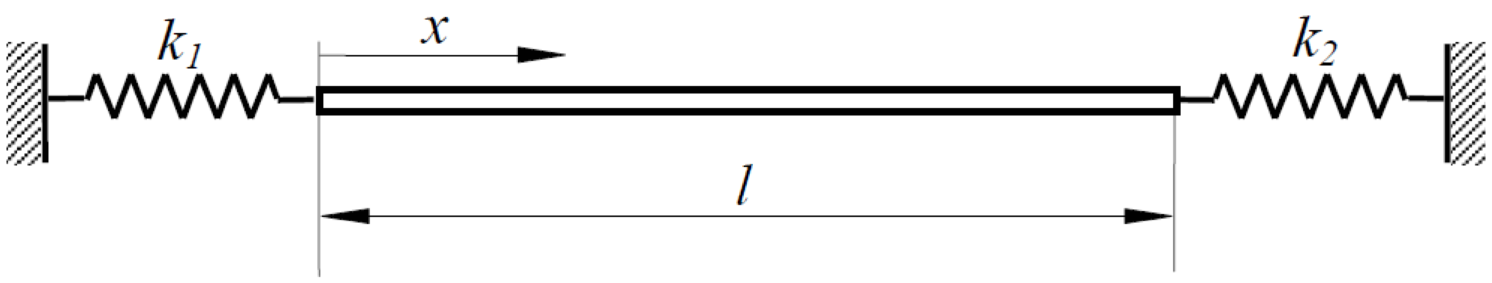


$f(t) := 1$

$$u(x,t) := \left(A\cdot cos\left(\frac{\omega\cdot x}{c}\right) + B\cdot sin\left(\frac{\omega\cdot x}{c}\right)\right)\cdot f(t)$$

Soluzione dell'eq. di D' Alembert

$$\frac{\partial}{\partial x}u(x,t)\ collect,\left(\frac{\omega}{c}\right) \rightarrow \left(B\cdot cos\left(\frac{\omega}{c}\cdot x\right) - A\cdot sin\left(\frac{\omega}{c}\cdot x\right)\right)\cdot\frac{\omega}{c}$$

$$N(x,t) := E\cdot A_s\cdot\frac{\partial}{\partial x}u(x,t)$$

$$N(0,t) - k_1\cdot u(0,t)\ collect,A,B \rightarrow (-k_1)\cdot A + \frac{A_s\cdot E\cdot\omega}{c}\cdot B$$

$$N(l,t) + k_2\cdot u(l,t)\ collect,A,B \rightarrow \left(k_2\cdot cos\left(\frac{\omega\cdot l}{c}\right) - \frac{A_s\cdot E\cdot\omega\cdot sin\left(\frac{\omega\cdot l}{c}\right)}{c}\right)\cdot A + \left(k_2\cdot sin\left(\frac{\omega\cdot l}{c}\right) + \frac{A_s\cdot E\cdot\omega\cdot cos\left(\frac{\omega\cdot l}{c}\right)}{c}\right)\cdot B$$

$$\beta = \frac{\omega\cdot l}{c} \qquad \frac{\omega}{c} = \frac{\beta}{l}$$

$$f_{11}(\beta) := -k_1$$

$$f_{12}(\beta) := \frac{E\cdot A_s}{l}\cdot\beta$$

$$f_{21}(\beta) := \left(k_2\cdot cos(\beta) - \frac{E\cdot A_s}{l}\cdot\beta\cdot sin(\beta)\right)$$

$$f_{22}(\beta) := k_2\cdot sin(\beta) + \frac{E\cdot A_s}{l}\cdot\beta\cdot cos(\beta)$$

$$\Delta(\beta) := \begin{pmatrix} f_{11}(\beta) & f_{12}(\beta) \\ f_{21}(\beta) & f_{22}(\beta) \end{pmatrix}$$

$$\Delta(\beta) \rightarrow \begin{pmatrix} -k_1 & \dfrac{A_s \cdot E \cdot \beta}{l} \\ k_2 \cdot cos(\beta) - \dfrac{A_s \cdot E \cdot \beta \cdot sin(\beta)}{l} & k_2 \cdot sin(\beta) + \dfrac{A_s \cdot E \cdot \beta \cdot cos(\beta)}{l} \end{pmatrix}$$

$$\left|\Delta(\beta)\right| \; simplify \; \rightarrow \frac{{A_s}^2 \cdot E^2 \cdot \beta^2 \cdot sin(\beta)}{l^2} - k_1 \cdot k_2 \cdot sin(\beta) - \frac{A_s \cdot E \cdot \beta \cdot k_1 \cdot cos(\beta)}{l} - \frac{A_s \cdot E \cdot \beta \cdot k_2 \cdot cos(\beta)}{l}$$

