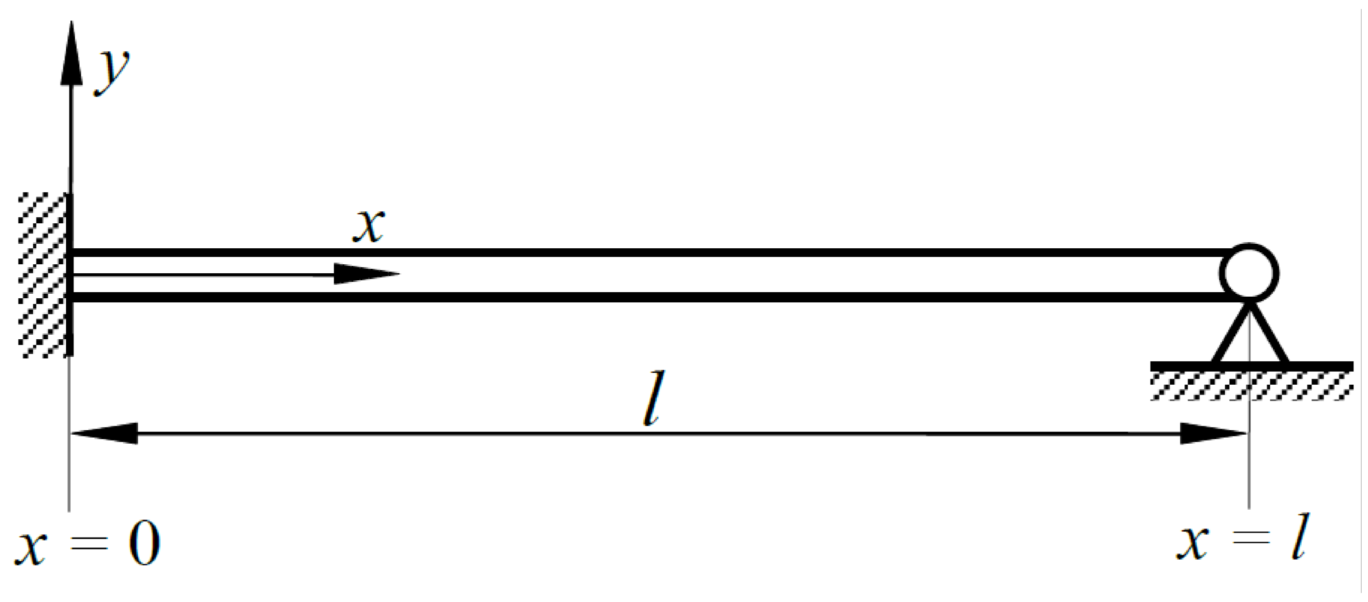
Trave incastro - appoggio



$Y(x) := A\cdot cos(\beta\cdot x) + B\cdot sin(\beta\cdot x) + C\cdot cosh(\beta\cdot x) + D\cdot sinh(\beta\cdot x)$

$Y'(x) := \frac{d}{dx}Y(x)$ $\qquad Y'(x)\ collect, \beta \ \rightarrow (B\cdot cos(\beta\cdot x) - A\cdot sin(\beta\cdot x) + C\cdot sinh(\beta\cdot x) + D\cdot cosh(\beta\cdot x))\cdot\beta$

$Y''(x) := \frac{d^2}{dx^2}Y(x)$ $\qquad Y''(x)\ collect, \beta \ \rightarrow (C\cdot cosh(\beta\cdot x) - B\cdot sin(\beta\cdot x) - A\cdot cos(\beta\cdot x) + D\cdot sinh(\beta\cdot x))\cdot\beta^2$

Condizioni al contorno:

in x = 0

$Y(0) \rightarrow A + C$ $\qquad A + C = 0$

$Y'(0)\ collect, \beta \ \rightarrow (B + D)\cdot\beta$ $\qquad B + D = 0$

in x = l

$Y(l) \rightarrow A\cdot cos(\beta\cdot l) + B\cdot sin(\beta\cdot l) + C\cdot cosh(\beta\cdot l) + D\cdot sinh(\beta\cdot l)$ $\qquad A\cdot cos(\beta\cdot l) + B\cdot sin(\beta\cdot l) + C\cdot cosh(\beta\cdot l) + D\cdot sinh(\beta\cdot l) = 0$

