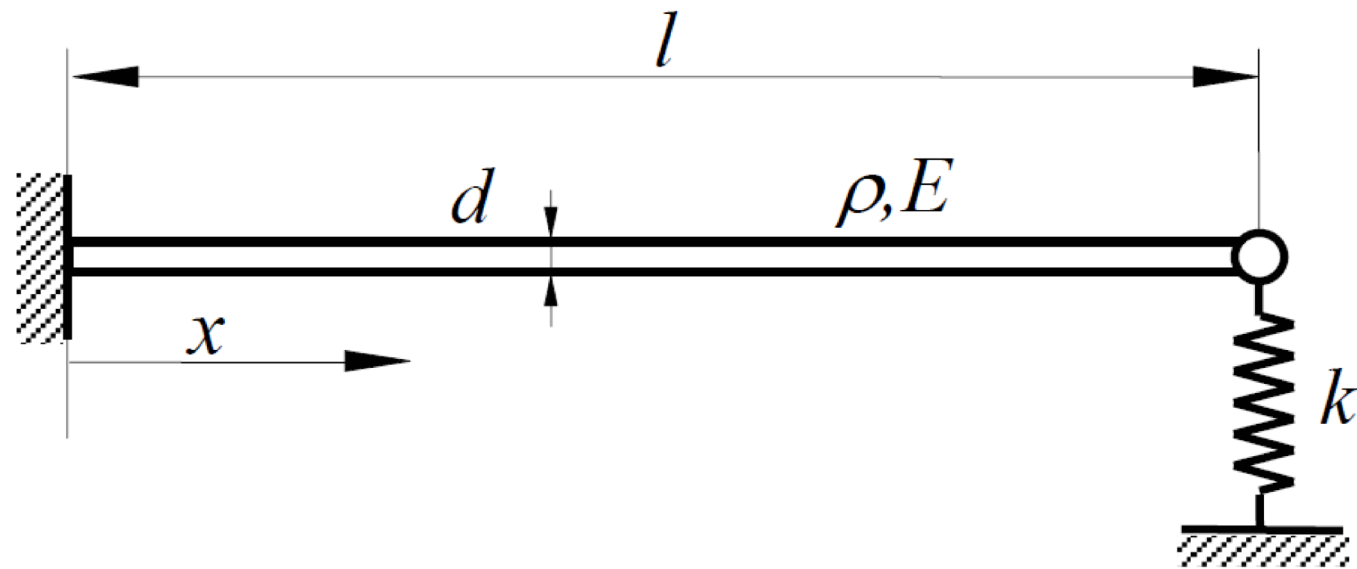


Trave incastro - molla all'estremità opposta



$$Y(x) := A \cdot \cos(\beta \cdot x) + B \cdot \sin(\beta \cdot x) + C \cdot \cosh(\beta \cdot x) + D \cdot \sinh(\beta \cdot x)$$

$$Y'(x) := \frac{d}{dx} Y(x) \quad Y'(x) \text{ collect } \beta \rightarrow (B \cdot \cos(\beta \cdot x) - A \cdot \sin(\beta \cdot x) + C \cdot \sinh(\beta \cdot x) + D \cdot \cosh(\beta \cdot x)) \cdot \beta$$

$$Y''(x) := \frac{d^2}{dx^2} Y(x) \quad Y''(x) \text{ collect } \beta \rightarrow (C \cdot \cosh(\beta \cdot x) - B \cdot \sin(\beta \cdot x) - A \cdot \cos(\beta \cdot x) + D \cdot \sinh(\beta \cdot x)) \cdot \beta^2$$

$$Y'''(x) := \frac{d^3}{dx^3} Y(x) \quad Y'''(x) \text{ collect } \beta \rightarrow (A \cdot \sin(\beta \cdot x) - B \cdot \cos(\beta \cdot x) + C \cdot \sinh(\beta \cdot x) + D \cdot \cosh(\beta \cdot x)) \cdot \beta^3$$

$$T(x) := E \cdot J \cdot \frac{d^3}{dx^3} Y(x) \quad \text{Azione di taglio}$$

Condizioni al contorno:

