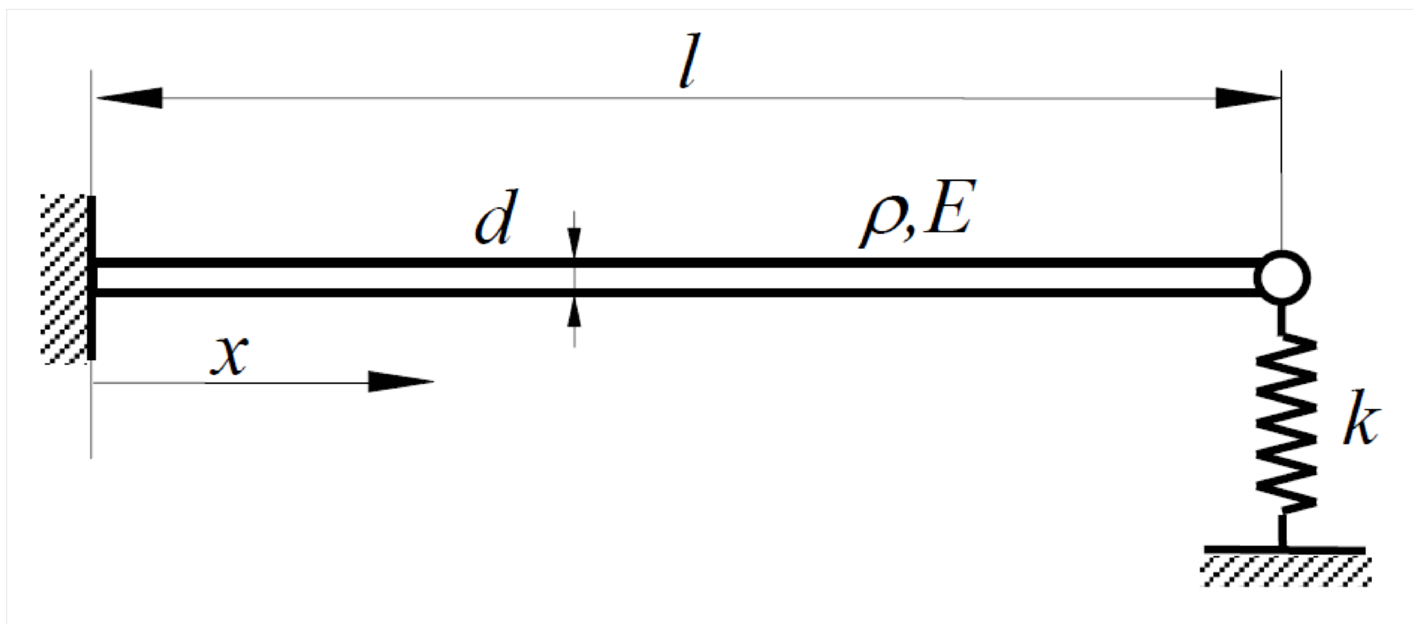
Trave incastro - molla all'estremità opposta



$Y(x) := A\cdot cos(\beta\cdot x) + B\cdot sin(\beta\cdot x) + C\cdot cosh(\beta\cdot x) + D\cdot sinh(\beta\cdot x)$

$Y'(x) := \frac{d}{dx}Y(x)$ $\qquad Y'(x)\ collect,\beta\ \rightarrow (B\cdot cos(\beta\cdot x) - A\cdot sin(\beta\cdot x) + C\cdot sinh(\beta\cdot x) + D\cdot cosh(\beta\cdot x))\cdot\beta$

$Y''(x) := \frac{d^2}{dx^2}Y(x)$ $\qquad Y''(x)\ collect,\beta\ \rightarrow (C\cdot cosh(\beta\cdot x) - B\cdot sin(\beta\cdot x) - A\cdot cos(\beta\cdot x) + D\cdot sinh(\beta\cdot x))\cdot\beta^2$

$Y'''(x) := \frac{d^3}{dx^3}Y(x)$ $\qquad Y'''(x)\ collect,\beta\ \rightarrow (A\cdot sin(\beta\cdot x) - B\cdot cos(\beta\cdot x) + C\cdot sinh(\beta\cdot x) + D\cdot cosh(\beta\cdot x))\cdot\beta^3$