$Y''(l)\ collect, \beta \ \rightarrow (C\cdot cosh(\beta\cdot l) - B\cdot sin(\beta\cdot l) - A\cdot cos(\beta\cdot l) + D\cdot sinh(\beta\cdot l))\cdot\beta^2$ $\qquad C\cdot cosh(\beta\cdot l) - B\cdot sin(\beta\cdot l) - A\cdot cos(\beta\cdot l) + D\cdot sinh(\beta\cdot l) = 0$

Riassunto delle condizioni al contorno (che formano un sistema lineare)

$A + C = 0$

$B + D = 0$

$A\cdot cos(\beta\cdot l) + B\cdot sin(\beta\cdot l) + C\cdot cosh(\beta\cdot l) + D\cdot sinh(\beta\cdot l) = 0$

$C\cdot cosh(\beta\cdot l) - B\cdot sin(\beta\cdot l) - A\cdot cos(\beta\cdot l) + D\cdot sinh(\beta\cdot l) = 0$

Poniamo per semplicità: α = β l e otteniamo:

$A + C = 0$

$B + D = 0$

$A\cdot cos(\alpha) + B\cdot sin(\alpha) + C\cdot cosh(\alpha) + D\cdot sinh(\alpha) = 0$

$C\cdot cosh(\alpha) - B\cdot sin(\alpha) - A\cdot cos(\alpha) + D\cdot sinh(\alpha) = 0$

Le quattro condizioni al contorno si possono scrivere in forma matriciale, come segue

$$\Delta(\alpha)\cdot\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Delta(\alpha) := \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ cos(\alpha) & sin(\alpha) & cosh(\alpha) & sinh(\alpha) \\ -cos(\alpha) & -sin(\alpha) & cosh(\alpha) & sinh(\alpha) \end{pmatrix}$$

Infatti, il prodotto matriciale seguente, fornisce:

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ cos(\alpha) & sin(\alpha) & cosh(\alpha) & sinh(\alpha) \\ -cos(\alpha) & -sin(\alpha) & cosh(\alpha) & sinh(\alpha) \end{pmatrix}\cdot\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} \rightarrow \begin{pmatrix} A + C \\ B + D \\ A\cdot cos(\alpha) + B\cdot sin(\alpha) + C\cdot cosh(\alpha) + D\cdot sinh(\alpha) \\ C\cdot cosh(\alpha) - B\cdot sin(\alpha) - A\cdot cos(\alpha) + D\cdot sinh(\alpha) \end{pmatrix}$$

Si calcola ora il determinante della matrice **Δ(α)** *e lo si pone uguale a zero, per ricavare l'equazione caratteristica:*

$det(\alpha) := |\Delta(\alpha)|$

$det(\alpha)\ factor \ \rightarrow 2\cdot(cosh(\alpha)\cdot sin(\alpha) - sinh(\alpha)\cdot cos(\alpha))$

Rielaborando l'equazione, si ricava:

