

$$f(t) := 1$$

$$u(x,t) := \left(A \cdot \cos\left(\frac{\omega \cdot x}{c}\right) + B \cdot \sin\left(\frac{\omega \cdot x}{c}\right) \right) \cdot f(t) \qquad \text{Soluzione dell'eq. di D' Alembert}$$

$$\frac{\partial}{\partial x} u(x,t) \text{ collect, } \left(\frac{\omega}{c}\right) \rightarrow \left(B \cdot \cos\left(\frac{\omega}{c} \cdot x\right) - A \cdot \sin\left(\frac{\omega}{c} \cdot x\right) \right) \cdot \frac{\omega}{c}$$

$$N(x,t) := E \cdot A_s \cdot \frac{\partial}{\partial x} \textcolor{red}{u}(x,t)$$

$$N(0,t) - k_1 \cdot u(0,t) \text{ collect, } A, B \rightarrow (-k_1) \cdot A + \frac{A_s \cdot E \cdot \omega}{c} \cdot B$$

$$N(l,t) + k_2 \cdot u(l,t) \text{ collect, } A, B \rightarrow \left(k_2 \cdot \cos\left(\frac{\omega \cdot l}{c}\right) - \frac{A_s \cdot E \cdot \omega \cdot \sin\left(\frac{\omega \cdot l}{c}\right)}{c} \right) \cdot A + \left(k_2 \cdot \sin\left(\frac{\omega \cdot l}{c}\right) + \frac{A_s \cdot E \cdot \omega \cdot \cos\left(\frac{\omega \cdot l}{c}\right)}{c} \right) \cdot B$$

$$\beta = \frac{\omega \cdot l}{c} \qquad \frac{\omega}{c} = \frac{\beta}{l}$$

$$f_{11}(\beta) := -\textcolor{red}{k}_1$$

$$f_{12}(\beta) := \frac{\textcolor{red}{E} \cdot A_s}{l} \cdot \beta$$

$$f_{21}(\beta) := \left(\textcolor{red}{k}_2 \cdot \cos(\beta) - \frac{E \cdot A_s}{l} \cdot \beta \cdot \sin(\beta) \right)$$

$$f_{22}(\beta) := \textcolor{red}{k}_2 \cdot \sin(\beta) + \frac{E \cdot A_s}{l} \cdot \beta \cdot \cos(\beta)$$

$$\Delta_{(\beta)} := \begin{pmatrix} f_{11}(\beta) & f_{12}(\beta) \\ f_{21}(\beta) & f_{22}(\beta) \end{pmatrix}$$

$$\Delta_{(\beta)} \rightarrow \begin{pmatrix} -k_I & \frac{A_s \cdot E \cdot \beta}{l} \\ k_2 \cdot \cos(\beta) - \frac{A_s \cdot E \cdot \beta \cdot \sin(\beta)}{l} & k_2 \cdot \sin(\beta) + \frac{A_s \cdot E \cdot \beta \cdot \cos(\beta)}{l} \end{pmatrix}$$

$$\left| \Delta_{(\beta)} \right| \text{ simplify } \rightarrow \frac{A_s^2 \cdot E^2 \cdot \beta^2 \cdot \sin(\beta)}{l^2} - k_I \cdot k_2 \cdot \sin(\beta) - \frac{A_s \cdot E \cdot \beta \cdot k_I \cdot \cos(\beta)}{l} - \frac{A_s \cdot E \cdot \beta \cdot k_2 \cdot \cos(\beta)}{l}$$

Pongo per
semplicità

$$a = \frac{E \cdot A_s}{l}$$

$$f_{11}(\beta) := -k_I$$

$$f_{12}(\beta) := a \cdot \beta$$

$$f_{21}(\beta) := \left(k_2 \cdot \cos(\beta) - a \cdot \beta \cdot \sin(\beta) \right)$$

$$f_{22}(\beta) := k_2 \cdot \sin(\beta) + a \cdot \beta \cdot \cos(\beta)$$

$$\Delta_{(\beta)} := \begin{pmatrix} f_{11}(\beta) & f_{12}(\beta) \\ f_{21}(\beta) & f_{22}(\beta) \end{pmatrix}$$

$$\Delta_{(\beta)} \rightarrow \begin{pmatrix} -k_I & a \cdot \beta \\ k_2 \cdot \cos(\beta) - a \cdot \beta \cdot \sin(\beta) & k_2 \cdot \sin(\beta) + a \cdot \beta \cdot \cos(\beta) \end{pmatrix}$$

