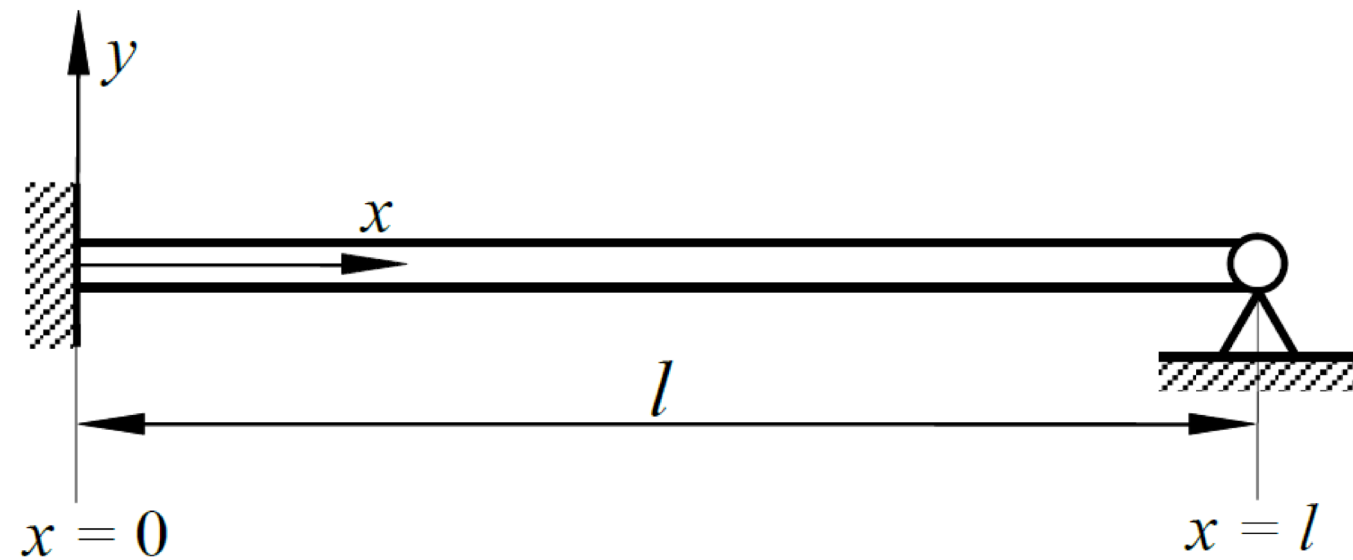


Trave incastro - appoggio



$$Y(x) := A \cdot \cos(\beta \cdot x) + B \cdot \sin(\beta \cdot x) + C \cdot \cosh(\beta \cdot x) + D \cdot \sinh(\beta \cdot x)$$

$$Y'(x) := \frac{d}{dx} Y(x) \quad Y'(x) \text{ collect } \beta \rightarrow (B \cdot \cos(\beta \cdot x) - A \cdot \sin(\beta \cdot x) + C \cdot \sinh(\beta \cdot x) + D \cdot \cosh(\beta \cdot x)) \cdot \beta$$

$$Y''(x) := \frac{d^2}{dx^2} Y(x) \quad Y''(x) \text{ collect } \beta \rightarrow (C \cdot \cosh(\beta \cdot x) - B \cdot \sin(\beta \cdot x) - A \cdot \cos(\beta \cdot x) + D \cdot \sinh(\beta \cdot x)) \cdot \beta^2$$

Condizioni al contorno:

in $x = 0$

$$Y(0) \rightarrow A + C \quad \mathbf{A + C = 0}$$

$$Y'(0) \text{ collect } \beta \rightarrow (B + D) \cdot \beta \quad \mathbf{B + D = 0}$$

in $x = l$

$$Y(l) \rightarrow A \cdot \cos(\beta \cdot l) + B \cdot \sin(\beta \cdot l) + C \cdot \cosh(\beta \cdot l) + D \cdot \sinh(\beta \cdot l) \quad \mathbf{A \cdot \cos(\beta \cdot l) + B \cdot \sin(\beta \cdot l) + C \cdot \cosh(\beta \cdot l) + D \cdot \sinh(\beta \cdot l) = 0}$$

$$Y''(l) \text{ collect } \beta \rightarrow (C \cdot \cosh(\beta \cdot l) - B \cdot \sin(\beta \cdot l) - A \cdot \cos(\beta \cdot l) + D \cdot \sinh(\beta \cdot l)) \cdot \beta^2 \quad \mathbf{C \cdot \cosh(\beta \cdot l) - B \cdot \sin(\beta \cdot l) - A \cdot \cos(\beta \cdot l) + D \cdot \sinh(\beta \cdot l) = 0}$$