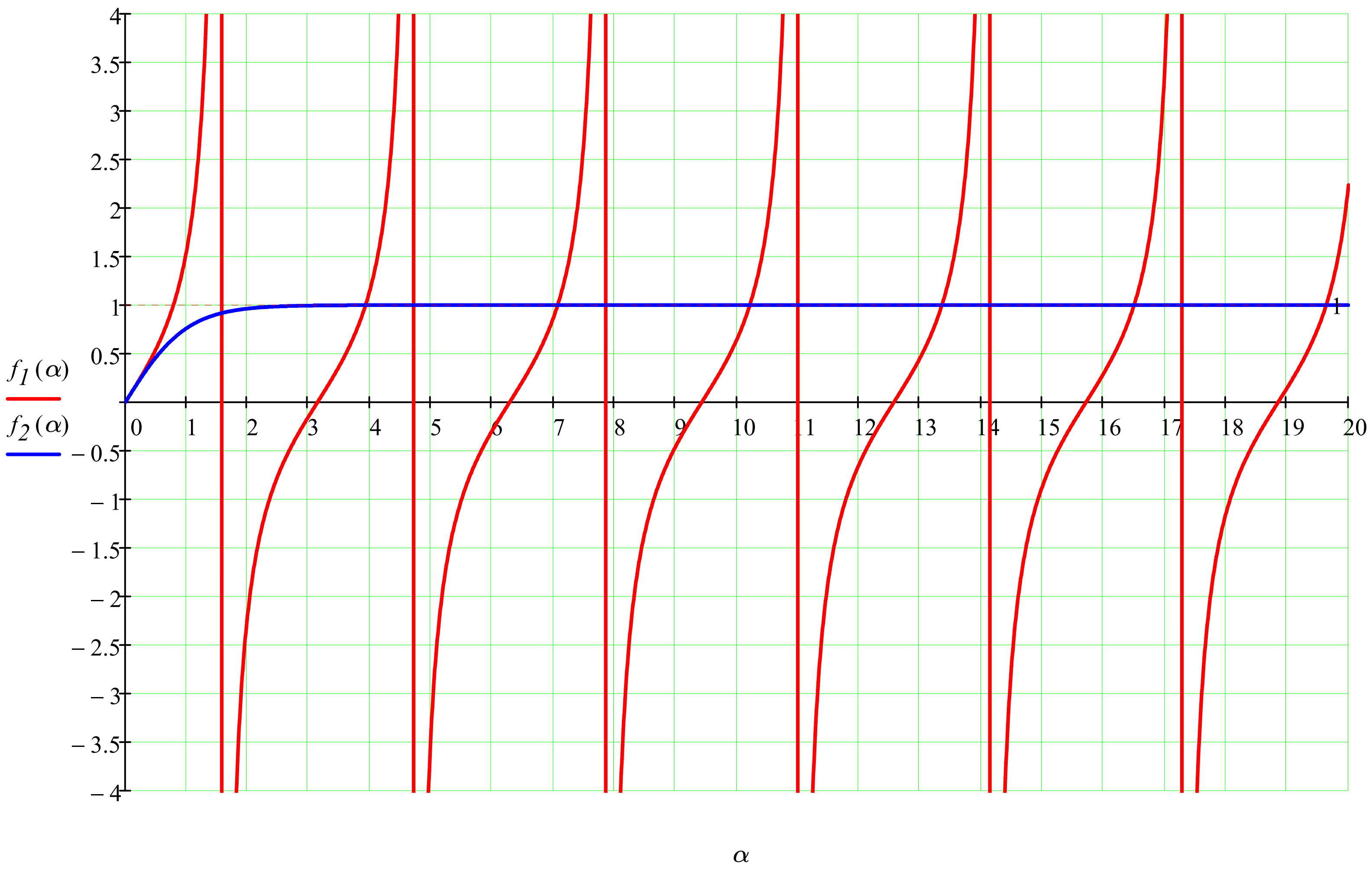
$tan(\alpha) = tanh(\alpha)$ *Equazione caratteristica (o equazione delle frequenze)*

Cerchiamo le soluzioni dell'equazione caratteristica per via grafica:

$f_1(\alpha) := tan(\alpha)$

$f_2(\alpha) := tanh(\alpha)$

$\alpha := 0, 0.01 .. 20$ $FS := 4$



$\alpha := 4$

$\alpha_1 := root(f_1(\alpha) - f_2(\alpha), \alpha) = 3.926602$

$\alpha := 7$

$\alpha_2 := root(f_1(\alpha) - f_2(\alpha), \alpha) = 7.068583$

$\alpha := 10$

$\alpha_3 := root(f_1(\alpha) - f_2(\alpha), \alpha) = 10.210176$

$\alpha := 13$

$\alpha_4 := root(f_1(\alpha) - f_2(\alpha), \alpha) = 13.351769$

Lunghezza

$l := 1200 \cdot mm = 1.2\,m$

Materiale: acciaio

$\rho := 7800 \cdot \frac{kg}{m^3}$

$E := 206000 \cdot MPa = 2.06 \times 10^{11}\,Pa$

Sezione circolare di diametro d:

$d := 15 \cdot mm = 0.015\,m$

$J := \frac{\pi \cdot d^4}{64} = 2.485 \times 10^{-9}\,m^4$

$A_{sez} := \frac{\pi \cdot d^2}{4} = 1.767 \times 10^{-4}\,m^2$

$c := \sqrt{\frac{E \cdot J}{\rho \cdot A_{sez}}} = 19.272\,\frac{m^2}{s}$

$\alpha = \beta \cdot l$ $\beta = \frac{\alpha}{l}$

$$\begin{pmatrix}\alpha_1\\\alpha_2\\\alpha_3\\\alpha_4\end{pmatrix} = \begin{pmatrix}3.926602\\7.068583\\10.210176\\13.351769\end{pmatrix} \qquad \begin{pmatrix}\beta_1\\\beta_2\\\beta_3\\\beta_4\end{pmatrix} := \begin{pmatrix}\alpha_1\\\alpha_2\\\alpha_3\\\alpha_4\end{pmatrix}\cdot\frac{1}{l} = \begin{pmatrix}3.272169\\5.890486\\8.50848\\11.126474\end{pmatrix}\frac{1}{m}$$

$\omega = \beta^2 \cdot c$

$\omega_1 := (\beta_1)^2 \cdot c = 206.343 \cdot \frac{rad}{s}$ $\qquad f_1 := \frac{\omega_1}{2\cdot\pi} = 32.84 \cdot Hz$

$\omega_2 := (\beta_2)^2 \cdot c = 668.682 \cdot \frac{rad}{s}$ $\qquad f_2 := \frac{\omega_2}{2\cdot\pi} = 106.424 \cdot Hz$

$\omega_3 := (\beta_3)^2 \cdot c = 1395.152 \cdot \frac{rad}{s}$ $\qquad f_3 := \frac{\omega_3}{2\cdot\pi} = 222.045 \cdot Hz$

$\omega_4 := (\beta_4)^2 \cdot c = 2385.793 \cdot \frac{rad}{s}$ $\qquad f_4 := \frac{\omega_4}{2\cdot\pi} = 379.711 \cdot Hz$

$l := l$

Given

$A + C = 0$

$B + D = 0$

$A\cdot cos(\beta\cdot l) + B\cdot sin(\beta\cdot l) + C\cdot cosh(\beta\cdot l) + D\cdot sinh(\beta\cdot l) = 0$

$$Find(B,C,D) \rightarrow \begin{pmatrix} \frac{A\cdot cos(\beta\cdot l) - A\cdot cosh(\beta\cdot l)}{sinh(\beta\cdot l) - sin(\beta\cdot l)} \\ -A \\ -\frac{A\cdot cos(\beta\cdot l) - A\cdot cosh(\beta\cdot l)}{sinh(\beta\cdot l) - sin(\beta\cdot l)} \end{pmatrix}$$

$A := 1$

$B(\beta) := \frac{A\cdot cos(\beta\cdot l) - A\cdot cosh(\beta\cdot l)}{sinh(\beta\cdot l) - sin(\beta\cdot l)}$