$T(x) := E\cdot J\cdot\frac{d^3}{dx^3}Y(x)$ $\qquad$ *Azione di taglio*

Condizioni al contorno:

in x = 0

$Y(0) \rightarrow A + C$

$Y'(0)\ collect,\beta\ \rightarrow (B + D)\cdot\beta$

in x = l

$Y''(l)\ collect,\beta\ \rightarrow (C\cdot cosh(\beta\cdot l) - B\cdot sin(\beta\cdot l) - A\cdot cos(\beta\cdot l) + D\cdot sinh(\beta\cdot l))\cdot\beta^2$

$T(l) - k\cdot Y(l)\ collect,A,B,C,D\ \rightarrow \left(E\cdot J\cdot\beta^3\cdot sin(\beta\cdot l) - k\cdot cos(\beta\cdot l)\right)\cdot A + \left(-k\cdot sin(\beta\cdot l) - E\cdot J\cdot\beta^3\cdot cos(\beta\cdot l)\right)\cdot B + \left(E\cdot J\cdot\beta^3\cdot sinh(\beta\cdot l) - k\cdot cosh(\beta\cdot l)\right)\cdot C + \left(E\cdot J\cdot\beta^3\cdot cosh(\beta\cdot l) - k\cdot sinh(\beta\cdot l)\right)\cdot D$

Riassunto delle condizioni al contorno (che formano un sistema lineare)

$A + C = 0$

$B + D = 0$

$C\cdot cosh(\beta\cdot l) - B\cdot sin(\beta\cdot l) - A\cdot cos(\beta\cdot l) + D\cdot sinh(\beta\cdot l) = 0$

$\left(E\cdot J\cdot\beta^3\cdot sin(\beta\cdot l) - k\cdot cos(\beta\cdot l)\right)\cdot A + \left(-k\cdot sin(\beta\cdot l) - E\cdot J\cdot\beta^3\cdot cos(\beta\cdot l)\right)\cdot B + \left(E\cdot J\cdot\beta^3\cdot sinh(\beta\cdot l) - k\cdot cosh(\beta\cdot l)\right)\cdot C + \left(E\cdot J\cdot\beta^3\cdot cosh(\beta\cdot l) - k\cdot sinh(\beta\cdot l)\right) = 0$

Poniamo per semplicità: α = β l e otteniamo:

$A + C = 0$

$B + D = 0$

$C\cdot cosh(\alpha) - B\cdot sin(\alpha) - A\cdot cos(\alpha) + D\cdot sinh(\alpha) = 0$

$\left[E\cdot J\cdot\left(\frac{\alpha}{l}\right)^3\cdot sin(\alpha) - k\cdot cos(\alpha)\right]\cdot A + \left[-k\cdot sin(\alpha) - E\cdot J\cdot\left(\frac{\alpha}{l}\right)^3\cdot cos(\alpha)\right]\cdot B + \left[E\cdot J\cdot\left(\frac{\alpha}{l}\right)^3\cdot sinh(\alpha) - k\cdot cosh(\alpha)\right]\cdot C + \left[E\cdot J\cdot\left(\frac{\alpha}{l}\right)^3\cdot cosh(\alpha) - k\cdot sinh(\alpha)\right] = 0$

Quest'ultima condizione si può riscrivere nella forma:

$f_A(\alpha)\cdot A + f_B(\alpha)\cdot B + f_C(\alpha)\cdot C + f_D(\alpha)\cdot D = 0$

dove si è posto:

$h = \frac{E\cdot J}{l^3}$

$f_A(\alpha) := h\cdot\alpha^3\cdot sin(\alpha) - k\cdot cos(\alpha)$

$f_B(\alpha) := -k\cdot sin(\alpha) - h\cdot\alpha^3\cdot cos(\alpha)$

$f_C(\alpha) := h\cdot\alpha^3\cdot sinh(\alpha) - k\cdot cosh(\alpha)$

$f_D(\alpha) := h\cdot\alpha^3\cdot cosh(\alpha) - k\cdot sinh(\alpha)$

Le quattro condizioni al contorno si possono scrivere in forma matriciale, come segue

$$\Delta(\alpha)\cdot\begin{pmatrix}A\\B\\C\\D\end{pmatrix} = \begin{pmatrix}0\\0\\0\\0\end{pmatrix}$$

$$\Delta(\alpha) := \begin{pmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\\ -cos(\alpha) & -sin(\alpha) & cosh(\alpha) & sinh(\alpha)\\ f_A(\alpha) & f_B(\alpha) & f_C(\alpha) & f_D(\alpha)\end{pmatrix}$$

Conviene procedere per via numerica, senza sviluppi simbolici...