Riassunto delle condizioni al contorno (che formano un sistema lineare)

$$A + C = 0$$

$$B + D = 0$$

$$A \cdot \cos(\beta \cdot l) + B \cdot \sin(\beta \cdot l) + C \cdot \cosh(\beta \cdot l) + D \cdot \sinh(\beta \cdot l) = 0$$

$$C \cdot \cosh(\beta \cdot l) - B \cdot \sin(\beta \cdot l) - A \cdot \cos(\beta \cdot l) + D \cdot \sinh(\beta \cdot l) = 0$$

Poniamo per semplicità: $\alpha = \beta l$ e otteniamo:

$$A + C = 0$$

$$B + D = 0$$

$$A \cdot \cos(\alpha) + B \cdot \sin(\alpha) + C \cdot \cosh(\alpha) + D \cdot \sinh(\alpha) = 0$$

$$C \cdot \cosh(\alpha) - B \cdot \sin(\alpha) - A \cdot \cos(\alpha) + D \cdot \sinh(\alpha) = 0$$

Le quattro condizioni al contorno si possono scrivere in forma matriciale, come segue

$$\Delta(\alpha) \cdot \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Delta(\alpha) := \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ \cos(\alpha) & \sin(\alpha) & \cosh(\alpha) & \sinh(\alpha) \\ -\cos(\alpha) & -\sin(\alpha) & \cosh(\alpha) & \sinh(\alpha) \end{pmatrix}$$

Infatti, il prodotto matriciale seguente, fornisce:

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ \cos(\alpha) & \sin(\alpha) & \cosh(\alpha) & \sinh(\alpha) \\ -\cos(\alpha) & -\sin(\alpha) & \cosh(\alpha) & \sinh(\alpha) \end{pmatrix} \cdot \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} \rightarrow \begin{pmatrix} A + C \\ B + D \\ A \cdot \cos(\alpha) + B \cdot \sin(\alpha) + C \cdot \cosh(\alpha) + D \cdot \sinh(\alpha) \\ C \cdot \cosh(\alpha) - B \cdot \sin(\alpha) - A \cdot \cos(\alpha) + D \cdot \sinh(\alpha) \end{pmatrix}$$

Si calcola ora il determinante della matrice $\Delta(\alpha)$ e lo si pone uguale a zero, per ricavare l'equazione caratteristica:

$$\det(\alpha) := |\Delta(\alpha)|$$

$$\det(\alpha) \text{ factor} \rightarrow 2 \cdot (\cosh(\alpha) \cdot \sin(\alpha) - \sinh(\alpha) \cdot \cos(\alpha))$$

Rielaborando l'equazione, si ricava:

$$\tan(\alpha) = \tanh(\alpha)$$

Equazione caratteristica (o equazione delle frequenze)

