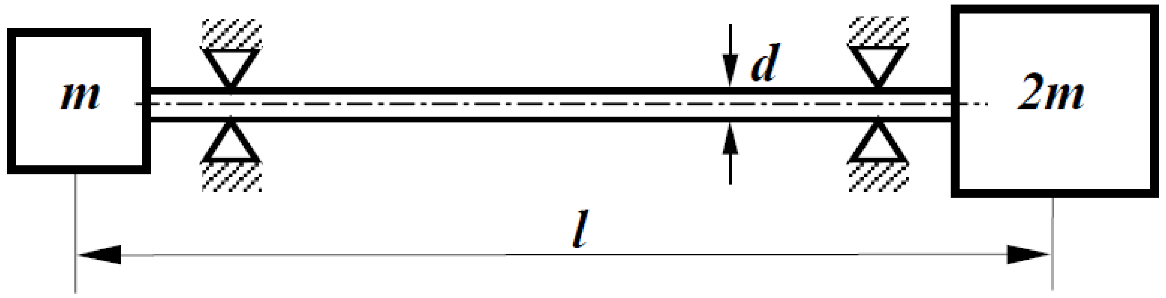


$f(t) := 1$

$u(x,t) := \left(A\cdot cos\left(\frac{\omega\cdot x}{c}\right) + B\cdot sin\left(\frac{\omega\cdot x}{c}\right)\right)\cdot f(t)$ Soluzione dell'eq. di D' Alembert

$\frac{\partial}{\partial x}u(x,t)\ collect,\left(\frac{\omega}{c}\right) \rightarrow \left(B\cdot cos\left(\frac{\omega}{c}\cdot x\right) - A\cdot sin\left(\frac{\omega}{c}\cdot x\right)\right)\cdot\frac{\omega}{c}$

$N(x,t) := E\cdot A_s\cdot\frac{\partial}{\partial x}u(x,t)$

$-m\cdot\omega^2\cdot u(0,t) - N(0,t)\ collect,A,B \rightarrow \left(-\omega^2\cdot m\right)\cdot A + \left(-\frac{A_s\cdot E\cdot\omega}{c}\right)\cdot B$

$-2\cdot m\cdot\omega^2\cdot u(l,t) + N(l,t)\ collect,A,B \rightarrow \left(-2\cdot\omega^2\cdot m\cdot cos\left(\frac{\omega\cdot l}{c}\right) - \frac{A_s\cdot E\cdot\omega\cdot sin\left(\frac{\omega\cdot l}{c}\right)}{c}\right)\cdot A + \left(\frac{A_s\cdot E\cdot\omega\cdot cos\left(\frac{\omega\cdot l}{c}\right)}{c} - 2\cdot\omega^2\cdot m\cdot sin\left(\frac{\omega\cdot l}{c}\right)\right)\cdot B$

$\beta = \frac{\omega\cdot l}{c}$ $\quad \frac{\omega}{c} = \frac{\beta}{l}$ $\quad \omega^2 = \left(\frac{\beta}{l}\cdot c\right)^2$

$f_{11}(\beta) := m\cdot\left(\frac{\beta}{l}\cdot c\right)^2$

$f_{12}(\beta) := E\cdot A_s\cdot\frac{\beta}{l}$

$f_{21}(\beta) := -2\cdot m\cdot\left(\frac{\beta}{l}\cdot c\right)^2\cdot cos(\beta) - \frac{E\cdot A_s}{l}\cdot\beta\cdot sin(\beta)$

$$f_{22}(\beta) := \frac{E \cdot A}{l} \cdot \beta \cdot \cos(\beta) - 2 \cdot m \cdot \left(\frac{\beta}{l} \cdot c\right)^2 \cdot \sin(\beta)$$

$$\Delta(\beta) := \begin{pmatrix} f_{11}(\beta) & f_{12}(\beta) \\ f_{21}(\beta) & f_{22}(\beta) \end{pmatrix}$$

$$\Delta(\beta) \rightarrow \begin{pmatrix} \frac{\beta^2 \cdot c^2 \cdot m}{l^2} & \frac{A_s \cdot E \cdot \beta}{l} \\ -\frac{A_s \cdot E \cdot \beta \cdot \sin(\beta)}{l} - \frac{2 \cdot \beta^2 \cdot c^2 \cdot m \cdot \cos(\beta)}{l^2} & \frac{A \cdot E \cdot \beta \cdot \cos(\beta)}{l} - \frac{2 \cdot \beta^2 \cdot c^2 \cdot m \cdot \sin(\beta)}{l^2} \end{pmatrix}$$