Lunghezza

$l := 1$

Materiale: acciaio

$\rho := 7800$

$E := 206000\cdot 10^{6} = 2.06\times 10^{11}$

Sezione circolare di diametro d:

$d := 20\cdot 10^{-3} = 0.02$

$J := \dfrac{\pi\cdot d^4}{64} = 7.854\times 10^{-9}$

$A_{sez} := \dfrac{\pi\cdot d^2}{4} = 3.142\times 10^{-4}$

Rigidezza della molla:

$k := 20000\cdot 1 = 2\times 10^{4}$

$c := \sqrt{\dfrac{E\cdot J}{\rho\cdot A_{sez}}} = 25.695$

$h := \dfrac{E\cdot J}{l^3} = 1617.92$

$f_A(\alpha) := h\cdot\alpha^3\cdot sin(\alpha) - k\cdot cos(\alpha)$

$f_B(\alpha) := -k\cdot sin(\alpha) - h\cdot\alpha^3\cdot cos(\alpha)$

$f_C(\alpha) := h\cdot\alpha^3\cdot sinh(\alpha) - k\cdot cosh(\alpha)$

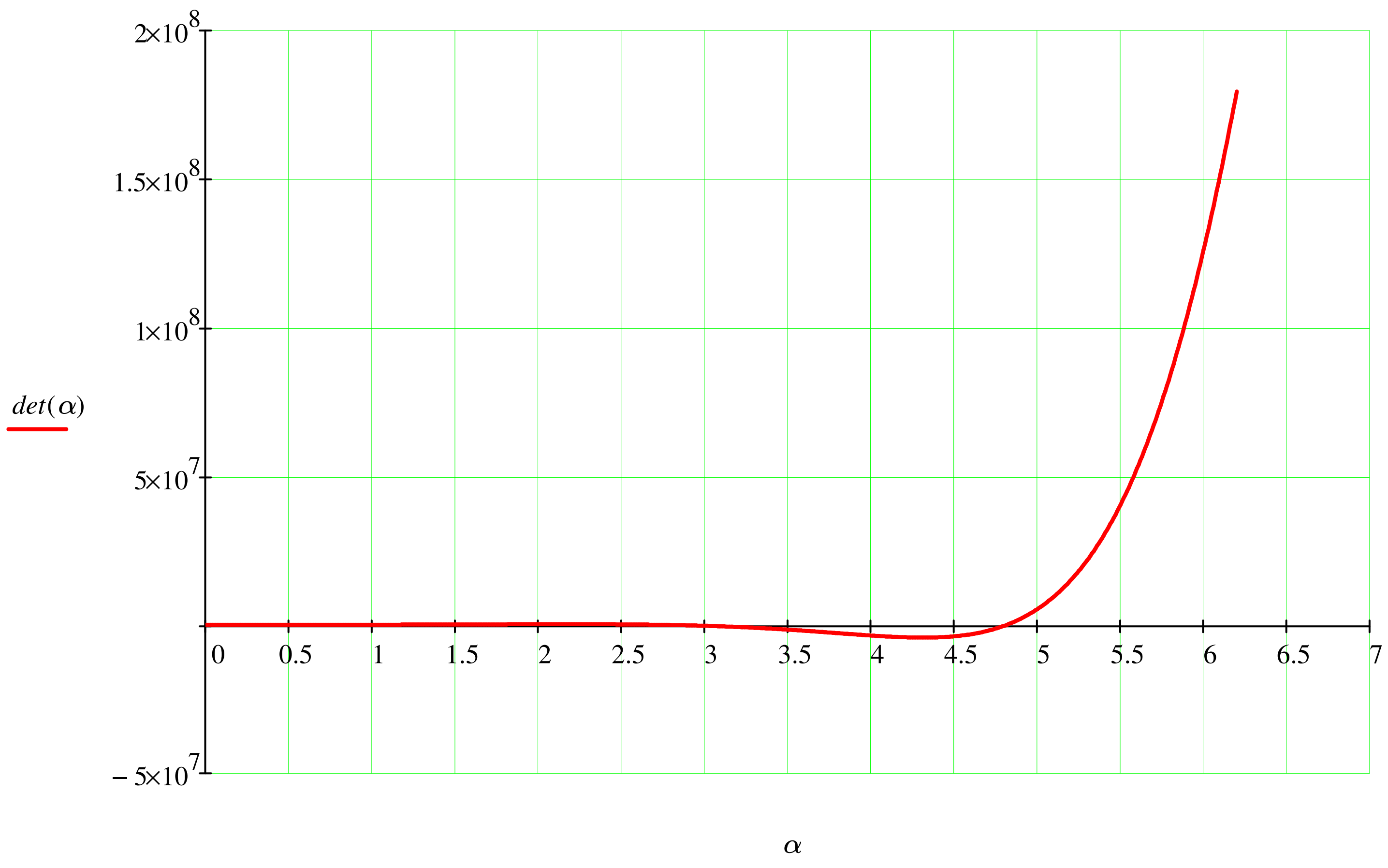
$f_D(\alpha) := h\cdot\alpha^3\cdot cosh(\alpha) - k\cdot sinh(\alpha)$

