$f(t) := 1$

$u(x,t) := \left(A \cdot \cos\left(\dfrac{\omega \cdot x}{c} \right) + B \cdot \sin\left(\dfrac{\omega \cdot x}{c} \right) \right) \cdot f(t)$ Soluzione dell'eq. di D'Alembert

$\dfrac{\partial}{\partial x} u(x,t) \ \text{collect}, \left(\dfrac{\omega}{c} \right) \ \rightarrow \left(B \cdot \cos\left(\dfrac{\omega}{c} \cdot x \right) - A \cdot \sin\left(\dfrac{\omega}{c} \cdot x \right) \right) \cdot \dfrac{\omega}{c}$

$N(x,t) := E \cdot A_s \cdot \dfrac{\partial}{\partial x} u(x,t)$

$N(0,t) - k_1 \cdot u(0,t) \ \text{collect}, A, B \ \rightarrow \left(-k_1 \right) \cdot A + \dfrac{A_s \cdot E \cdot \omega}{c} \cdot B$

$N(l,t) + k_2 \cdot u(l,t) \ \text{collect}, A, B \ \rightarrow \left(k_2 \cdot \cos\left(\dfrac{\omega \cdot l}{c} \right) - \dfrac{A_s \cdot E \cdot \omega \cdot \sin\left(\dfrac{\omega \cdot l}{c} \right)}{c} \right) \cdot A + \left(k_2 \cdot \sin\left(\dfrac{\omega \cdot l}{c} \right) + \dfrac{A_s \cdot E \cdot \omega \cdot \cos\left(\dfrac{\omega \cdot l}{c} \right)}{c} \right) \cdot B$

$\beta = \dfrac{\omega \cdot l}{c} \qquad\qquad \dfrac{\omega}{c} = \dfrac{\beta}{l}$

$f_{11}(\beta) := -k_1$

$f_{12}(\beta) := \dfrac{E \cdot A_s}{l} \cdot \beta$

$f_{21}(\beta) := \left(k_2 \cdot \cos(\beta) - \dfrac{E \cdot A_s}{l} \cdot \beta \cdot \sin(\beta) \right)$

$f_{22}(\beta) := k_2 \cdot \sin(\beta) + \dfrac{E \cdot A_s}{l} \cdot \beta \cdot \cos(\beta)$

$$\Delta_{(\beta)} := \begin{pmatrix} f_{11}(\beta) & f_{12}(\beta) \\ f_{21}(\beta) & f_{22}(\beta) \end{pmatrix}$$

$$\Delta_{(\beta)} \rightarrow \begin{pmatrix} -k_1 & \dfrac{A_s \cdot E \cdot \beta}{l} \\ k_2 \cdot cos(\beta) - \dfrac{A_s \cdot E \cdot \beta \cdot sin(\beta)}{l} & k_2 \cdot sin(\beta) + \dfrac{A_s \cdot E \cdot \beta \cdot cos(\beta)}{l} \end{pmatrix}$$

$$\left| \Delta_{(\beta)} \right| \; simplify \; \rightarrow \frac{A_s^2 \cdot E^2 \cdot \beta^2 \cdot sin(\beta)}{l^2} - k_1 \cdot k_2 \cdot sin(\beta) - \frac{A_s \cdot E \cdot \beta \cdot k_1 \cdot cos(\beta)}{l} - \frac{A_s \cdot E \cdot \beta \cdot k_2 \cdot cos(\beta)}{l}$$

Pongo per
semplicità

$$a = \frac{E \cdot A_s}{l}$$

$$f_{11}(\beta) := -k_1$$

$$f_{12}(\beta) := a \cdot \beta$$

$$f_{21}(\beta) := \left(k_2 \cdot cos(\beta) - a \cdot \beta \cdot sin(\beta) \right)$$

$$f_{22}(\beta) := k_2 \cdot sin(\beta) + a \cdot \beta \cdot cos(\beta)$$

$$\Delta_{(\beta)} := \begin{pmatrix} f_{11}(\beta) & f_{12}(\beta) \\ f_{21}(\beta) & f_{22}(\beta) \end{pmatrix}$$

$$\Delta(\beta) \rightarrow \begin{pmatrix} -k_1 & a \cdot \beta \\ k_2 \cdot \cos(\beta) - a \cdot \beta \cdot \sin(\beta) & k_2 \cdot \sin(\beta) + a \cdot \beta \cdot \cos(\beta) \end{pmatrix}$$