in $x = 0$

$$Y(0) \rightarrow A + C$$

$$Y'(0) \text{ collect } \beta \rightarrow (B + D) \cdot \beta$$

in $x = l$

$$Y''(l) \text{ collect } \beta \rightarrow (C \cdot \cosh(\beta \cdot l) - B \cdot \sin(\beta \cdot l) - A \cdot \cos(\beta \cdot l) + D \cdot \sinh(\beta \cdot l)) \cdot \beta^2$$

$$T(l) - k \cdot Y(l) \text{ collect } A, B, C, D \rightarrow (E \cdot J \cdot \beta^3 \cdot \sin(\beta \cdot l) - k \cdot \cos(\beta \cdot l)) \cdot A + (-k \cdot \sin(\beta \cdot l) - E \cdot J \cdot \beta^3 \cdot \cos(\beta \cdot l)) \cdot B + (E \cdot J \cdot \beta^3 \cdot \sinh(\beta \cdot l) - k \cdot \cosh(\beta \cdot l)) \cdot C + (E \cdot J \cdot \beta^3 \cdot \cosh(\beta \cdot l) - k \cdot \sinh(\beta \cdot l)) \cdot D$$

Riassunto delle condizioni al contorno (che formano un sistema lineare)

$$A + C = 0$$

$$B + D = 0$$

$$C \cdot \cosh(\beta \cdot l) - B \cdot \sin(\beta \cdot l) - A \cdot \cos(\beta \cdot l) + D \cdot \sinh(\beta \cdot l) = 0$$

$$(E \cdot J \cdot \beta^3 \cdot \sin(\beta \cdot l) - k \cdot \cos(\beta \cdot l)) \cdot A + (-k \cdot \sin(\beta \cdot l) - E \cdot J \cdot \beta^3 \cdot \cos(\beta \cdot l)) \cdot B + (E \cdot J \cdot \beta^3 \cdot \sinh(\beta \cdot l) - k \cdot \cosh(\beta \cdot l)) \cdot C + (E \cdot J \cdot \beta^3 \cdot \cosh(\beta \cdot l) - k \cdot \sinh(\beta \cdot l)) \cdot D = 0$$

Poniamo per semplicità: $\alpha = \beta l$ e otteniamo:

$$A + C = 0$$

$$B + D = 0$$

$$C \cdot \cosh(\alpha) - B \cdot \sin(\alpha) - A \cdot \cos(\alpha) + D \cdot \sinh(\alpha) = 0$$

$$\left[E \cdot J \cdot \left(\frac{\alpha}{l} \right)^3 \cdot \sin(\alpha) - k \cdot \cos(\alpha) \right] \cdot A + \left[-k \cdot \sin(\alpha) - E \cdot J \cdot \left(\frac{\alpha}{l} \right)^3 \cdot \cos(\alpha) \right] \cdot B + \left[E \cdot J \cdot \left(\frac{\alpha}{l} \right)^3 \cdot \sinh(\alpha) - k \cdot \cosh(\alpha) \right] \cdot C + \left[E \cdot J \cdot \left(\frac{\alpha}{l} \right)^3 \cdot \cosh(\alpha) - k \cdot \sinh(\alpha) \right] \cdot D = 0$$

Quest'ultima condizione si può riscrivere nella forma:

$$f_A(\alpha) \cdot A + f_B(\alpha) \cdot B + f_C(\alpha) \cdot C + f_D(\alpha) \cdot D = 0$$

dove si è posto:

$$h = \frac{E \cdot J}{l^3}$$

$$f_A(\alpha) := h \cdot \alpha^3 \cdot \sin(\alpha) - k \cdot \cos(\alpha)$$

$$f_B(\alpha) := -k \cdot \sin(\alpha) - h \cdot \alpha^3 \cdot \cos(\alpha)$$

$$f_C(\alpha) := h \cdot \alpha^3 \cdot \sinh(\alpha) - k \cdot \cosh(\alpha)$$

$$f_D(\alpha) := h \cdot \alpha^3 \cdot \cosh(\alpha) - k \cdot \sinh(\alpha)$$

Le quattro condizioni al contorno si possono scrivere in forma matriciale, come segue

$$\Delta(\alpha) \cdot \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Delta(\alpha) := \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -\cos(\alpha) & -\sin(\alpha) & \cosh(\alpha) & \sinh(\alpha) \\ f_A(\alpha) & f_B(\alpha) & f_C(\alpha) & f_D(\alpha) \end{pmatrix}$$

Conviene procedere per via numerica, senza sviluppi simbolici...