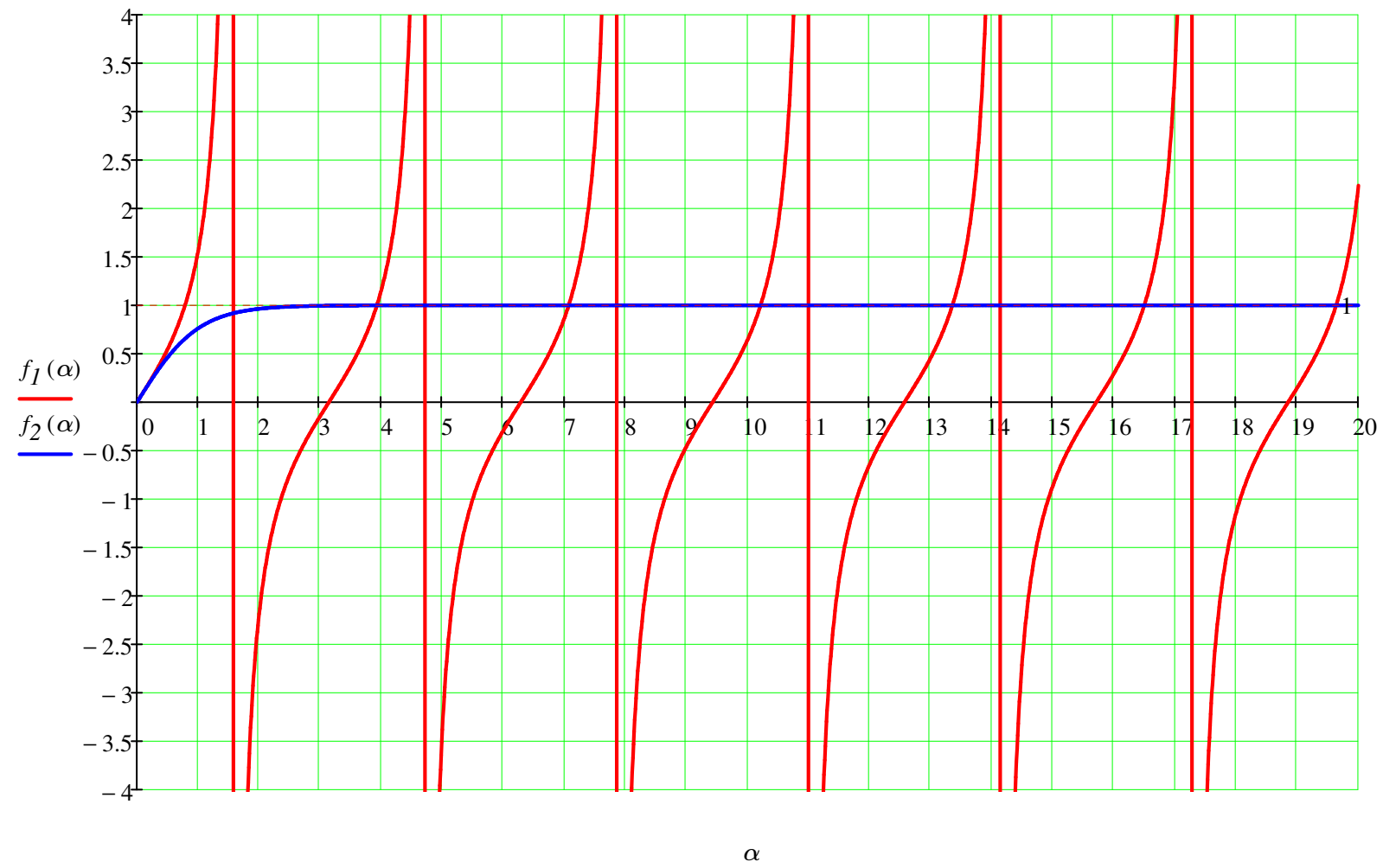
Cerchiamo le soluzioni dell'equazione caratteristica per via grafica:

$$f_1(\alpha) := \tan(\alpha)$$

$$f_2(\alpha) := \tanh(\alpha)$$

$$\alpha := 0, 0.01 \dots 20$$

$$FS := 4$$



$\alpha := 4$

$$\alpha_1 := \text{root}(f_1(\alpha) - f_2(\alpha), \alpha) = 3.926602$$

$\alpha := 7$

$$\alpha_2 := \text{root}(f_1(\alpha) - f_2(\alpha), \alpha) = 7.068583$$

$\alpha := 10$

$$\alpha_3 := \text{root}(f_1(\alpha) - f_2(\alpha), \alpha) = 10.210176$$

$$\alpha := 13$$

$$\alpha_4 := \text{root}(f_1(\alpha) - f_2(\alpha), \alpha) = 13.351769$$

Lunghezza

$$l := 1200 \cdot \text{mm} = 1.2 \text{ m}$$

Materiale: acciaio

$$\rho := 7800 \cdot \frac{\text{kg}}{\text{m}^3}$$

$$E := 206000 \cdot \text{MPa} = 2.06 \times 10^{11} \text{ Pa}$$

Sezione circolare di diametro d :