Pongo per
semplicità

$$a = \frac{E \cdot A_s}{l}$$

$$f_{11}(\beta) := -k_1$$

$$f_{12}(\beta) := a \cdot \beta$$

$$f_{21}(\beta) := \left(k_2 \cdot cos(\beta) - a \cdot \beta \cdot sin(\beta)\right)$$

$$f_{22}(\beta) := k_2 \cdot sin(\beta) + a \cdot \beta \cdot cos(\beta)$$

$$\Delta(\beta) := \begin{pmatrix} f_{11}(\beta) & f_{12}(\beta) \\ f_{21}(\beta) & f_{22}(\beta) \end{pmatrix}$$

$$\Delta(\beta) \rightarrow \begin{pmatrix} -k_1 & a\cdot\beta \\ k_2\cdot cos(\beta) - a\cdot\beta\cdot sin(\beta) & k_2\cdot sin(\beta) + a\cdot\beta\cdot cos(\beta) \end{pmatrix}$$

$$\left|\Delta(\beta)\right| \; simplify \; \rightarrow a^2\cdot\beta^2\cdot sin(\beta) - k_1\cdot k_2\cdot sin(\beta) - a\cdot\beta\cdot k_1\cdot cos(\beta) - a\cdot\beta\cdot k_2\cdot cos(\beta)$$

$$\left|\Delta(\beta)\right| \; \begin{vmatrix} simplify \\ collect, sin(\beta), cos(\beta) \end{matrix} \rightarrow \left(a^2\cdot\beta^2 - k_1\cdot k_2\right)\cdot sin(\beta) + \left(-a\cdot\beta\cdot k_1 - a\cdot\beta\cdot k_2\right)\cdot cos(\beta)$$

$$\left(a^2\cdot\beta^2 - k_1\cdot k_2\right)\cdot sin(\beta) + \left(-a\cdot\beta\cdot k_1 - a\cdot\beta\cdot k_2\right)\cdot cos(\beta) = 0$$

Uguagliando a zero il determinante si ha:

$$tan(\beta) = -\left[\frac{a\cdot\beta\cdot\left(k_1 + k_2\right)}{k_1\cdot k_2 - a^2\cdot\beta^2}\right]$$

oppure

$$tan(\beta) = -\left[\frac{a\cdot\beta\cdot\left(1 + \frac{k_2}{k_1}\right)}{k_2 - \frac{a^2\cdot\beta^2}{k_1}}\right]$$

$E := 206000\cdot 10^6 = 2.06 \times 10^{11}$

$l := 1.5$

$\rho := 7800$

$d := 10\cdot 10^{-3} = 0.01$

$A_{sez} := \frac{\pi\cdot d^2}{4} = 7.854 \times 10^{-5}$

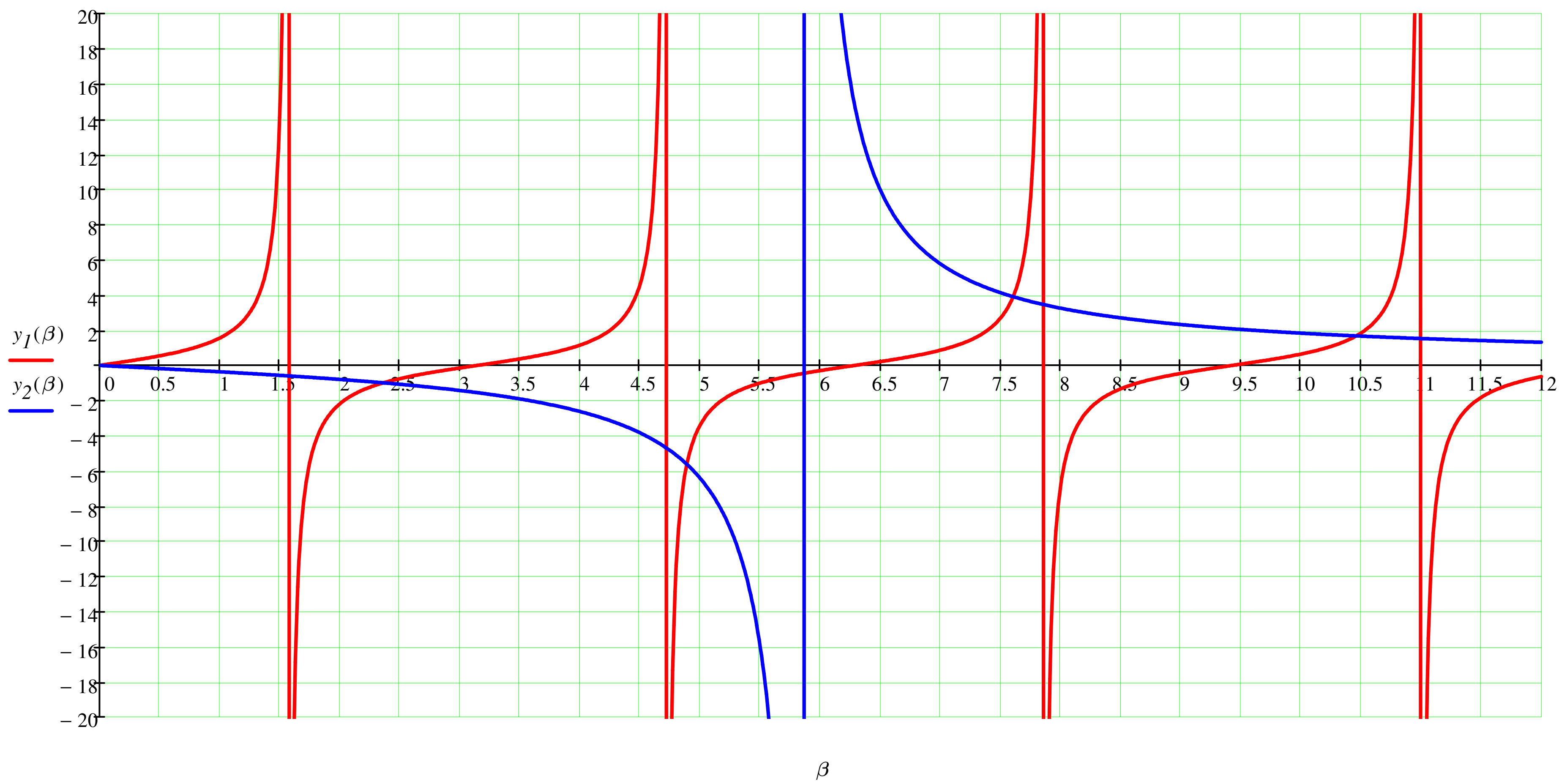
$a := \frac{E\cdot A_{sez}}{l} = 1.079 \times 10^7$