Lunghezza

$$l_w := 1$$

Materiale: acciaio

$$\rho := 7800$$

$$E := 206000 \cdot 10^6 = 2.06 \times 10^{11}$$

Sezione circolare di diametro d:

$$d := 20 \cdot 10^{-3} = 0.02$$

$$J := \frac{\pi \cdot d^4}{64} = 7.854 \times 10^{-9}$$

$$A_{sez} := \frac{\pi \cdot d^2}{4} = 3.142 \times 10^{-4}$$

Rigidità della molla:

$$k := 20000 \cdot 1 = 2 \times 10^4$$

$$\xi := \sqrt{\frac{E \cdot J}{\rho \cdot A_{sez}}} = 25.695$$

$$h := \frac{E \cdot J}{l^3} = 1617.92$$

$$f_A(\alpha) := h \cdot \alpha^3 \cdot \sin(\alpha) - k \cdot \cos(\alpha)$$

$$f_B(\alpha) := -k \cdot \sin(\alpha) - h \cdot \alpha^3 \cdot \cos(\alpha)$$

$$f_C(\alpha) := h \cdot \alpha^3 \cdot \sinh(\alpha) - k \cdot \cosh(\alpha)$$

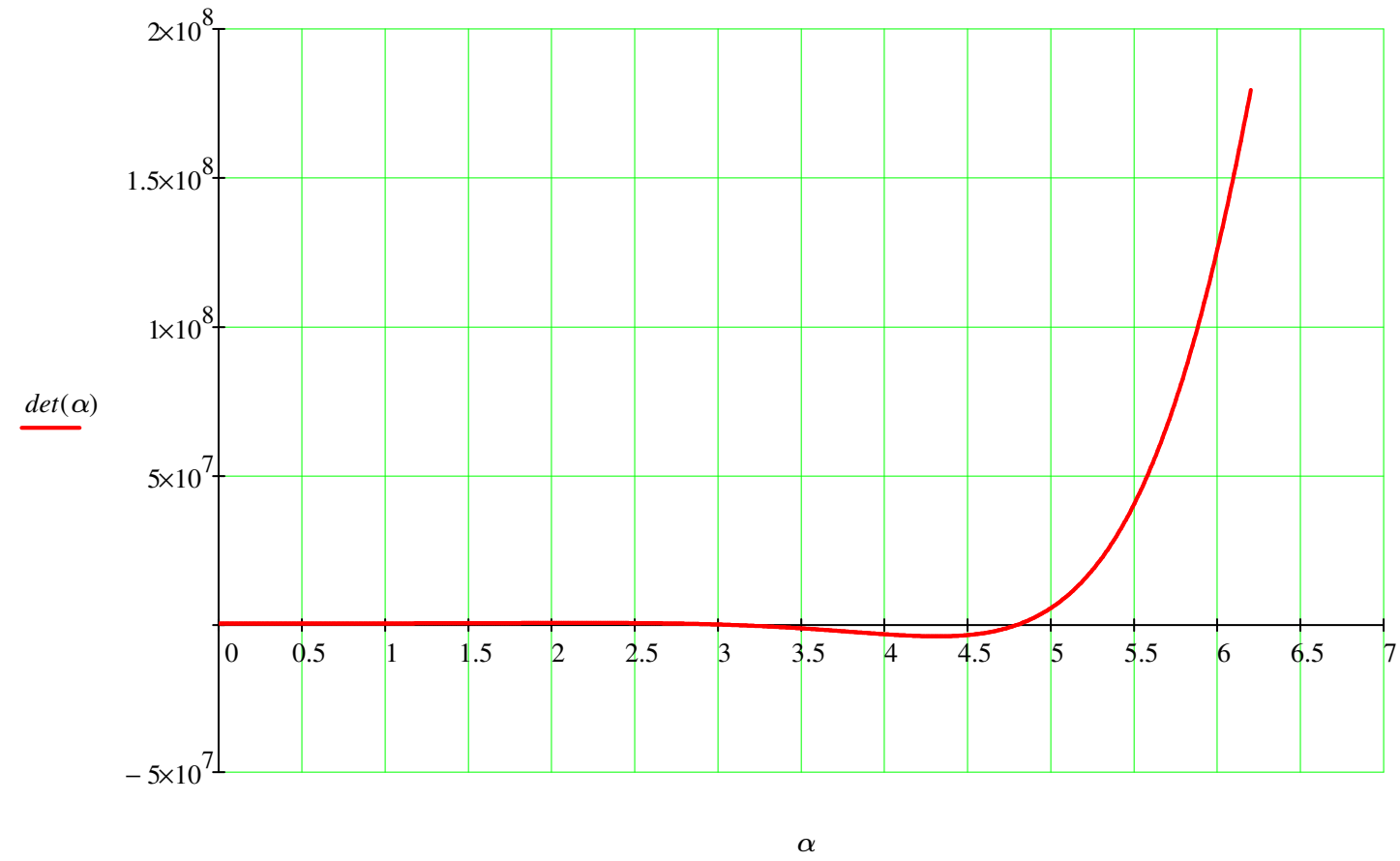
$$f_D(\alpha) := h \cdot \alpha^3 \cdot \cosh(\alpha) - k \cdot \sinh(\alpha)$$