$$d := 15 \cdot \text{mm} = 0.015 \text{ m}$$

$$J := \frac{\pi \cdot d^4}{64} = 2.485 \times 10^{-9} \text{ m}^4$$

$$A_{sez} := \frac{\pi \cdot d^2}{4} = 1.767 \times 10^{-4} \text{ m}^2$$

$$\xi := \sqrt{\frac{E \cdot J}{\rho \cdot A_{sez}}} = 19.272 \frac{\text{m}^2}{\text{s}}$$

$$\alpha = \beta \cdot l$$

$$\beta = \frac{\alpha}{l}$$

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} = \begin{pmatrix} 3.926602 \\ 7.068583 \\ 10.210176 \\ 13.351769 \end{pmatrix}$$

$$\begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix} := \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} \cdot \frac{1}{l} = \begin{pmatrix} 3.272169 \\ 5.890486 \\ 8.50848 \\ 11.126474 \end{pmatrix} \frac{l}{m}$$

$$\omega = \beta^2 \cdot c$$

$$\omega_1 := (\beta_1)^2 \cdot c = 206.343 \cdot \frac{\text{rad}}{\text{s}}$$

$$f_{1\text{Hz}} := \frac{\omega_1}{2 \cdot \pi} = 32.84 \cdot \text{Hz}$$

$$\omega_2 := (\beta_2)^2 \cdot c = 668.682 \cdot \frac{\text{rad}}{\text{s}}$$

$$f_{2\text{Hz}} := \frac{\omega_2}{2 \cdot \pi} = 106.424 \cdot \text{Hz}$$

$$\omega_3 := (\beta_3)^2 \cdot c = 1395.152 \cdot \frac{\text{rad}}{\text{s}}$$

$$f_3 := \frac{\omega_3}{2 \cdot \pi} = 222.045 \cdot \text{Hz}$$

$$\omega_4 := (\beta_4)^2 \cdot c = 2385.793 \cdot \frac{\text{rad}}{\text{s}}$$

$$f_4 := \frac{\omega_4}{2 \cdot \pi} = 379.711 \cdot \text{Hz}$$

$$l := l$$

Given

$$A + C = 0$$

$$B + D = 0$$

$$A \cdot \cos(\beta \cdot l) + B \cdot \sin(\beta \cdot l) + C \cdot \cosh(\beta \cdot l) + D \cdot \sinh(\beta \cdot l) = 0$$

$$\text{Find}(B, C, D) \rightarrow \begin{pmatrix} \frac{A \cdot \cos(\beta \cdot l) - A \cdot \cosh(\beta \cdot l)}{\sinh(\beta \cdot l) - \sin(\beta \cdot l)} \\ -A \\ \frac{A \cdot \cos(\beta \cdot l) - A \cdot \cosh(\beta \cdot l)}{\sinh(\beta \cdot l) - \sin(\beta \cdot l)} \end{pmatrix}$$

$$A := 1$$

$$B(\beta) := \frac{A \cdot \cos(\beta \cdot l) - A \cdot \cosh(\beta \cdot l)}{\sinh(\beta \cdot l) - \sin(\beta \cdot l)}$$

$$C := -A$$

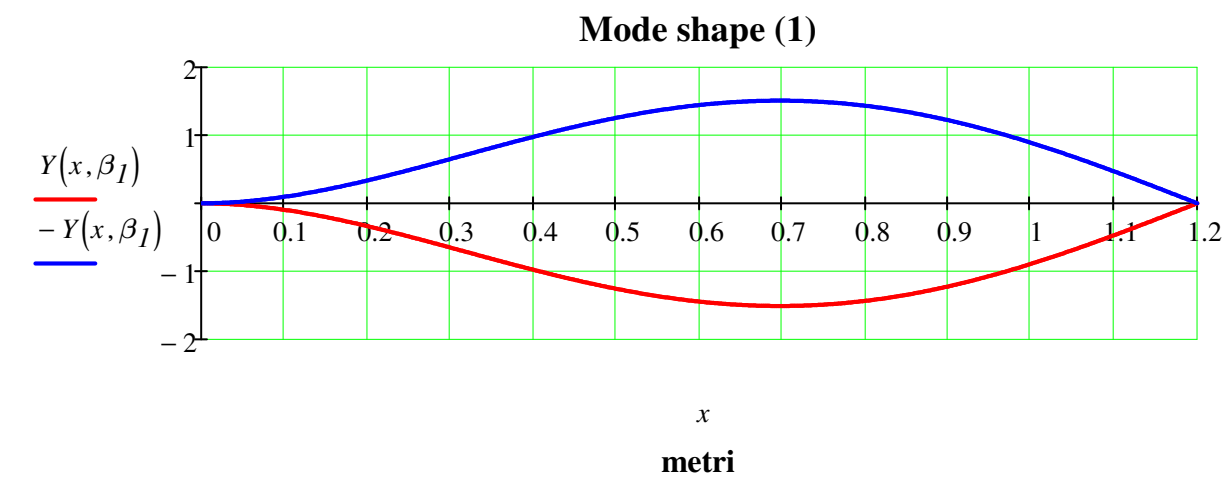
$$D(\beta) := -\frac{A \cdot \cos(\beta \cdot l) - A \cdot \cosh(\beta \cdot l)}{\sinh(\beta \cdot l) - \sin(\beta \cdot l)}$$