$C := -A$

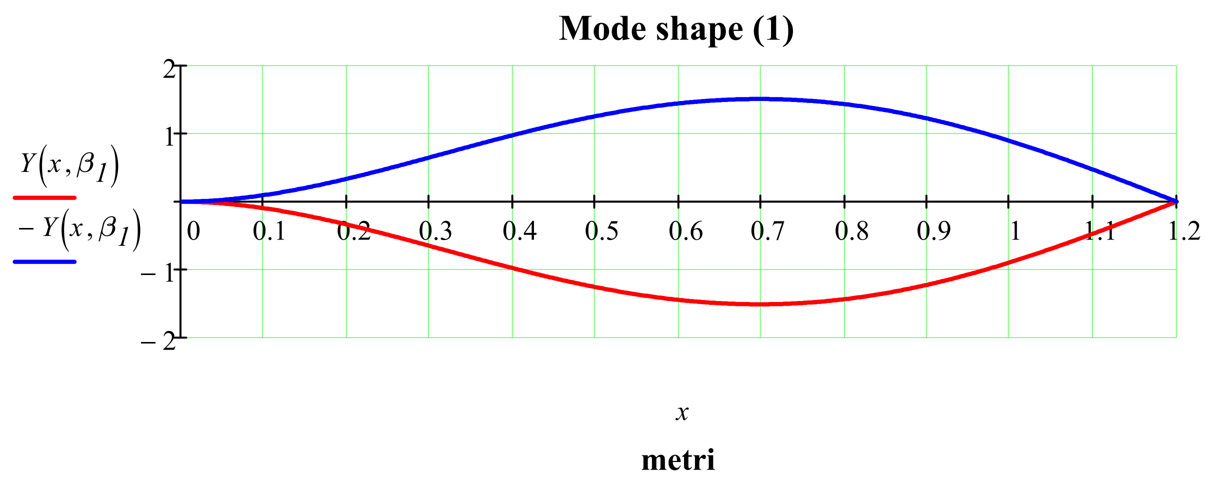
$D(\beta) := -\frac{A\cdot cos(\beta\cdot l) - A\cdot cosh(\beta\cdot l)}{sinh(\beta\cdot l) - sin(\beta\cdot l)}$

$Y(x,\beta) := A\cdot cos(\beta\cdot x) + B(\beta)\cdot sin(\beta\cdot x) + C\cdot cosh(\beta\cdot x) + D(\beta)\cdot sinh(\beta\cdot x)$