$$\Delta(\alpha) := \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -\cos(\alpha) & -\sin(\alpha) & \cosh(\alpha) & \sinh(\alpha) \\ f_A(\alpha) & f_B(\alpha) & f_C(\alpha) & f_D(\alpha) \end{pmatrix}$$

Cerchiamo le soluzioni dell'equazione caratteristica per via grafica:

$$\det(\alpha) := |\Delta(\alpha)|$$

$$\alpha := 0, 0.01 \dots 6.2$$



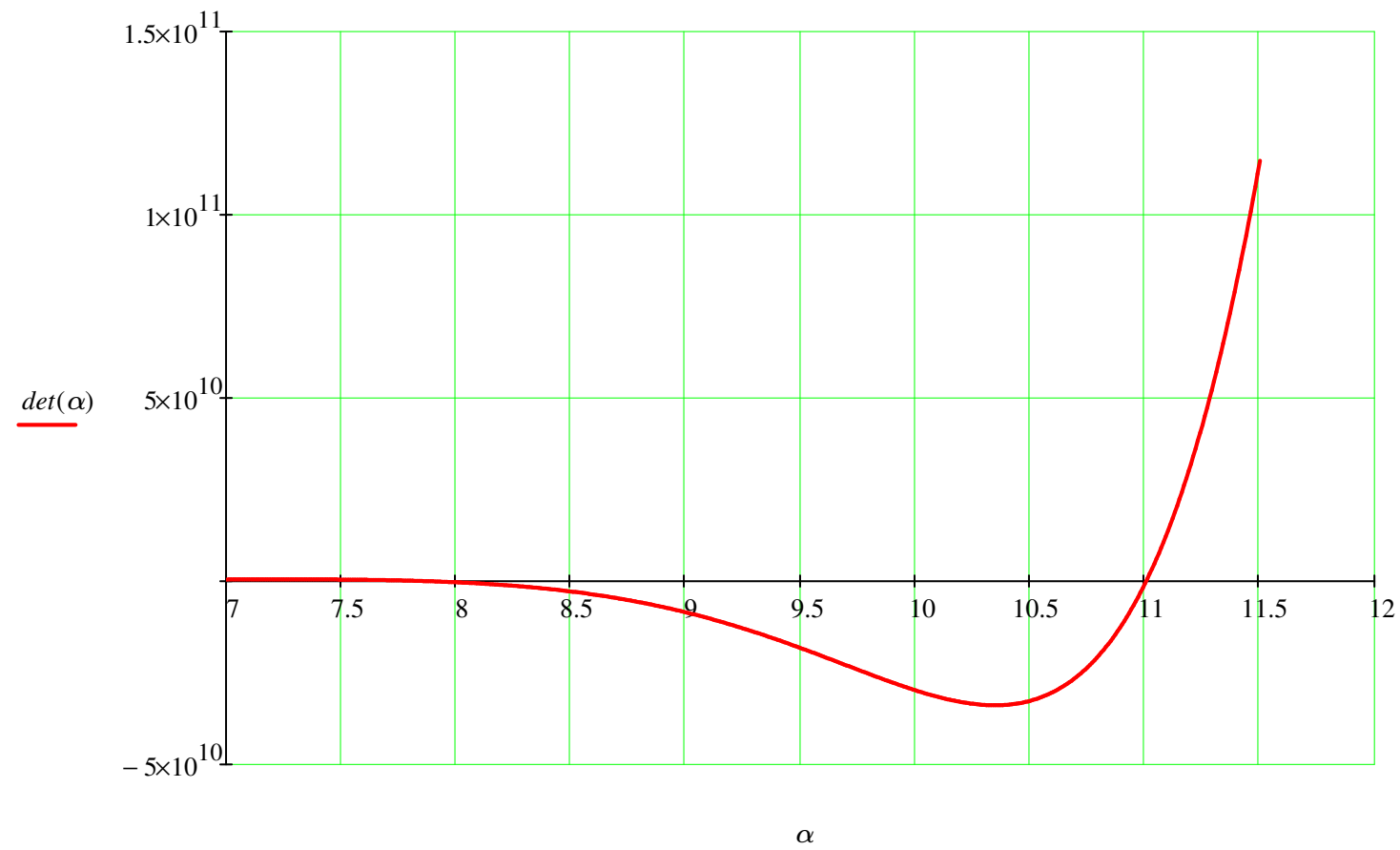
$$\alpha := 3.5$$

$$\alpha_1 := \text{root}(\det(\alpha), \alpha) = 2.735026$$

$$\alpha := 7$$

$$\alpha_2 := \text{root}(\det(\alpha), \alpha) = 4.817982$$

$$\alpha := 7, 7.01.. 11.5$$



$$\alpha := 10$$

$$\alpha_3 := \text{root}(\text{det}(\alpha), \alpha) = 7.880663$$

$$\alpha := 13$$

$$\alpha_4 := \text{root}(\text{det}(\alpha), \alpha) = 11.004902$$

$$\alpha = \beta \cdot l \quad \beta = \frac{\alpha}{l}$$

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} = \begin{pmatrix} 2.735026 \\ 4.817982 \\ 7.880663 \\ 11.004902 \end{pmatrix}$$

$$\begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix} := \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} \cdot \frac{1}{l} = \begin{pmatrix} 2.735026 \\ 4.817982 \\ 7.880663 \\ 11.004902 \end{pmatrix}$$