$$|\Delta(\beta)| \ simplify \ \rightarrow \frac{\beta^2 \cdot \left(A_s^{\ 2} \cdot E^2 \cdot l^2 \cdot \sin(\beta) - 2 \cdot \beta^2 \cdot c^4 \cdot m^2 \cdot \sin(\beta) + A \cdot E \cdot \beta \cdot c^2 \cdot l \cdot m \cdot \cos(\beta) + 2 \cdot A_s \cdot E \cdot \beta \cdot c^2 \cdot l \cdot m \cdot \cos(\beta)\right)}{l^4}$$

Pongo per semplicità

$$a = \frac{E \cdot A_s}{l}$$

$$f_{11}(\beta) := m \cdot \left(\frac{\beta}{l} \cdot c\right)^2$$

$$f_{12}(\beta) := a \cdot \beta$$

$$f_{21}(\beta) := -2 \cdot m \cdot \left(\frac{\beta}{l} \cdot c\right)^2 \cdot \cos(\beta) - a \cdot \beta \cdot \sin(\beta)$$

$$f_{22}(\beta) := a \cdot \beta \cdot \cos(\beta) - 2 \cdot m \cdot \left(\frac{\beta}{l} \cdot c\right)^2 \cdot \sin(\beta)$$

$$\Delta(\beta) := \begin{pmatrix} f_{11}(\beta) & f_{12}(\beta) \\ f_{21}(\beta) & f_{22}(\beta) \end{pmatrix}$$

$$\Delta(\beta) \rightarrow \begin{pmatrix} \dfrac{\beta^2 \cdot c^2 \cdot m}{l^2} & a \cdot \beta \\ -a \cdot \beta \cdot sin(\beta) - \dfrac{2 \cdot \beta^2 \cdot c^2 \cdot m \cdot cos(\beta)}{l^2} & a \cdot \beta \cdot cos(\beta) - \dfrac{2 \cdot \beta^2 \cdot c^2 \cdot m \cdot sin(\beta)}{l^2} \end{pmatrix}$$

$$|\Delta(\beta)| \; simplify \; \rightarrow \frac{\beta^2 \cdot \left(a^2 \cdot l^4 \cdot sin(\beta) - 2 \cdot \beta^2 \cdot c^4 \cdot m^2 \cdot sin(\beta) + 3 \cdot a \cdot \beta \cdot c^2 \cdot l^2 \cdot m \cdot cos(\beta)\right)}{l^4}$$

$$|\Delta(\beta)| \; \begin{vmatrix} simplify \\ collect, sin(\beta), cos(\beta) \end{vmatrix} \rightarrow \frac{\beta^2 \cdot \left(a^2 \cdot l^4 - 2 \cdot \beta^2 \cdot c^4 \cdot m^2\right)}{l^4} \cdot sin(\beta) + \frac{3 \cdot a \cdot \beta^3 \cdot c^2 \cdot m}{l^2} \cdot cos(\beta)$$

Uguagliando a zero il determinante si ha:

$$tan(\beta) = -\frac{3 \cdot a \cdot \beta \cdot c^2 \cdot l^2 \cdot m}{a^2 \cdot l^4 - 2 \cdot \beta^2 \cdot c^4 \cdot m^2}$$