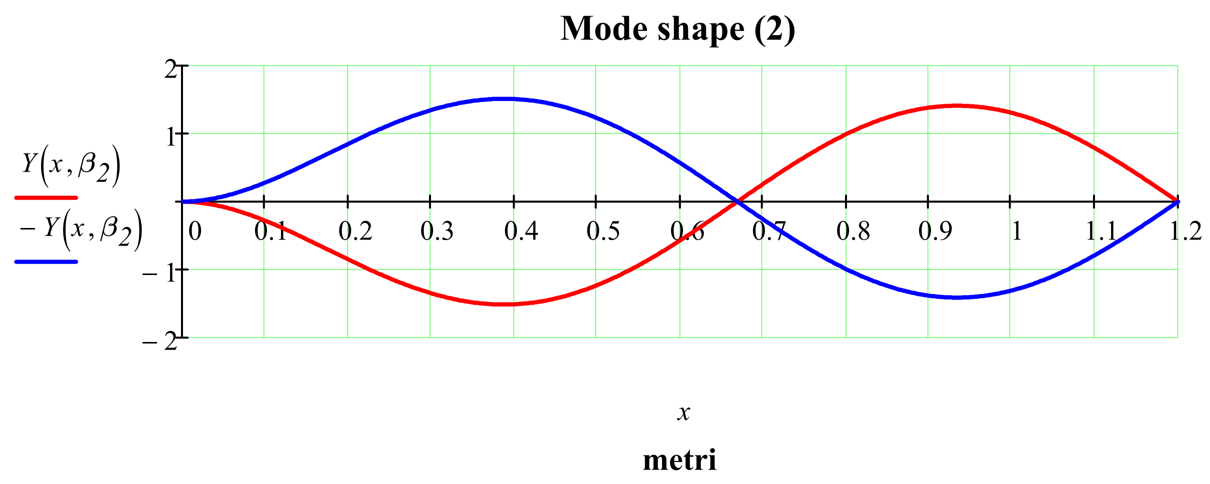
$x := 0, \frac{l}{500} .. l$

$FS := 2$ *Fondo scala per i grafici*

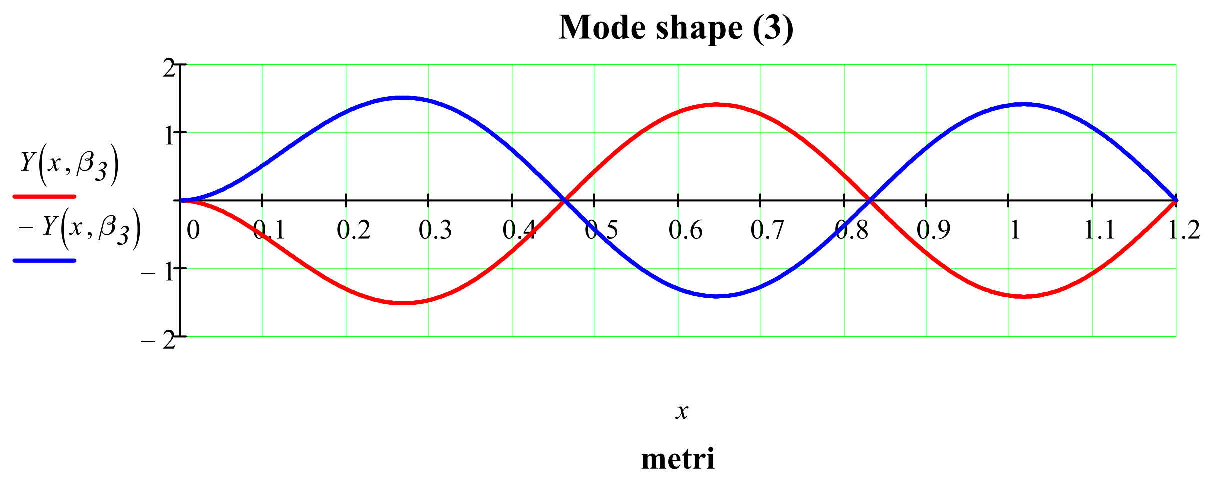
$f_1 = 32.84 \cdot Hz$



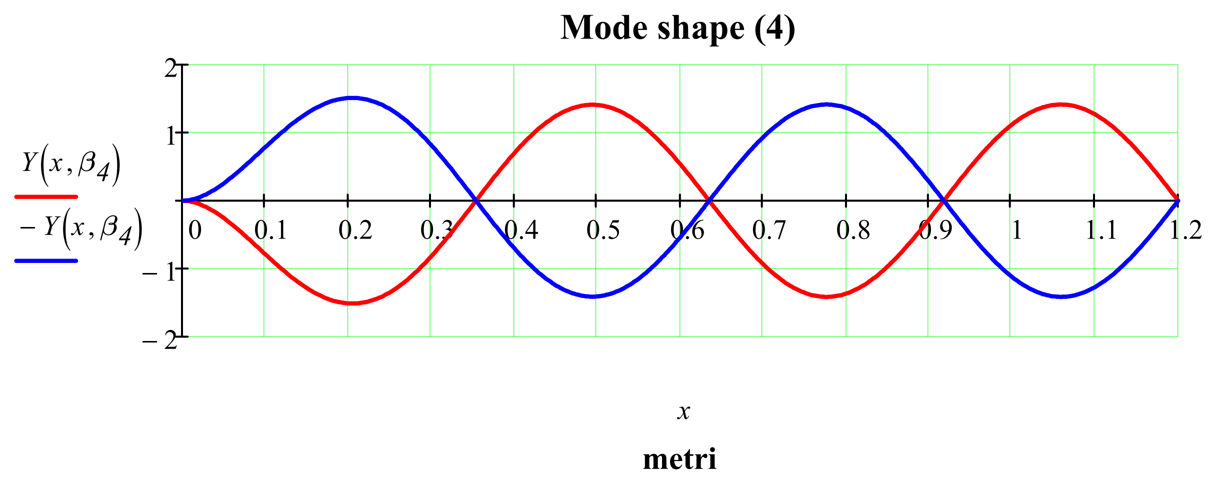
$f_2 = 106.424 \cdot Hz$



$f_3 = 222.045 \cdot Hz$