$$\Delta(\alpha) := \begin{pmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\\ -cos(\alpha) & -sin(\alpha) & cosh(\alpha) & sinh(\alpha)\\ f_A(\alpha) & f_B(\alpha) & f_C(\alpha) & f_D(\alpha)\end{pmatrix}$$

Cerchiamo le soluzioni dell'equazione caratteristica per via grafica:

$det(\alpha) := |\Delta(\alpha)|$

$\alpha := 0, 0.01 .. 6.2$



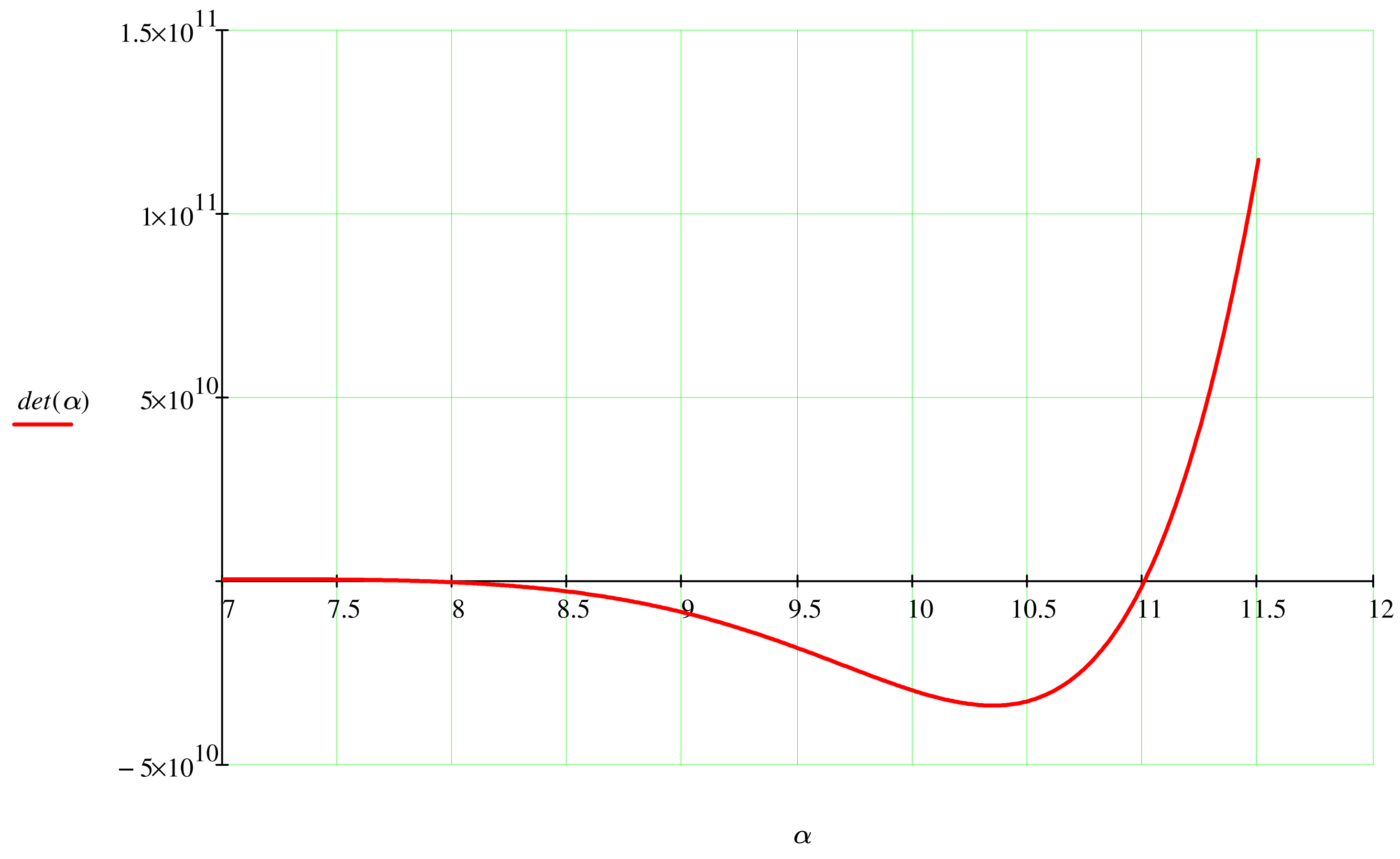
$\alpha := 3.5$

$\alpha_1 := root(det(\alpha), \alpha) = 2.735026$

$\alpha := 7$

$\alpha_2 := root(det(\alpha), \alpha) = 4.817982$

$\alpha := 7, 7.01 .. 11.5$



$\alpha := 10$

$\alpha_3 := root(det(\alpha), \alpha) = 7.880663$

$\alpha := 13$

$\alpha_4 := root(det(\alpha), \alpha) = 11.004902$

$\alpha = \beta \cdot l$ $\qquad \beta = \frac{\alpha}{l}$

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} = \begin{pmatrix} 2.735026 \\ 4.817982 \\ 7.880663 \\ 11.004902 \end{pmatrix} \qquad \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix} := \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} \cdot \frac{1}{l} = \begin{pmatrix} 2.735026 \\ 4.817982 \\ 7.880663 \\ 11.004902 \end{pmatrix}$$