$E := 206000 \cdot 10^6 = 2.06 \times 10^{11}$

$l := 0.8$

$\rho := 7800$

$d := 10 \cdot 10^{-3} = 0.01$

$A_{sez} := \dfrac{\pi \cdot d^2}{4} = 7.854 \times 10^{-5}$

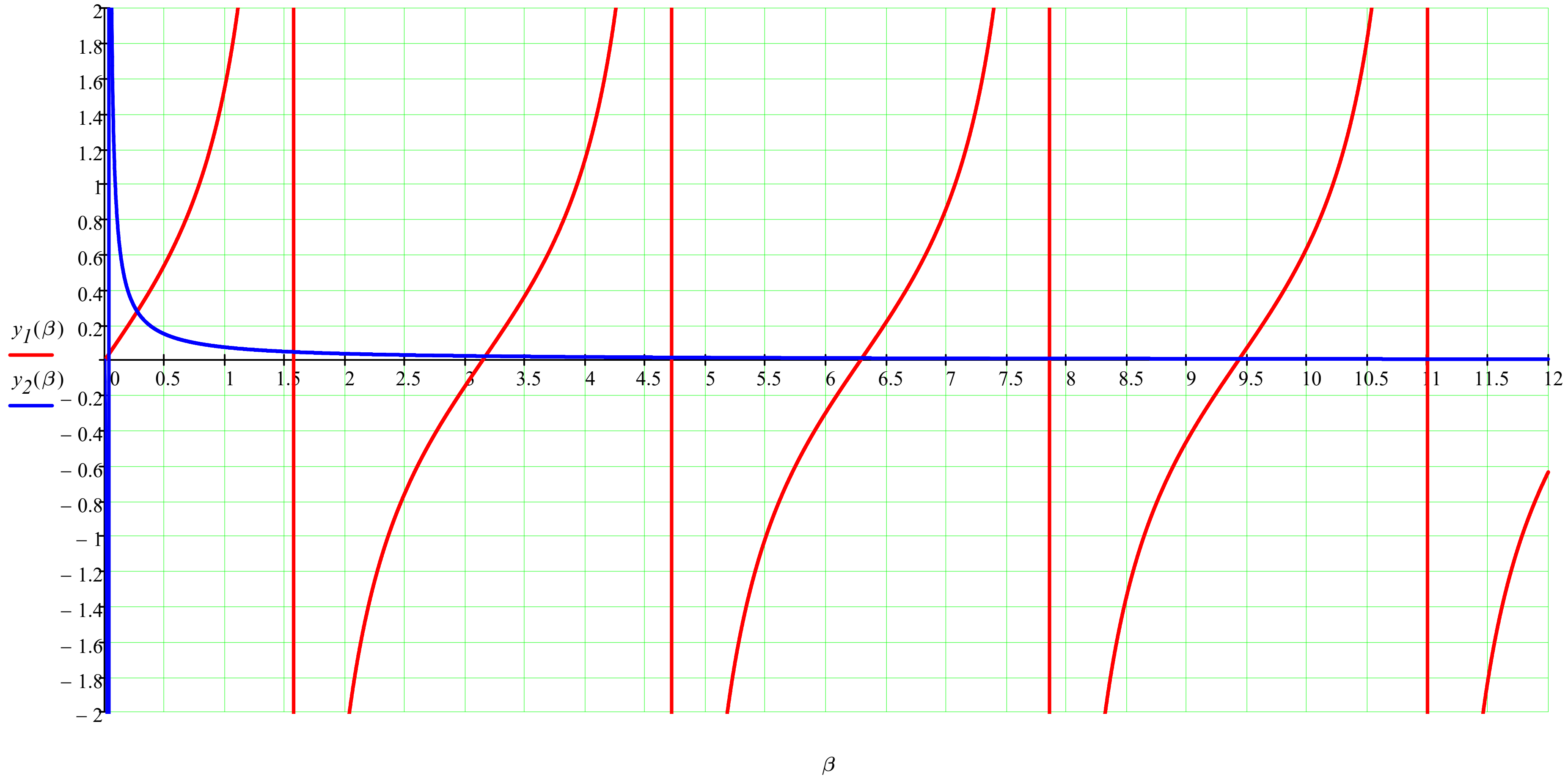
$a := \dfrac{E \cdot A_{sez}}{l} = 2.022 \times 10^7$

$m := 10$

$c := \sqrt{\frac{E}{\rho}} = 5139.091$

$y_1(\beta) := \tan(\beta)$ $\qquad$ $y_2(\beta) := -\frac{3 \cdot a \cdot \beta \cdot c^2 \cdot l^2 \cdot m}{a^2 \cdot l^4 - 2 \cdot \beta^2 \cdot c^4 \cdot m^2}$