$$\omega = \beta^2 \cdot c$$

$$\omega_1 := (\beta_1)^2 \cdot c = 192.211$$

$$f_1 := \frac{\omega_1}{2 \cdot \pi} = 30.591$$

$$\omega_2 := (\beta_2)^2 \cdot c = 596.467$$

$$f_2 := \frac{\omega_2}{2 \cdot \pi} = 94.931$$

$$\omega_3 := (\beta_3)^2 \cdot c = 1595.812$$

$$f_3 := \frac{\omega_3}{2 \cdot \pi} = 253.981$$

$$\omega_4 := (\beta_4)^2 \cdot c = 3111.922$$

$$f_4 := \frac{\omega_4}{2 \cdot \pi} = 495.278$$

$$l := l$$

Given

$$A + C = 0$$

$$B + D = 0$$

$$C \cdot \cosh(\beta \cdot l) - B \cdot \sin(\beta \cdot l) - A \cdot \cos(\beta \cdot l) + D \cdot \sinh(\beta \cdot l) = 0$$

$$\text{Find}(B, C, D) \rightarrow \begin{pmatrix} \frac{A \cdot \cos(\beta \cdot l) + A \cdot \cosh(\beta \cdot l)}{\sinh(\beta \cdot l) + \sin(\beta \cdot l)} \\ -A \\ \frac{A \cdot \cos(\beta \cdot l) + A \cdot \cosh(\beta \cdot l)}{\sinh(\beta \cdot l) + \sin(\beta \cdot l)} \end{pmatrix}$$

$$\underline{A} := 1$$

$$B(\beta) := -\frac{A \cdot \cos(\beta \cdot l) + A \cdot \cosh(\beta \cdot l)}{\sinh(\beta \cdot l) + \sin(\beta \cdot l)}$$