$\omega = \beta^2 \cdot c$

$\omega_1 := (\beta_1)^2 \cdot c = 192.211$ $\qquad f_1 := \frac{\omega_1}{2 \cdot \pi} = 30.591$

$\omega_2 := (\beta_2)^2 \cdot c = 596.467$ $\qquad f_2 := \frac{\omega_2}{2 \cdot \pi} = 94.931$

$\omega_3 := (\beta_3)^2 \cdot c = 1595.812$ $\qquad f_3 := \frac{\omega_3}{2 \cdot \pi} = 253.981$

$\omega_4 := (\beta_4)^2 \cdot c = 3111.922$ $\qquad f_4 := \frac{\omega_4}{2 \cdot \pi} = 495.278$

$l := l$

Given

$A + C = 0$

$B + D = 0$

$C \cdot cosh(\beta \cdot l) - B \cdot sin(\beta \cdot l) - A \cdot cos(\beta \cdot l) + D \cdot sinh(\beta \cdot l) = 0$

$$Find(B, C, D) \rightarrow \begin{pmatrix} -\frac{A \cdot cos(\beta \cdot l) + A \cdot cosh(\beta \cdot l)}{sinh(\beta \cdot l) + sin(\beta \cdot l)} \\ -A \\ \frac{A \cdot cos(\beta \cdot l) + A \cdot cosh(\beta \cdot l)}{sinh(\beta \cdot l) + sin(\beta \cdot l)} \end{pmatrix}$$