$$Y(x, \beta) := A \cdot \cos(\beta \cdot x) + B(\beta) \cdot \sin(\beta \cdot x) + C \cdot \cosh(\beta \cdot x) + D(\beta) \cdot \sinh(\beta \cdot x)$$

$$x := 0, \frac{l}{500} .. l$$

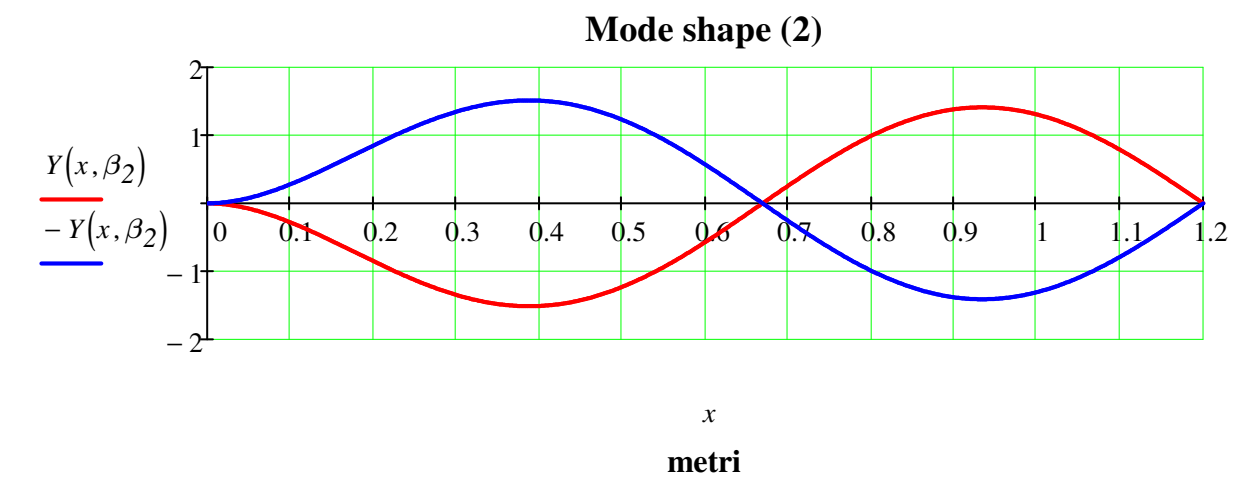
$$FS := 2 \quad \text{Fondo scala per i grafici}$$

