10^7$$

$$f_{11}(\beta) := \left( -\frac{M \cdot \beta^2 \cdot c^2}{l^2} \right)$$

$$f_{12}(\beta) := \left( -\frac{A_s \cdot E \cdot \beta}{l} \right)$$

$$f_{21}(\beta) := \left( k \cdot \cos(\beta) - \frac{A_s \cdot E \cdot \beta \cdot \sin(\beta)}{l} \right)$$

$$f_{22}(\beta) := \left( k \cdot \sin(\beta) + \frac{A_s \cdot E \cdot \beta \cdot \cos(\beta)}{l} \right)$$

$$\Delta(\beta) := \begin{pmatrix} f_{11}(\beta) & f_{12}(\beta) \\ f_{21}(\beta) & f_{22}(\beta) \end{pmatrix}$$

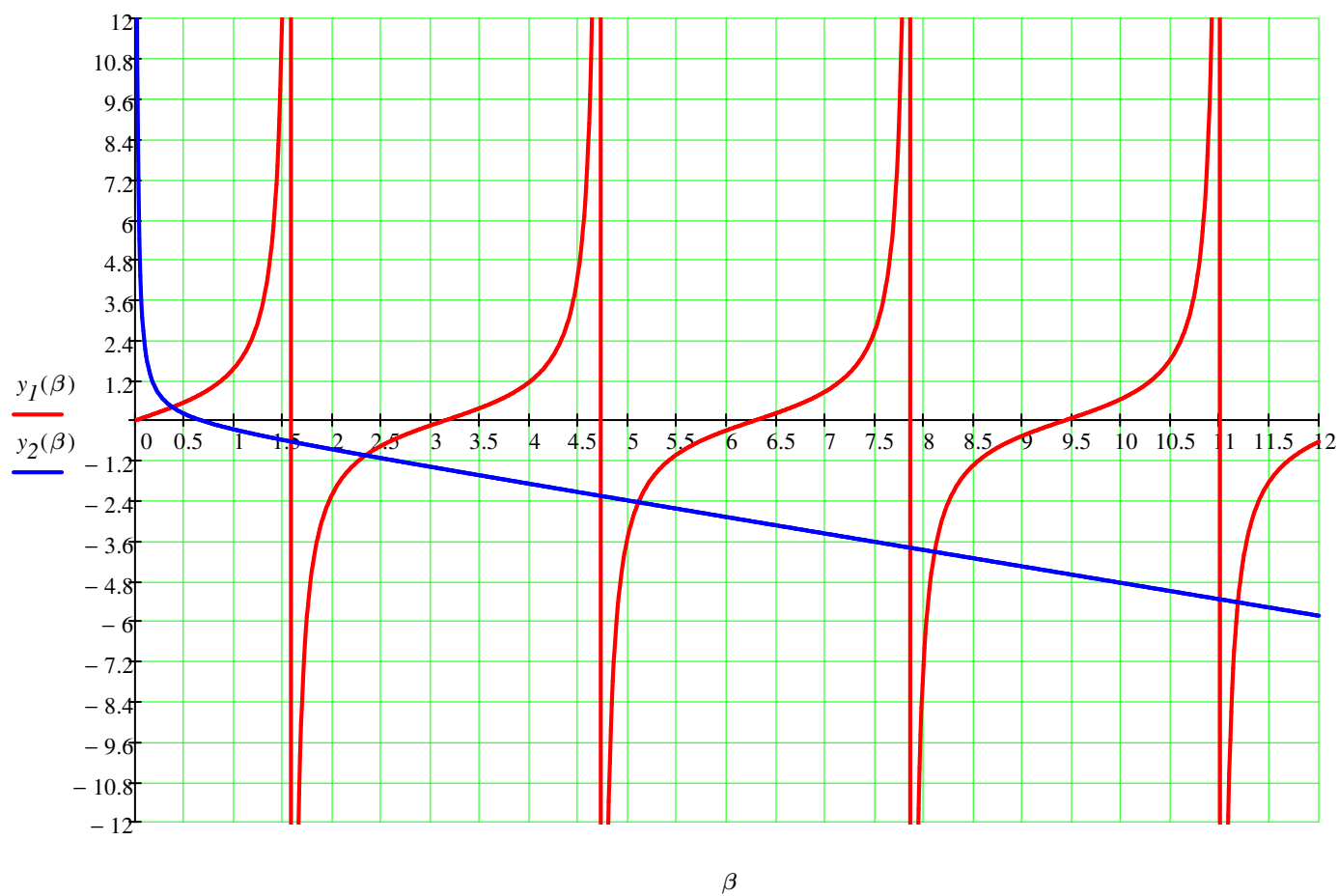
Equazione delle frequenze

$$\tan(\beta) = \frac{\frac{k \cdot \mu}{\beta} - a \cdot \beta}{k + \mu \cdot a}$$

$$y_1(\beta) := \tan(\beta)$$

$$y_2(\beta) := \frac{\frac{k \cdot \mu}{\beta} - a \cdot \beta}{k + \mu \cdot a}$$