$A := 1$

$B(\beta) := -\frac{A \cdot cos(\beta \cdot l) + A \cdot cosh(\beta \cdot l)}{sinh(\beta \cdot l) + sin(\beta \cdot l)}$

$C := -A$

$D(\beta) := \frac{A \cdot cos(\beta \cdot l) + A \cdot cosh(\beta \cdot l)}{sinh(\beta \cdot l) + sin(\beta \cdot l)}$

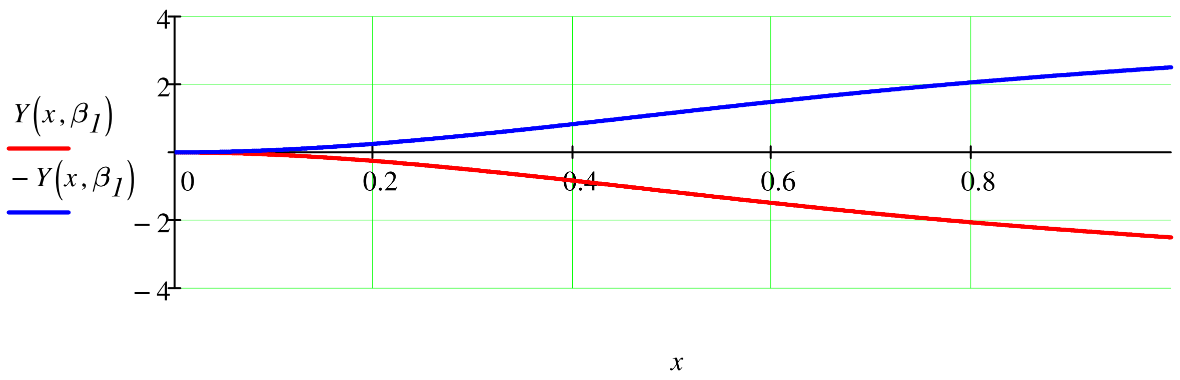
$Y(x, \beta) := A \cdot cos(\beta \cdot x) + B(\beta) \cdot sin(\beta \cdot x) + C \cdot cosh(\beta \cdot x) + D(\beta) \cdot sinh(\beta \cdot x)$

$x := 0, \frac{l}{500} .. l$

$FS := 4$ *Fondo scala per i grafici*

$f_1 = 30.591\, s \cdot Hz$

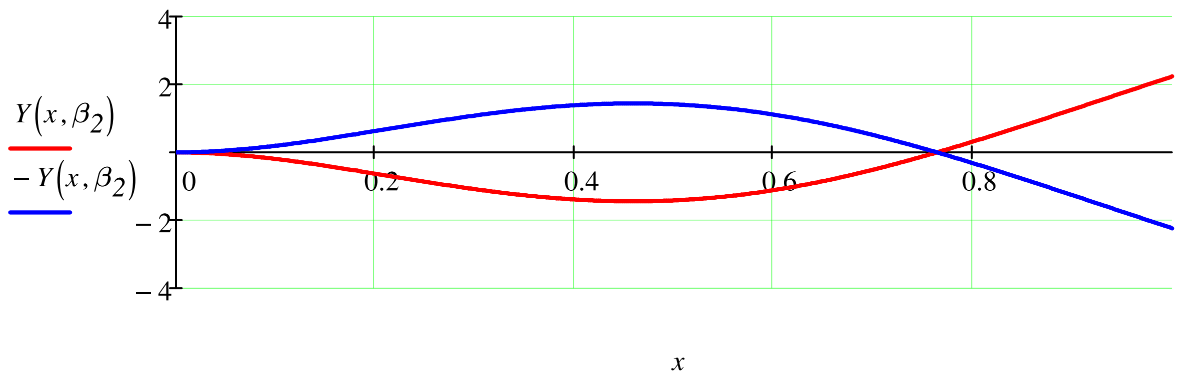
Mode shape (1)