$$\left|\Delta_{(\beta)}\right| \mathit{simplify} \rightarrow a^2 \cdot \beta^2 \cdot \sin(\beta) - k_I \cdot k_2 \cdot \sin(\beta) - a \cdot \beta \cdot k_I \cdot \cos(\beta) - a \cdot \beta \cdot k_2 \cdot \cos(\beta)$$

$$\left|\Delta_{(\beta)}\right| \begin{array}{l} \mathit{simplify} \\ \mathit{collect, \sin(\beta), \cos(\beta)} \end{array} \rightarrow \left(a^2 \cdot \beta^2 - k_I \cdot k_2\right) \cdot \sin(\beta) + \left(-a \cdot \beta \cdot k_I - a \cdot \beta \cdot k_2\right) \cdot \cos(\beta)$$

$$\left(a^2 \cdot \beta^2 - k_I \cdot k_2\right) \cdot \sin(\beta) + \left(-a \cdot \beta \cdot k_I - a \cdot \beta \cdot k_2\right) \cdot \cos(\beta) = 0$$

Uguagliando a zero il determinante si ha:

$$\tan(\beta) = - \left[\frac{a \cdot \beta \cdot \left(k_I + k_2\right)}{k_I \cdot k_2 - a^2 \cdot \beta^2} \right]$$

oppure

$$\tan(\beta) = - \left[\frac{a \cdot \beta \cdot \left(1 + \frac{k_2}{k_I}\right)}{k_2 - \frac{a^2 \cdot \beta^2}{k_I}} \right]$$

$$E := 206000 \cdot 10^6 = 2.06 \times 10^{11}$$

$$\overset{\textcolor{green}{l}}{\omega} := 1.5$$

$$\rho := 7800$$

$$d := 10 \cdot 10^{-3} = 0.01$$

$$A_{sez} := \frac{\pi \cdot d^2}{4} = 7.854 \times 10^{-5}$$

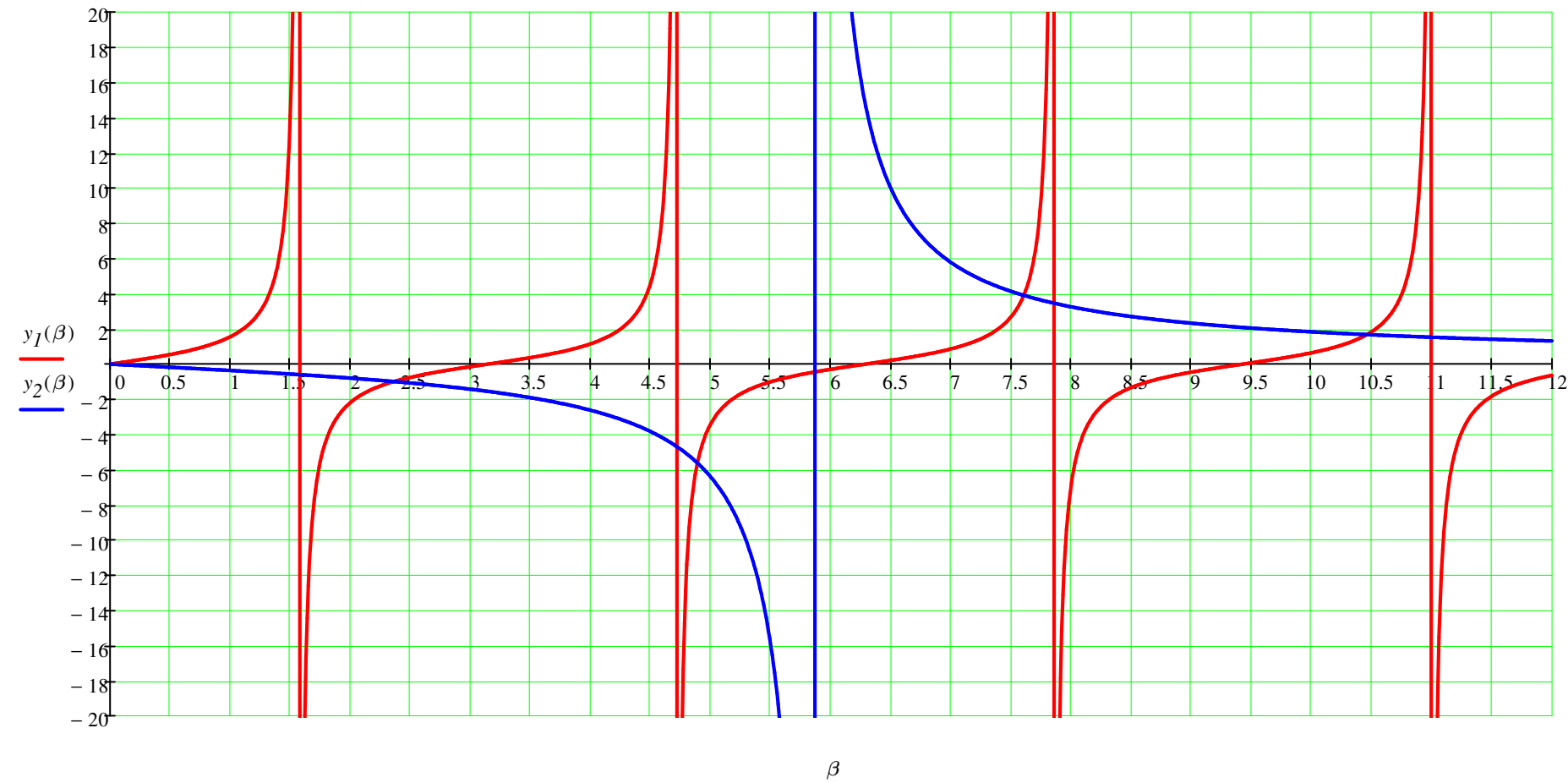
$$a := \frac{E \cdot A_{sez}}{l} = 1.079 \times 10^7$$