$$\underline{C} := -A$$

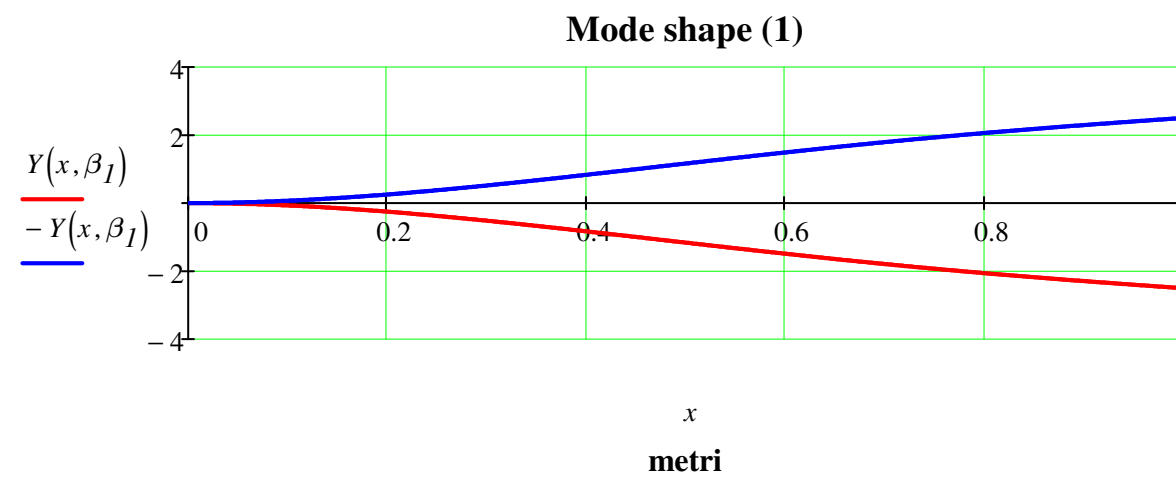
$$D(\beta) := \frac{A \cdot \cos(\beta \cdot l) + A \cdot \cosh(\beta \cdot l)}{\sinh(\beta \cdot l) + \sin(\beta \cdot l)}$$

$$Y(x, \beta) := A \cdot \cos(\beta \cdot x) + B(\beta) \cdot \sin(\beta \cdot x) + C \cdot \cosh(\beta \cdot x) + D(\beta) \cdot \sinh(\beta \cdot x)$$