metri

$f_2 = 94.931\, s \cdot Hz$

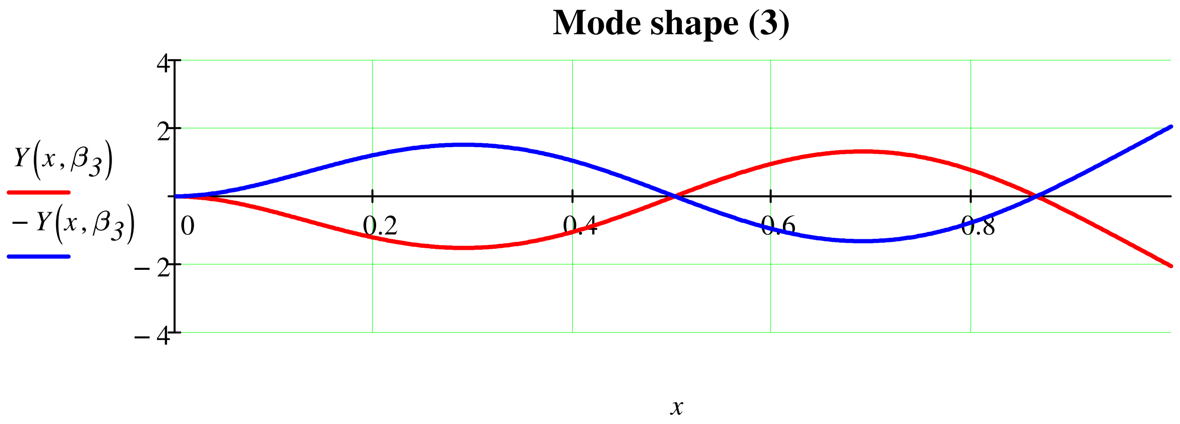
Mode shape (2)



metri

$f_3 = 253.981\, s \cdot Hz$

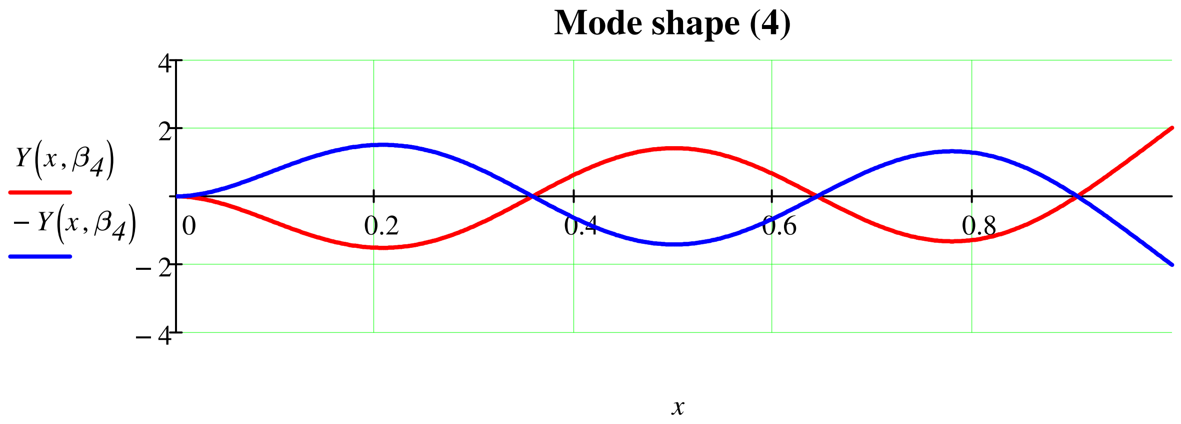
Mode shape (3)



metri

$f_4 = 495.278\, s \cdot Hz$

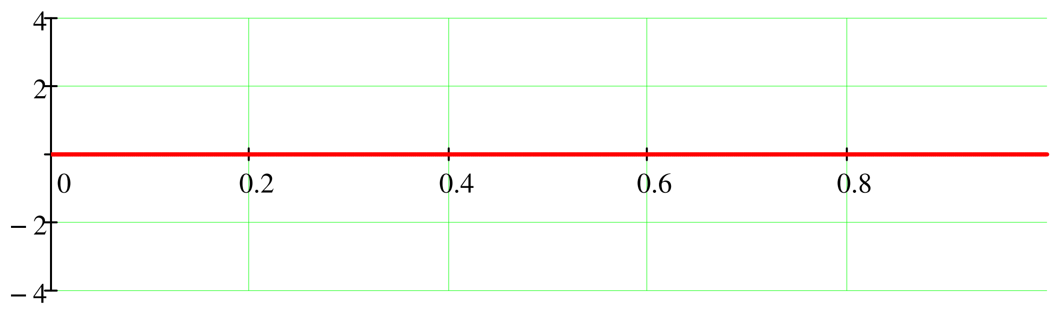
Mode shape (4)