$\beta := 0, 0.001 .. 12$ $\qquad$ $FS := 2$



$F(\beta) := y_1(\beta) - y_2(\beta)$

$\beta := 0.5$

$\beta_1 := root(F(\beta), \beta) = 0.27$

$\beta := 4$

$\beta_2 := root(F(\beta), \beta) = 3.165$

$\beta := 7$

$\beta_3 := root(F(\beta), \beta) = 6.295$

$\beta := 10$

$\beta_4 := root(F(\beta), \beta) = 9.433$

	rad/s	Hz
$\beta_1 = 0.27$	$\omega_1 := \frac{c}{l} \cdot \beta_1 = 1734.665$	$f_1 := \frac{\omega_1}{2 \cdot \pi} = 276.08$
$\beta_2 = 3.165$	$\omega_2 := \frac{c}{l} \cdot \beta_2 = 20330.369$	$f_2 := \frac{\omega_2}{2 \cdot \pi} = 3235.679$
$\beta_3 = 6.295$	$\omega_3 := \frac{c}{l} \cdot \beta_3 = 40437.345$	$f_3 := \frac{\omega_3}{2 \cdot \pi} = 6435.803$
$\beta_4 = 9.433$	$\omega_4 := \frac{c}{l} \cdot \beta_4 = 60593.554$	$f_4 := \frac{\omega_4}{2 \cdot \pi} = 9643.764$

Calcolo diretto del determinante in forma numerica, senza sviluppi di calcolo simbolico

$$f_{11}(\beta) := m\cdot\left(\frac{\beta}{l}\cdot c\right)^2$$

$$f_{12}(\beta) := a\cdot\beta$$

$$f_{21}(\beta) := -2\cdot m\cdot\left(\frac{\beta}{l}\cdot c\right)^2\cdot cos(\beta) - a\cdot\beta\cdot sin(\beta)$$

$$f_{22}(\beta) := a\cdot\beta\cdot cos(\beta) - 2\cdot m\cdot\left(\frac{\beta}{l}\cdot c\right)^2\cdot sin(\beta)$$

$$\Delta(\beta) := \begin{pmatrix} f_{11}(\beta) & f_{12}(\beta) \\ f_{21}(\beta) & f_{22}(\beta) \end{pmatrix}$$

$$det(\beta) := \left|\Delta(\beta)\right|$$

$\beta := 0.5$ $\quad root(det(\beta), \beta) = 3.24 \times 10^{-7} + 1.366i \times 10^{-7}$

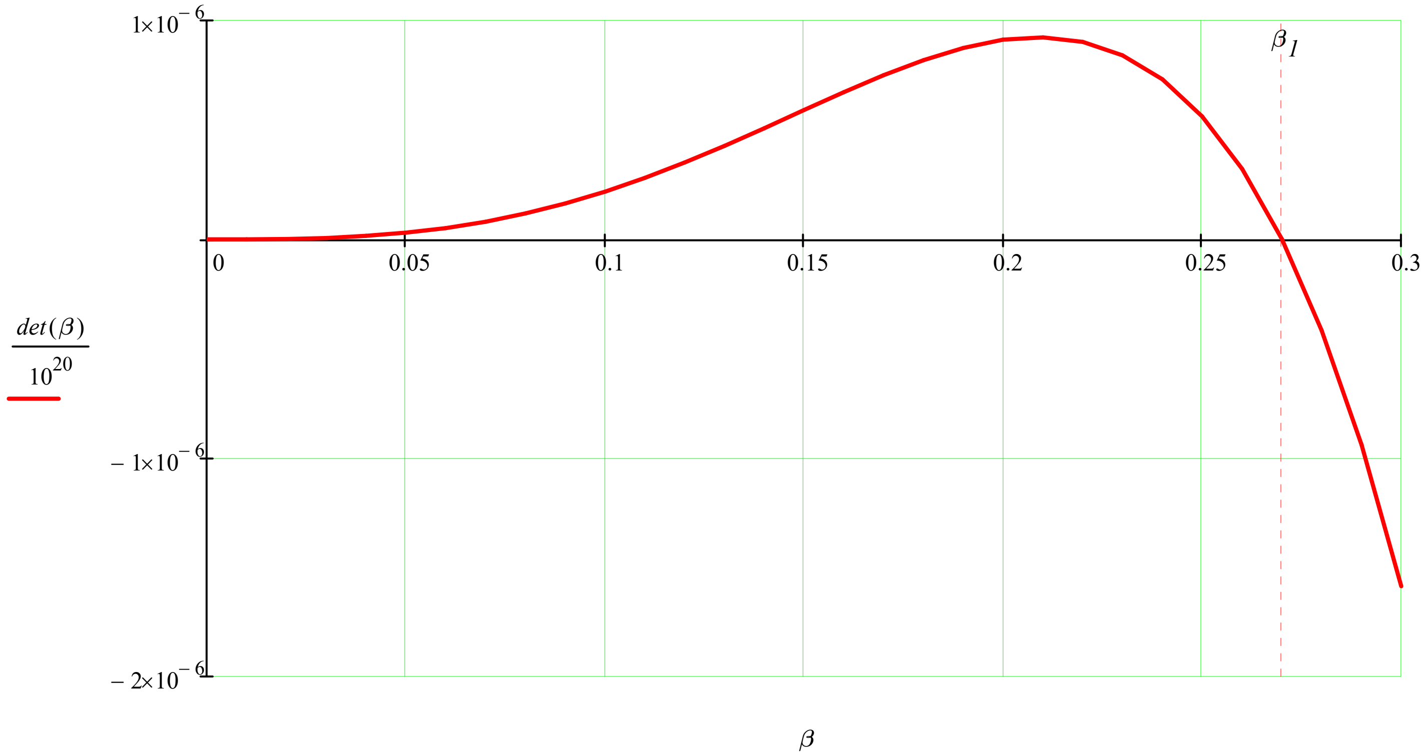
$\beta := 4$ $\quad root(det(\beta), \beta) = 3.165$