$k_1 := 5 \cdot 10^7$
 $k_2 := 8 \cdot 10^7$

$\xi := \sqrt{\frac{E}{\rho}} = 5139.091$

$y_1(\beta) := \tan(\beta)$
 $y_2(\beta) := \frac{a \cdot \beta \cdot \left(1 + \frac{k_2}{k_1}\right)}{k_2 - \frac{a^2 \cdot \beta^2}{k_1}}$

$\beta := 0,0.01..12$
 $FS := 20$



$\textcolor{green}{\beta}(\beta) := y_1(\beta) - y_2(\beta)$

$\beta := 2$

$\beta_1 := \textit{root}(F(\beta), \beta) = 2.362$

$\textcolor{green}{\beta} := 5$

$\beta_2 := \textit{root}(F(\beta), \beta) = 4.888$

$\textcolor{green}{\beta} := 7$

$\beta_3 := \textit{root}(F(\beta), \beta) = 7.604$

$\textcolor{green}{\beta} := 10$

$\beta_4 := \textit{root}(F(\beta), \beta) = 10.459$

rad/s

Hz

$\beta_1 = 2.362$

$\omega_1 := \frac{c}{l} \cdot \beta_1 = 8092.458$

$f_1 := \frac{\omega_1}{2 \cdot \pi} = 1287.955$

$\beta_2 = 4.888$

$\omega_2 := \frac{c}{l} \cdot \beta_2 = 16748.274$

$f_2 := \frac{\omega_2}{2 \cdot \pi} = 2665.571$

$\beta_3 = 7.604$

$\omega_3 := \frac{c}{l} \cdot \beta_3 = 26050.534$

$f_3 := \frac{\omega_3}{2 \cdot \pi} = 4146.071$

$\beta_4 = 10.459$

$\omega_4 := \frac{c}{l} \cdot \beta_4 = 35832.569$

$f_4 := \frac{\omega_4}{2 \cdot \pi} = 5702.93$

Calcolo diretto del determinante in forma numerica, senza sviluppi di calcolo simbolico

$$f_{11}(\beta) := -k_I$$

$$f_{12}(\beta) := a \cdot \beta$$

$$f_{21}(\beta) := \left(k_2 \cdot \cos(\beta) - a \cdot \beta \cdot \sin(\beta)\right)$$

$$f_{22}(\beta) := k_2 \cdot \sin(\beta) + a \cdot \beta \cdot \cos(\beta)$$

$$\Delta_{(\beta)} := \begin{pmatrix} f_{11}(\beta) & f_{12}(\beta) \\ f_{21}(\beta) & f_{22}(\beta) \end{pmatrix}$$

$$det(\beta) := \left| \Delta_{(\beta)} \right|$$

$\beta_{\text{root}} := 2$ $root(det(\beta), \beta) = 2.362$

$\beta_{\text{root}} := 4$ $root(det(\beta), \beta) = 4.888$

$\beta_{\text{root}} := 7$ $root(det(\beta), \beta) = 7.604$

$\beta_{\text{root}} := 10$ $root(det(\beta), \beta) = 10.459$

$\beta_1 = 2.362$

$\beta_2 = 4.888$

$\beta_3 = 7.604$

$\beta_4 = 10.459$

$\beta := 0, 0.01..12$