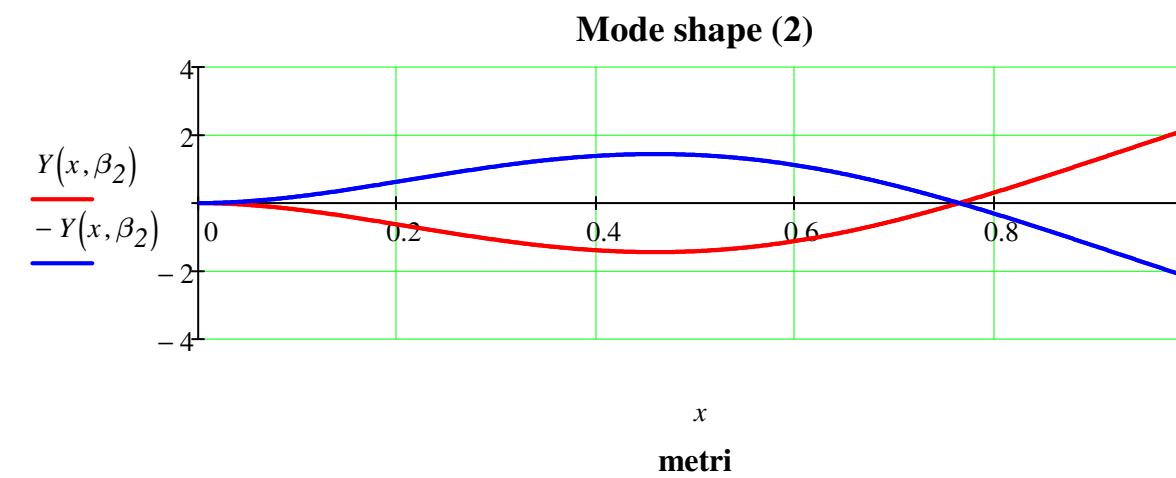
$$x := 0, \frac{l}{500} .. l$$

$FS := 4$ *Fondo scala per i grafici*

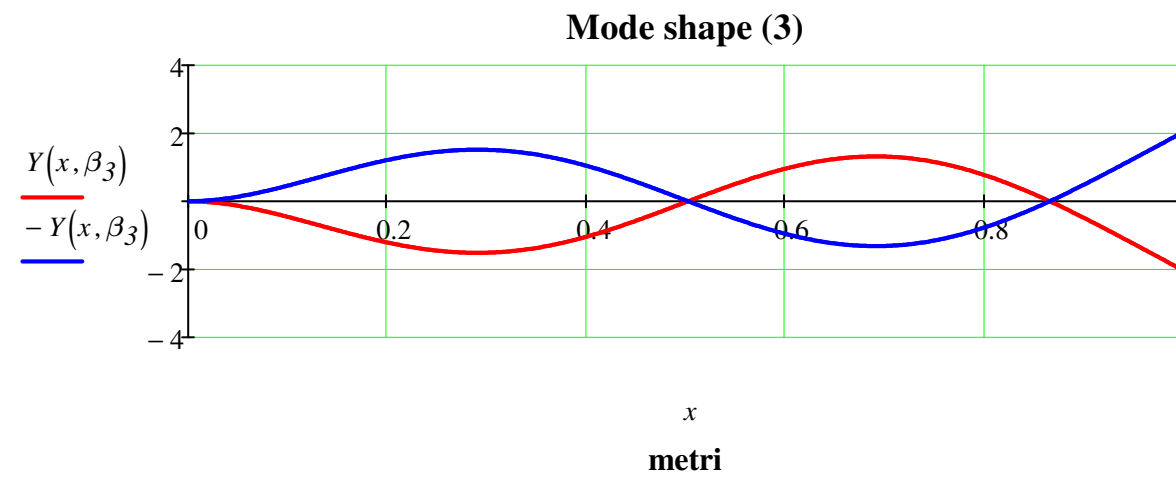
$$f_1 = 30.591 \text{ s}\cdot\text{Hz}$$



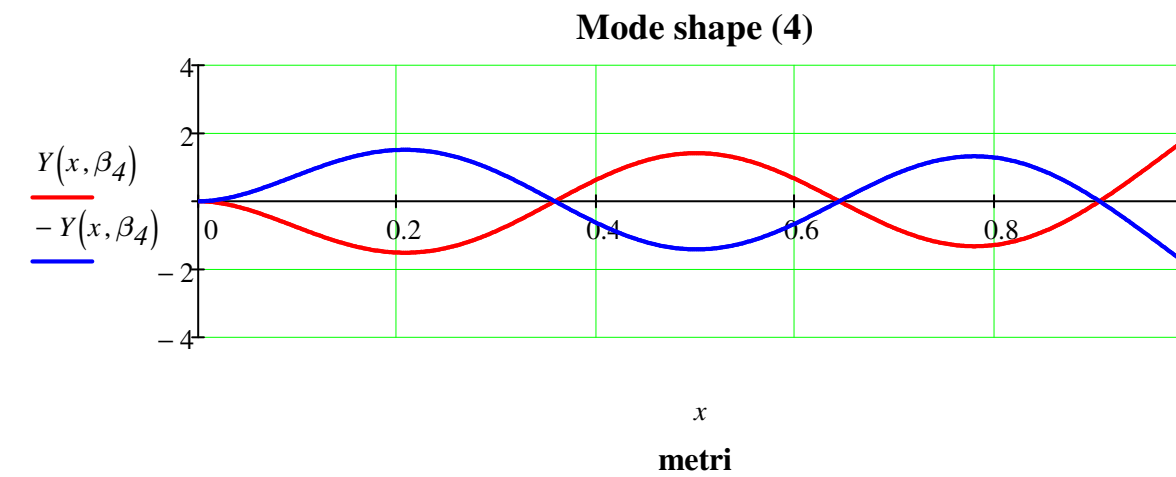
$$f_2 = 94.931 \text{ s}\cdot\text{Hz}$$