$\beta := 7$ $\quad root(det(\beta), \beta) = 6.295$

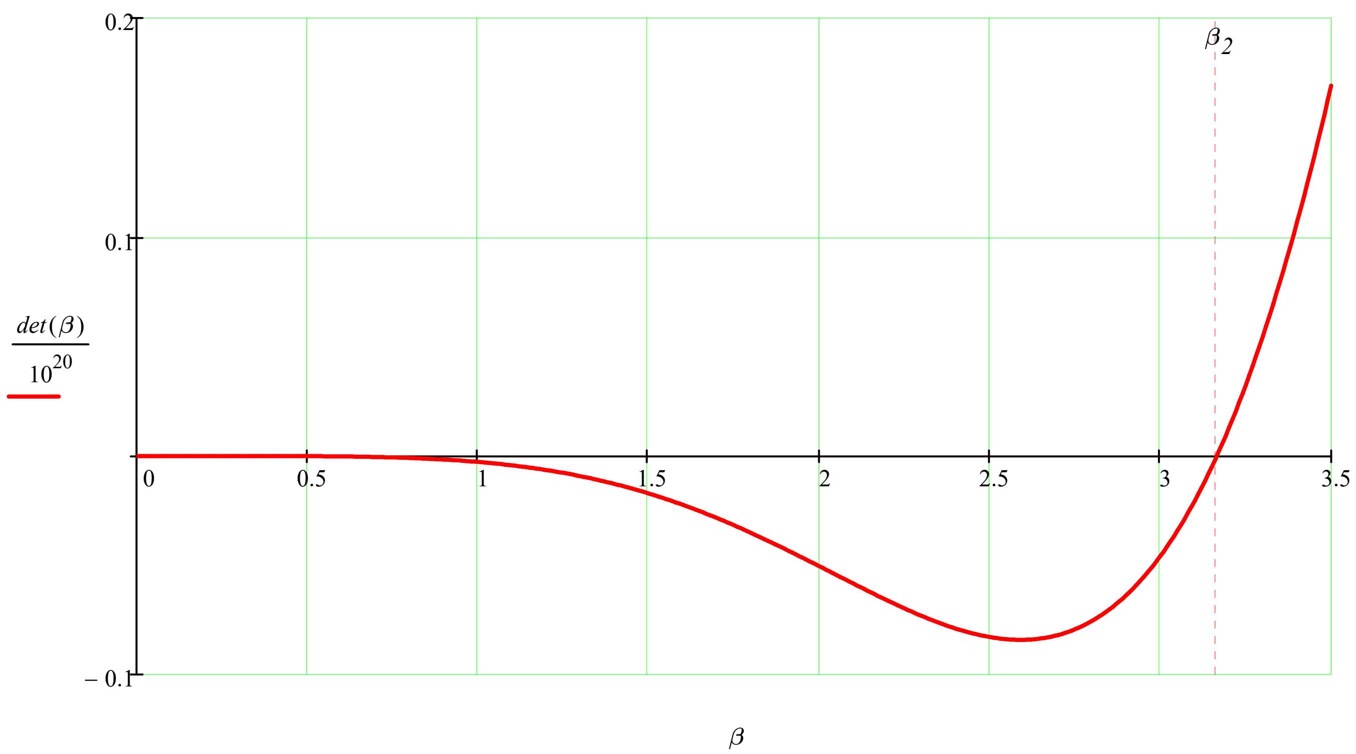
$\beta := 10$ $\quad root(det(\beta), \beta) = 9.433$