$$\left| \Delta(\beta) \right| \; simplify \; \rightarrow a^2 \cdot \beta^2 \cdot \sin(\beta) - k_1 \cdot k_2 \cdot \sin(\beta) - a \cdot \beta \cdot k_1 \cdot \cos(\beta) - a \cdot \beta \cdot k_2 \cdot \cos(\beta)$$

$$\left| \Delta(\beta) \right| \; \begin{vmatrix} simplify \\ collect, \sin(\beta), \cos(\beta) \end{vmatrix} \rightarrow \left(a^2 \cdot \beta^2 - k_1 \cdot k_2 \right) \cdot \sin(\beta) + \left(-a \cdot \beta \cdot k_1 - a \cdot \beta \cdot k_2 \right) \cdot \cos(\beta)$$

$$\left(a^2 \cdot \beta^2 - k_1 \cdot k_2 \right) \cdot \sin(\beta) + \left(-a \cdot \beta \cdot k_1 - a \cdot \beta \cdot k_2 \right) \cdot \cos(\beta) = 0$$

Uguagliando a zero il determinante si ha:

$$\tan(\beta) = -\left[\frac{a \cdot \beta \cdot \left(k_1 + k_2 \right)}{k_1 \cdot k_2 - a^2 \cdot \beta^2} \right] \qquad \text{oppure} \qquad \tan(\beta) = -\left[\frac{a \cdot \beta \cdot \left(1 + \dfrac{k_2}{k_1} \right)}{k_2 - \dfrac{a^2 \cdot \beta^2}{k_1}} \right]$$

$$E := 206000 \cdot 10^6 = 2.06 \times 10^{11}$$

$$l := 1.5$$

$$\rho := 7800$$

$$d := 10 \cdot 10^{-3} = 0.01$$

$$A_{sez} := \frac{\pi \cdot d^2}{4} = 7.854 \times 10^{-5}$$

$$a := \frac{E \cdot A_{sez}}{l} = 1.079 \times 10^7$$
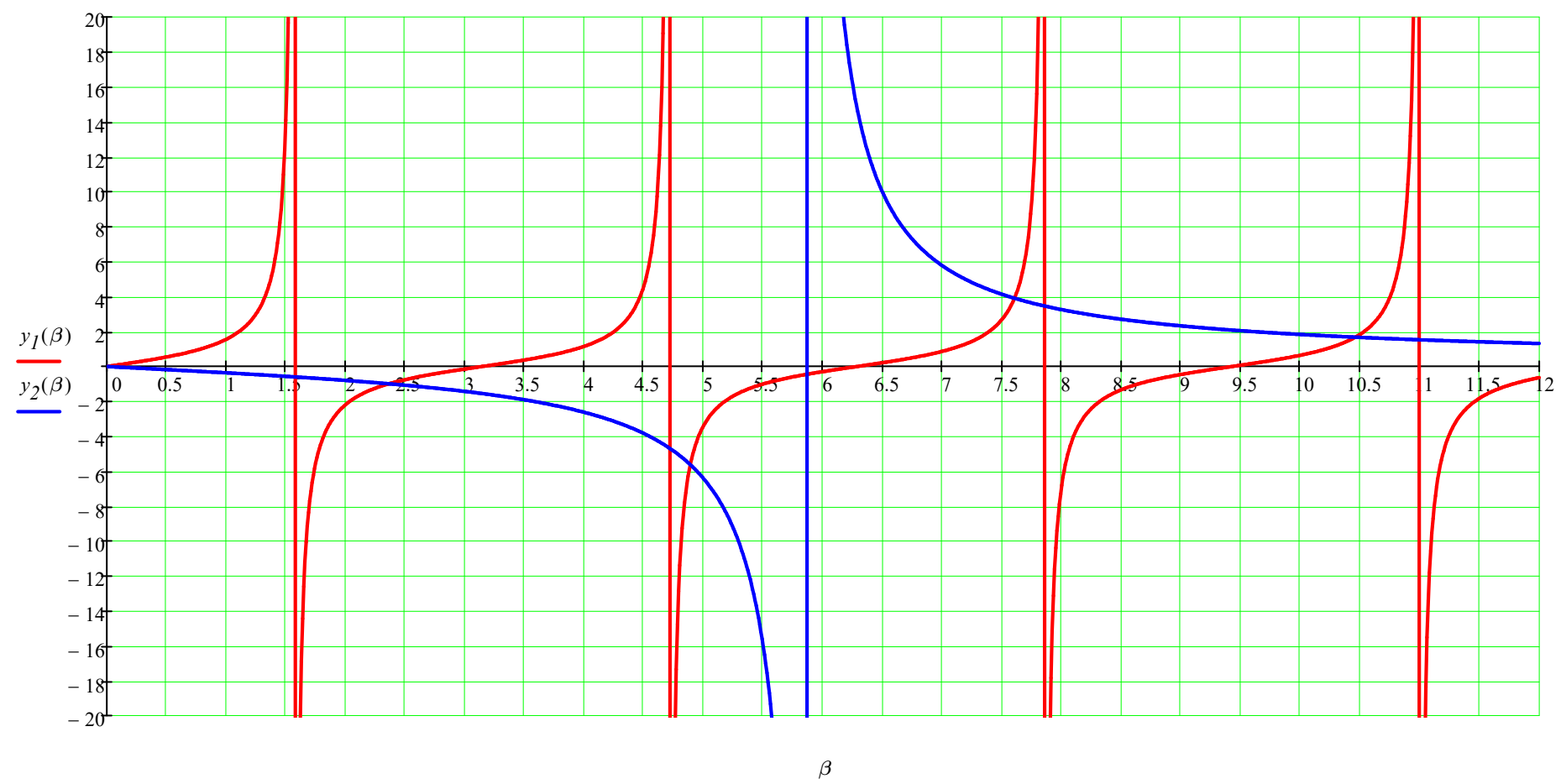
$k_1 := 5 \cdot 10^7$ $\qquad$ $k_2 := 8 \cdot 10^7$