$$f_1 = 32.84 \cdot \text{Hz}$$



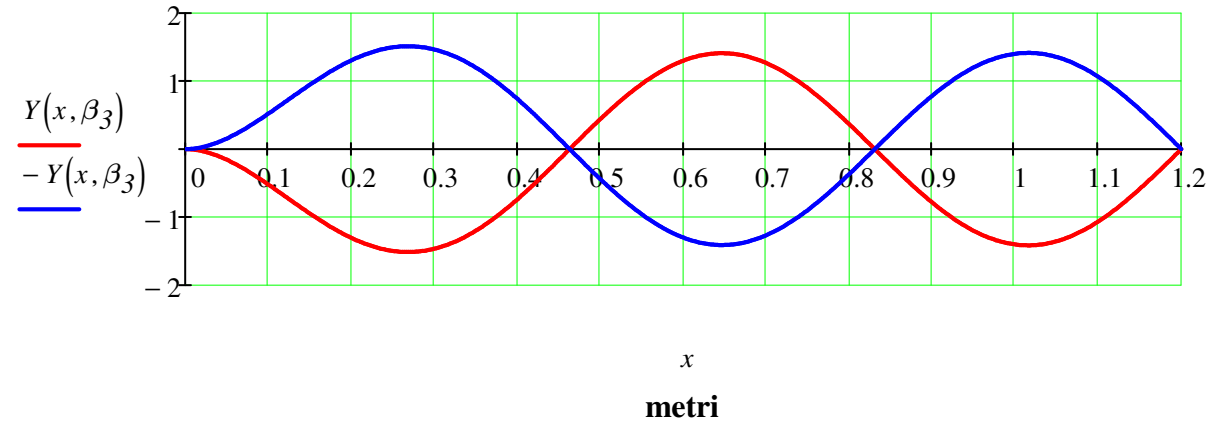
$$f_3 = 222.045 \cdot \text{Hz}$$

$$f_2 = 106.424 \cdot \text{Hz}$$

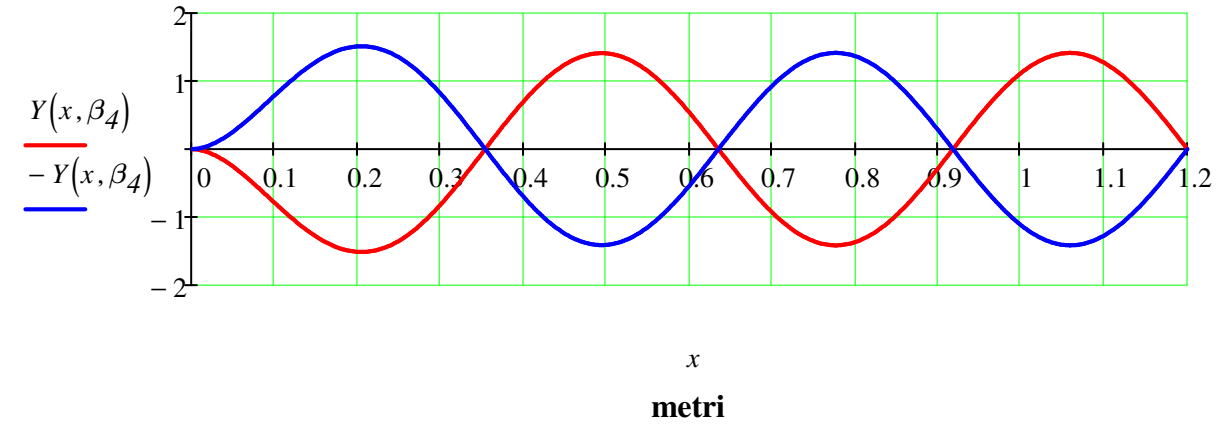


$$f_4 = 379.711 \cdot \text{Hz}$$

Mode shape (3)

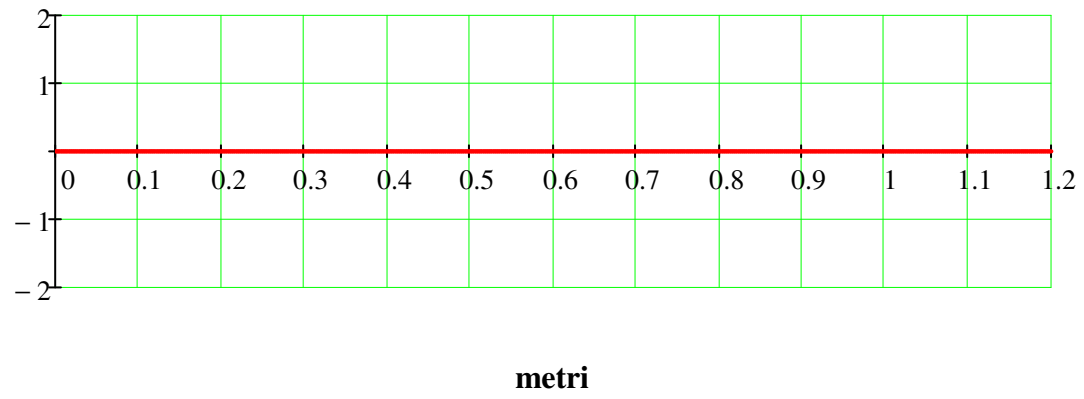


Mode shape (4)