$\beta_1 = 0.27$ $\quad \beta_2 = 3.165$ $\quad \beta_3 = 6.295$ $\quad \beta_4 = 9.433$

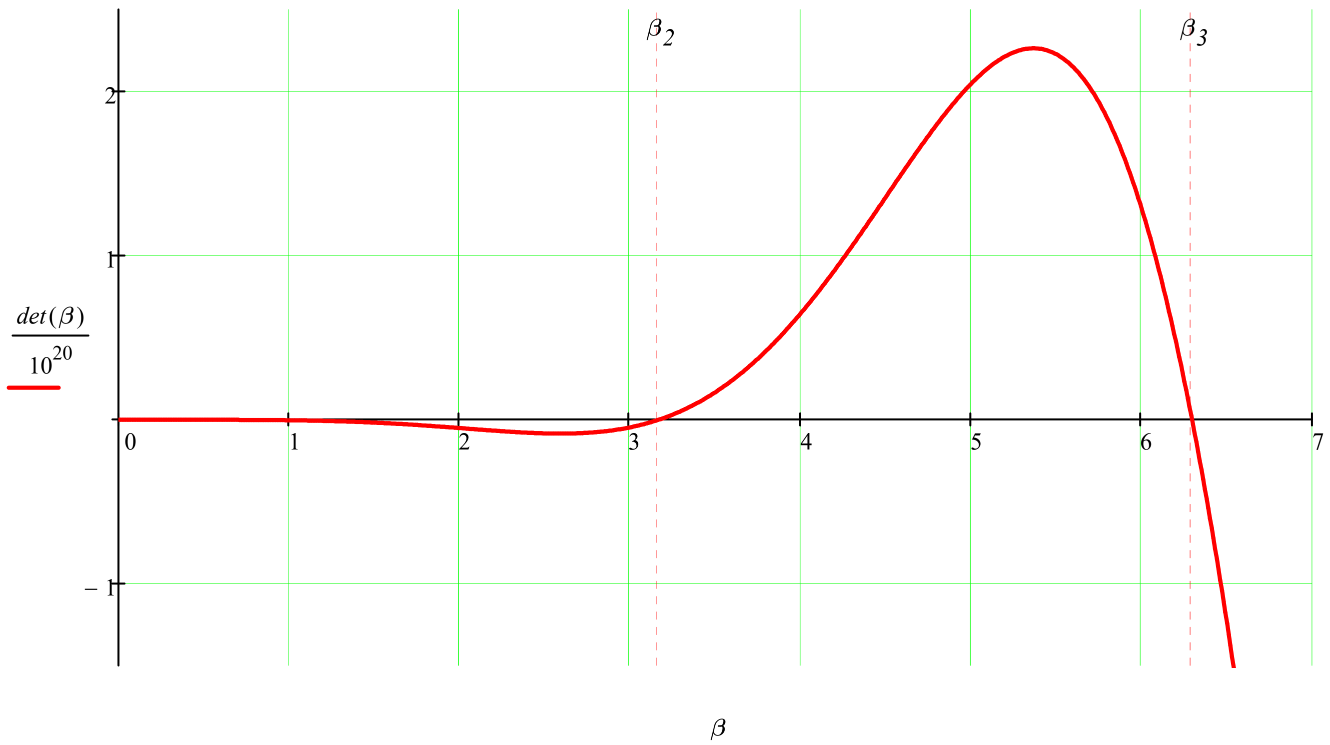
$\beta := 0, 0.01 .. 0.3$


1×10⁻⁶
−1×10⁻⁶
−2×10⁻⁶
0
0.05
0.1
0.15
0.2
0.25
0.3
β1
det(β) / 10^20
β

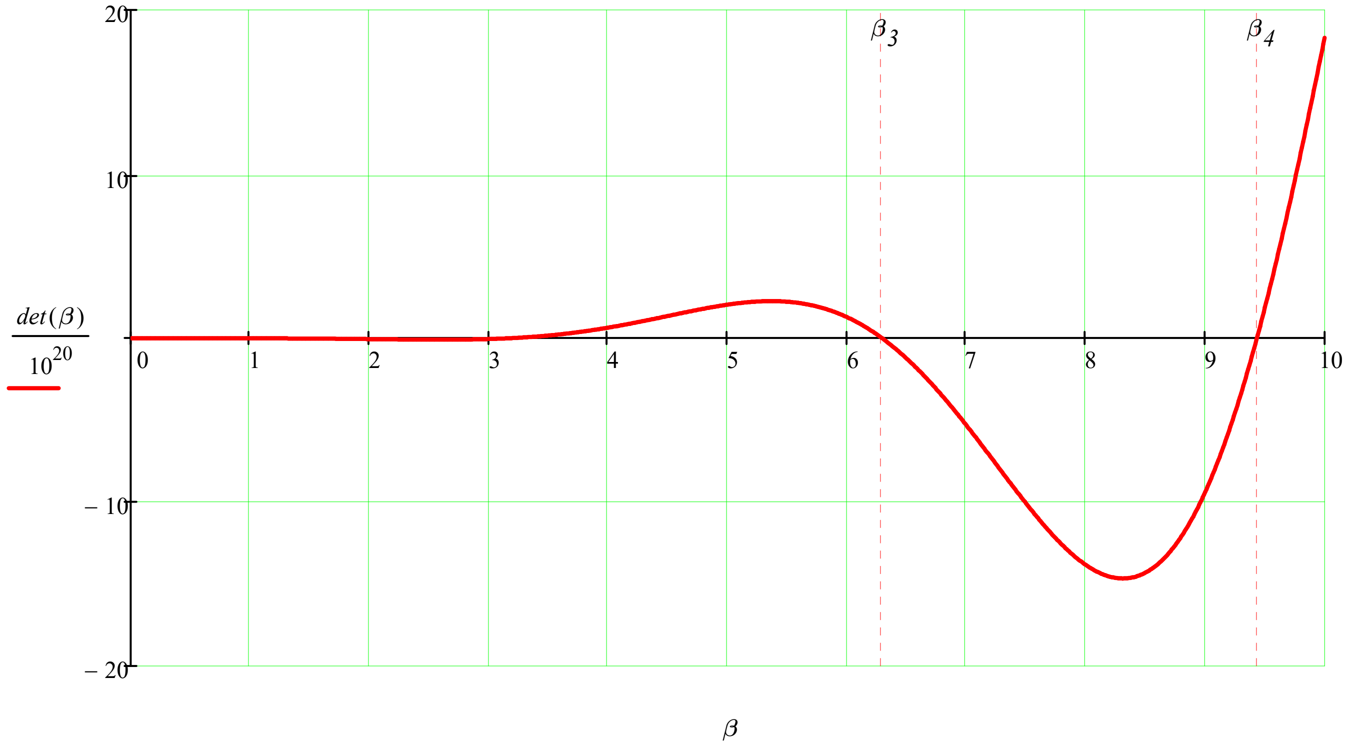
$\beta := 0, 0.01 .. 3.5$


0.2
0.1
−0.1
0
0.5
1
1.5
2
2.5
3
3.5
β2
det(β) / 10^20
β

$\beta := 0, 0.01 .. 7$



$\beta := 0, 0.01 .. 10$



Modello a parametri concentrati

$k := \frac{E \cdot A_{sez}}{l}$ $\quad k = 2.022 \times 10^7$

$M := \begin{pmatrix} m & 0 \\ 0 & 2 \cdot m \end{pmatrix}$ $\quad M = \begin{pmatrix} 10 & 0 \\ 0 & 20 \end{pmatrix}$

$K := \begin{pmatrix} k & -k \\ -k & k \end{pmatrix}$

$\omega := sort\left(\sqrt{genvals(K, M)}\right)$ $\quad \omega = \begin{pmatrix} 0 \\ 1741.723 \end{pmatrix}$ Pulsazioni proprie del sistema discreto [rad/s]

$f := \frac{\omega}{2 \cdot \pi}$ $\quad f = \begin{pmatrix} 0 \\ 277.204 \end{pmatrix}$ Frequenze proprie del sistema discreto [Hz]

$$f_{appr} := \frac{1}{2\cdot\pi}\cdot\sqrt{\frac{3\cdot k}{2\cdot m}} = 277.204$$

Frequenza propria del sistema discretizzato

$$f_1 = 276.08$$

Prima frequenza propria del sistema continuo

$$errore := \left(\frac{f_{appr} - f_1}{f_1}\right) = 0.407\cdot\%$$

Errore sulla prima frequenza propria