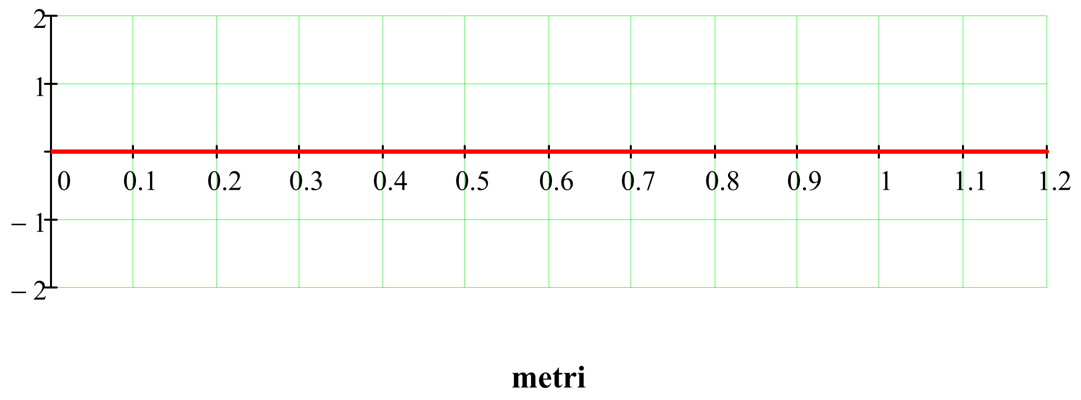


$f_4 = 379.711 \cdot Hz$

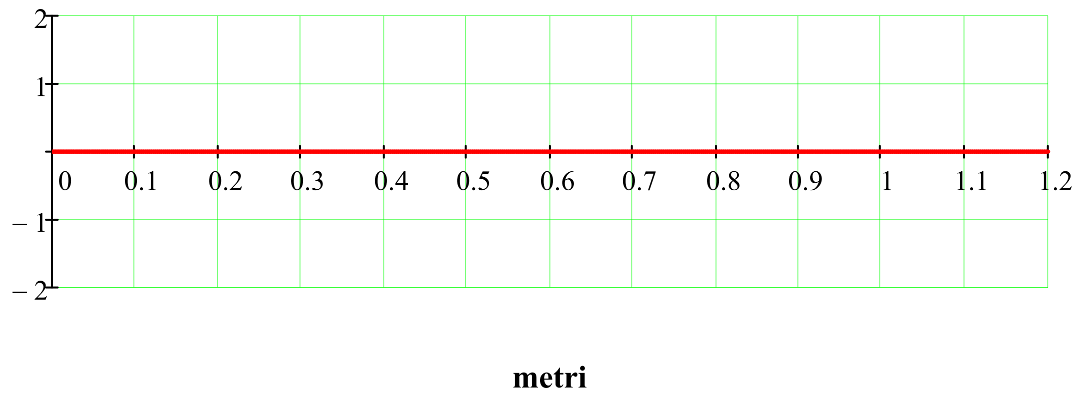


Animazioni delle deformate (Usare il menù: Strumenti - Animazione - Registra)

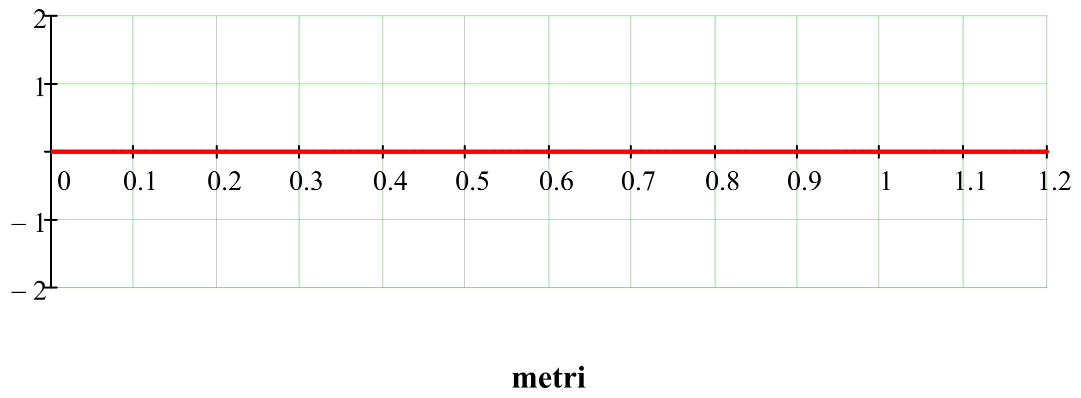
Mode shape (1)



Mode shape (2)



Mode shape (3)



Mode shape (4)