metri

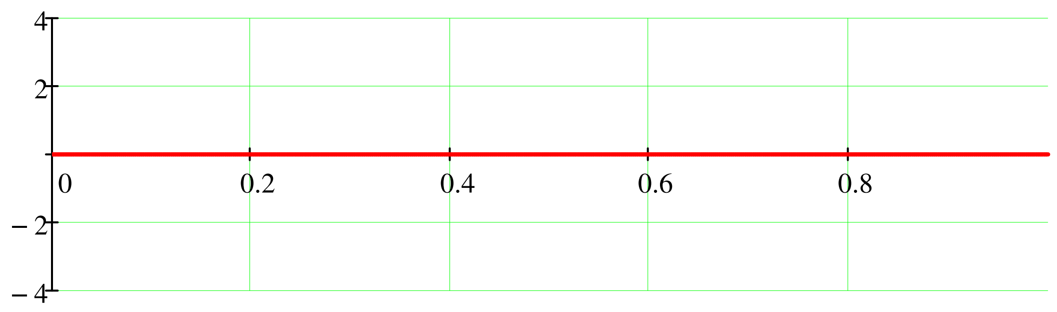
Animazioni delle deformate (Usare il menù: Strumenti - Aminazione - Registra)

Mode shape (1)



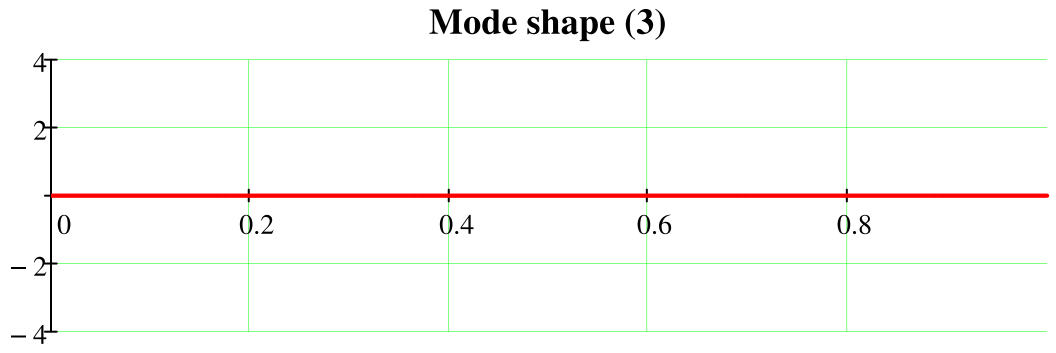
metri

Mode shape (2)



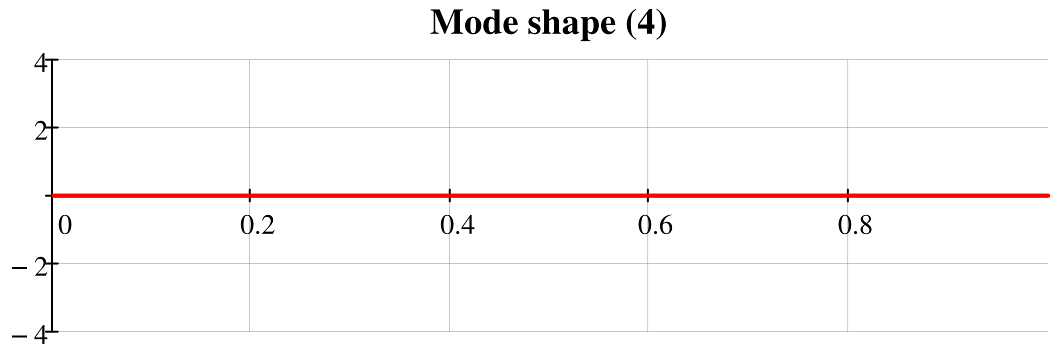
metri

Mode shape (3)



metri

Mode shape (4)



metri