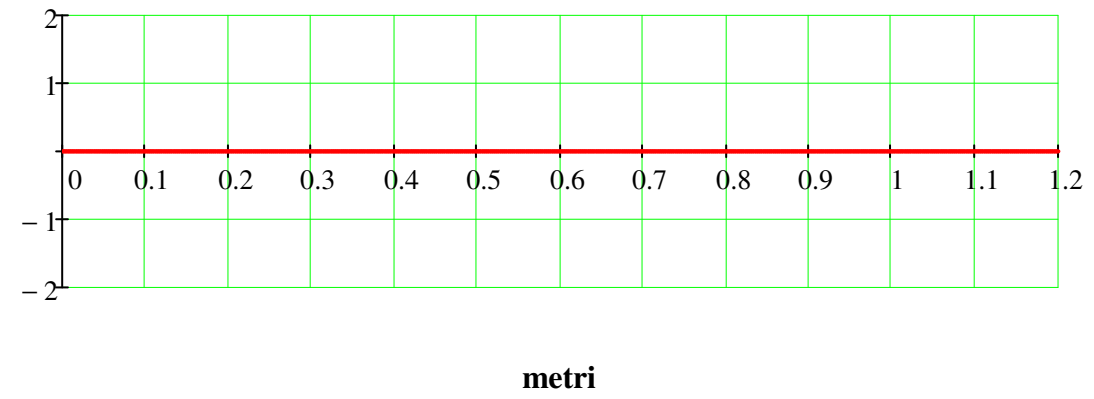


Animazioni delle deformate (Usare il menù: Strumenti - Animazione - Registra)

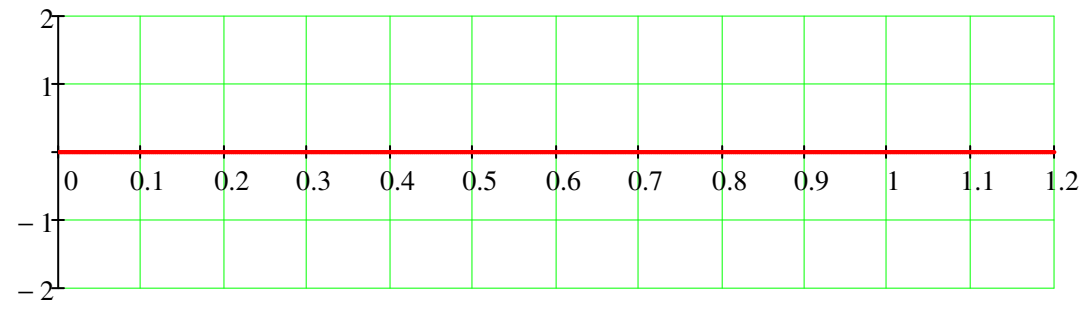
Mode shape (1)



Mode shape (2)

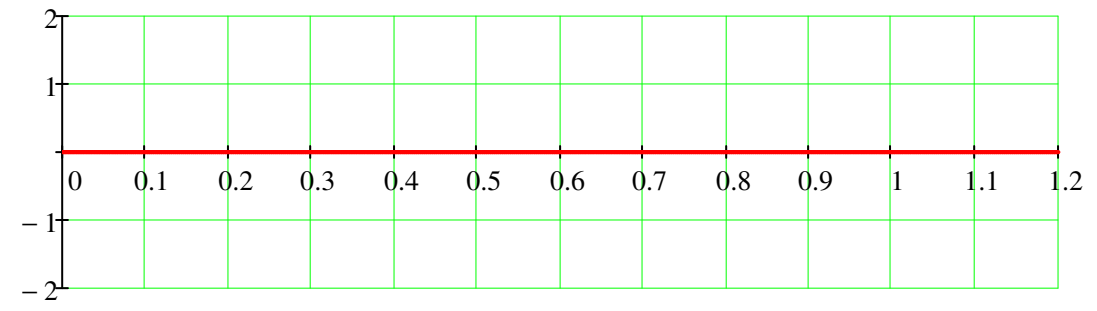


Mode shape (3)



metri

Mode shape (4)



metri