$$\beta := 0,01..12 \qquad FS := 12$$



$$F(\beta) := y_1(\beta) - y_2(\beta)$$

$$\beta := 0.5$$

$$\beta_1 := \text{root}(F(\beta), \beta) = 0.375856$$

$$\beta := 2.5$$

$$\beta_2 := \text{root}(F(\beta), \beta) = 2.333921$$

$$\beta := 5.2$$

$$\beta_3 := \text{root}(F(\beta), \beta) = 5.100515$$

$$\beta := 8.2$$

$$\beta_4 := \text{root}(F(\beta), \beta) = 8.103356$$

$$\beta := 11$$

$$\beta_5 := \text{root}(F(\beta), \beta) = 11.177573$$

|                       | rad/s                                               | Hz                                               |
|-----------------------|-----------------------------------------------------|--------------------------------------------------|
| $\beta_1 = 0.375856$  | $\omega_1 := \frac{c}{l} \cdot \beta_1 = 1379.685$  | $f_1 := \frac{\omega_1}{2 \cdot \pi} = 219.584$  |
| $\beta_2 = 2.333921$  | $\omega_2 := \frac{c}{l} \cdot \beta_2 = 8567.31$   | $f_2 := \frac{\omega_2}{2 \cdot \pi} = 1363.53$  |
| $\beta_3 = 5.100515$  | $\omega_3 := \frac{c}{l} \cdot \beta_3 = 18722.864$ | $f_3 := \frac{\omega_3}{2 \cdot \pi} = 2979.836$ |
| $\beta_4 = 8.103356$  | $\omega_4 := \frac{c}{l} \cdot \beta_4 = 29745.631$ | $f_4 := \frac{\omega_4}{2 \cdot \pi} = 4734.164$ |
| $\beta_5 = 11.177573$ | $\omega_5 := \frac{c}{l} \cdot \beta_5 = 41030.402$ | $f_5 := \frac{\omega_5}{2 \cdot \pi} = 6530.191$ |

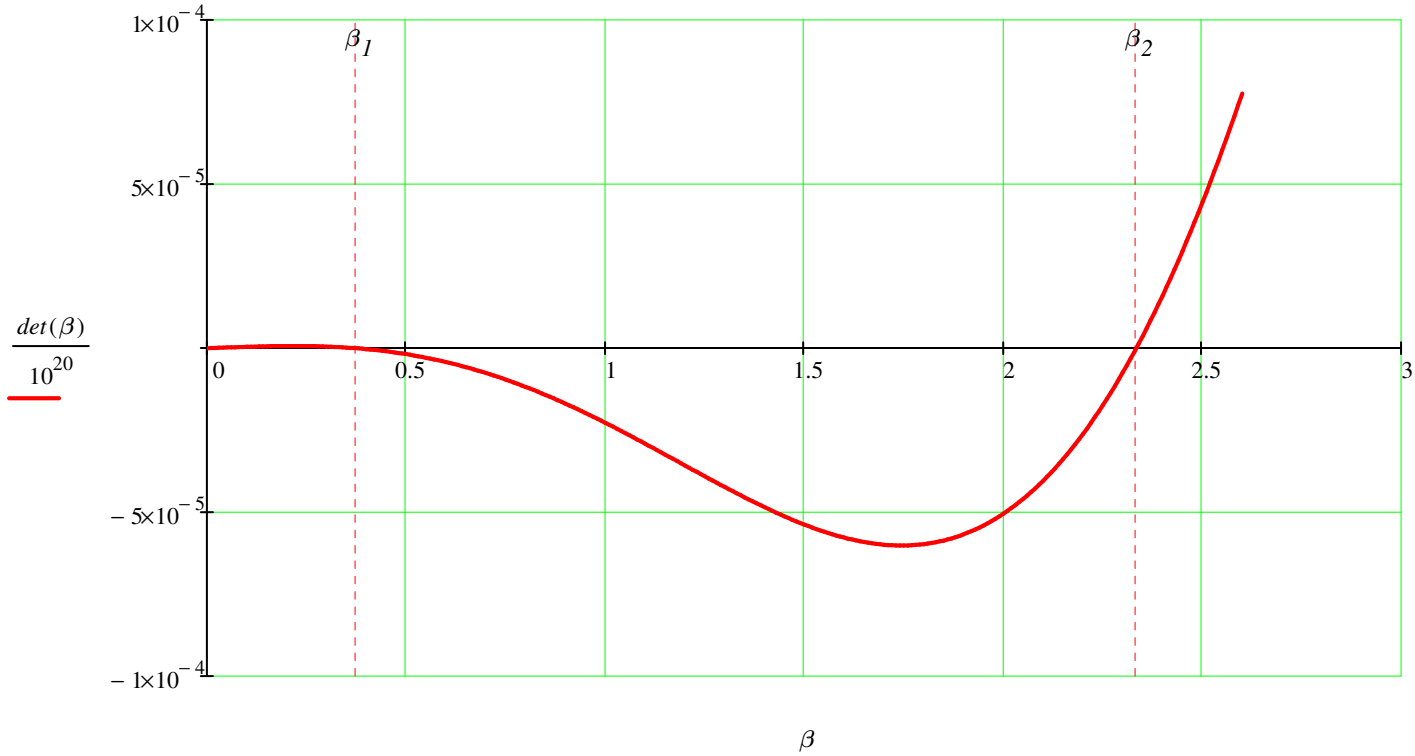
Calcolo diretto del determinante in forma numerica, senza sviluppi di calcolo simbolico

$$det(\beta) := \left| \Delta(\beta) \right|$$

|                |                                       |
|----------------|---------------------------------------|
| $\beta := 0.5$ | $root(det(\beta), \beta) = 0.375856$  |
| $\beta := 2.5$ | $root(det(\beta), \beta) = 2.333921$  |
| $\beta := 5.2$ | $root(det(\beta), \beta) = 5.100515$  |
| $\beta := 8.2$ | $root(det(\beta), \beta) = 8.103356$  |
| $\beta := 11$  | $root(det(\beta), \beta) = 11.177573$ |

$$\beta_1 = 0.375856 \quad \beta_2 = 2.333921 \quad \beta_3 = 5.100515 \quad \beta_4 = 8.103356 \quad \beta_5 = 11.177573$$

$\beta := 0,0.01..2.6$



$\beta := 4,4.01..12$