$c := \sqrt{\dfrac{E}{\rho}} = 5139.091$

$y_1(\beta) := \tan(\beta)$ $\qquad$ $y_2(\beta) := -\left[\dfrac{a \cdot \beta \cdot \left(1 + \dfrac{k_2}{k_1}\right)}{k_2 - \dfrac{a^2 \cdot \beta^2}{k_1}} \right]$

$\beta := 0, 0.01 .. 12$ $\qquad$ $FS := 20$

$F(\beta) := y_1(\beta) - y_2(\beta)$

$\beta := 2$

$\beta_1 := root(F(\beta), \beta) = 2.362$

$\beta := 5$

$\beta_2 := root(F(\beta), \beta) = 4.888$

$\beta := 7$

$\beta_3 := root(F(\beta), \beta) = 7.604$

$\beta := 10$

$\beta_4 := root(F(\beta), \beta) = 10.459$

rad/s   Hz

$\beta_1 = 2.362$ $\omega_1 := \dfrac{c}{l} \cdot \beta_1 = 8092.458$ $f_1 := \dfrac{\omega_1}{2\cdot\pi} = 1287.955$

$\beta_2 = 4.888$ $\omega_2 := \dfrac{c}{l} \cdot \beta_2 = 16748.274$ $f_2 := \dfrac{\omega_2}{2\cdot\pi} = 2665.571$

$\beta_3 = 7.604$ $\omega_3 := \dfrac{c}{l} \cdot \beta_3 = 26050.534$ $f_3 := \dfrac{\omega_3}{2\cdot\pi} = 4146.071$

$\beta_4 = 10.459$ $\omega_4 := \dfrac{c}{l} \cdot \beta_4 = 35832.569$ $f_4 := \dfrac{\omega_4}{2\cdot\pi} = 5702.93$

$f_{11}(\beta) := -k_1$

$f_{12}(\beta) := a \cdot \beta$

$f_{21}(\beta) := \left(k_2 \cdot \cos(\beta) - a \cdot \beta \cdot \sin(\beta) \right)$

$f_{22}(\beta) := k_2 \cdot \sin(\beta) + a \cdot \beta \cdot \cos(\beta)$

$\Delta(\beta) := \begin{pmatrix} f_{11}(\beta) & f_{12}(\beta) \\ f_{21}(\beta) & f_{22}(\beta) \end{pmatrix}$

$det(\beta) := \left| \Delta(\beta) \right|$

$\beta := 2 \qquad root(det(\beta), \beta) = 2.362$

$\beta := 4 \qquad root(det(\beta), \beta) = 4.888$

$\beta := 7 \qquad root(det(\beta), \beta) = 7.604$

$\beta := 10 \qquad root(det(\beta), \beta) = 10.459$

$\beta_1 = 2.362 \qquad \beta_2 = 4.888 \qquad \beta_3 = 7.604 \qquad \beta_4 = 10.459$

$\beta := 0, 0.01 .. 12$