$$f_3 = 253.981 \text{ s}\cdot\text{Hz}$$

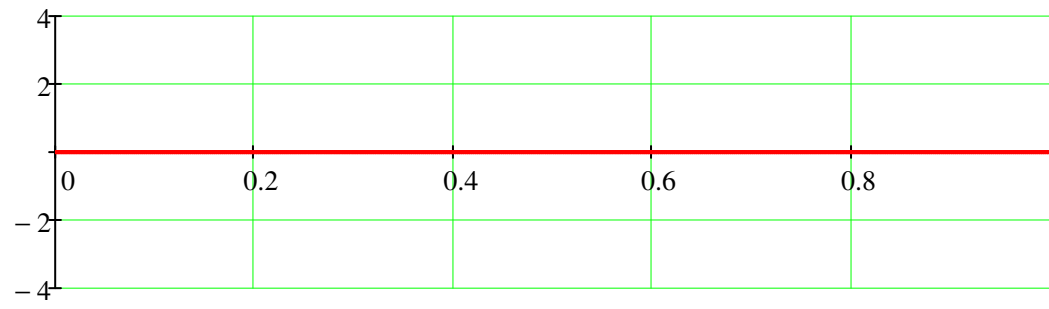


$$f_4 = 495.278 \text{ s}\cdot\text{Hz}$$



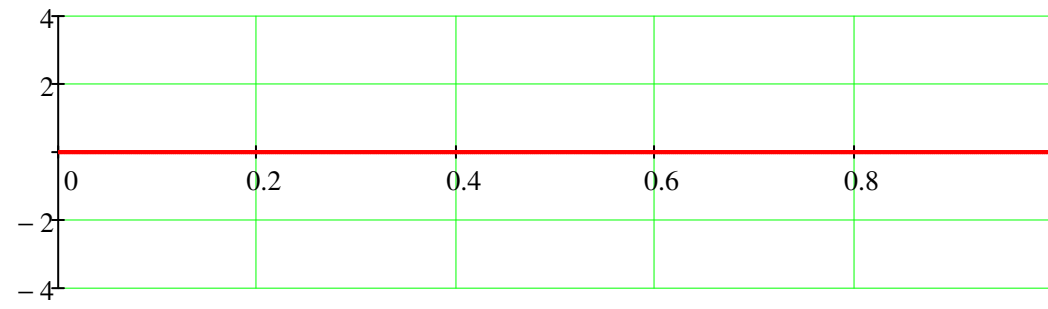
Animazioni delle deformate (Usare il menù: Strumenti - Animazione - Registra)

Mode shape (1)



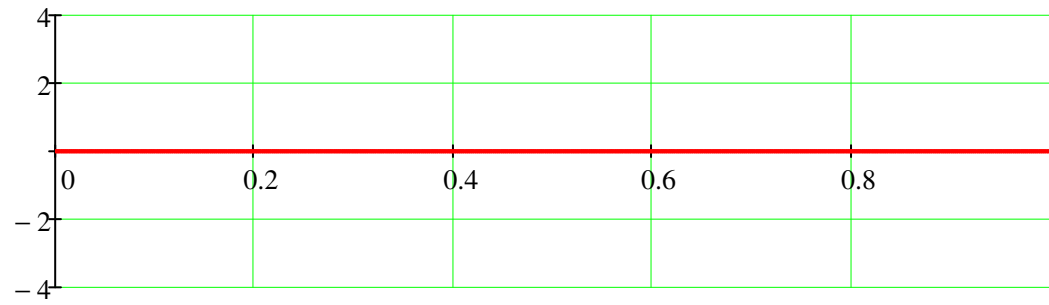
metri

Mode shape (2)



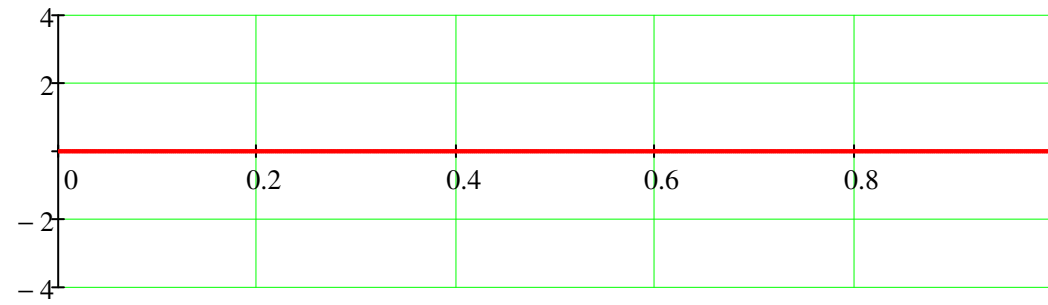
metri

Mode shape (3)



metri

Mode shape (4)



metri