$k_1 := 5 \cdot 10^7$ $k_2 := 8 \cdot 10^7$

$c := \sqrt{\frac{E}{\rho}} = 5139.091$

$y_1(\beta) := tan(\beta)$ $y_2(\beta) := -\left[\frac{a \cdot \beta \cdot \left(1 + \frac{k_2}{k_1}\right)}{k_2 - \frac{a^2 \cdot \beta^2}{k_1}}\right]$

$\beta := 0, 0.01 .. 12$ $FS := 20$



$F(\beta) := y_1(\beta) - y_2(\beta)$

$\beta := 2$

$\beta_1 := root(F(\beta), \beta) = 2.362$

$\beta := 5$

$\beta_2 := root(F(\beta), \beta) = 4.888$

$\beta := 7$

$\beta_3 := root(F(\beta), \beta) = 7.604$

$\beta := 10$

$\beta_4 := root(F(\beta), \beta) = 10.459$

	rad/s	Hz
$\beta_1 = 2.362$	$\omega_1 := \frac{c}{l} \cdot \beta_1 = 8092.458$	$f_1 := \frac{\omega_1}{2 \cdot \pi} = 1287.955$
$\beta_2 = 4.888$	$\omega_2 := \frac{c}{l} \cdot \beta_2 = 16748.274$	$f_2 := \frac{\omega_2}{2 \cdot \pi} = 2665.571$
$\beta_3 = 7.604$	$\omega_3 := \frac{c}{l} \cdot \beta_3 = 26050.534$	$f_3 := \frac{\omega_3}{2 \cdot \pi} = 4146.071$
$\beta_4 = 10.459$	$\omega_4 := \frac{c}{l} \cdot \beta_4 = 35832.569$	$f_4 := \frac{\omega_4}{2 \cdot \pi} = 5702.93$

Calcolo diretto del determinante in forma numerica, senza sviluppi di calcolo simbolico

$f_{11}(\beta) := -k_1$

$f_{12}(\beta) := a \cdot \beta$

$f_{21}(\beta) := \left(k_2 \cdot cos(\beta) - a \cdot \beta \cdot sin(\beta)\right)$

$f_{22}(\beta) := k_2 \cdot sin(\beta) + a \cdot \beta \cdot cos(\beta)$

$$\Delta(\beta) := \begin{pmatrix} f_{11}(\beta) & f_{12}(\beta) \\ f_{21}(\beta) & f_{22}(\beta) \end{pmatrix}$$

$det(\beta) := \left|\Delta(\beta)\right|$

$\beta := 2$ $root(det(\beta), \beta) = 2.362$

$\beta := 4$ $root(det(\beta), \beta) = 4.888$

$\beta := 7$ $root(det(\beta), \beta) = 7.604$

$\beta := 10$ $root(det(\beta), \beta) = 10.459$

$\beta_1 = 2.362$ $\beta_2 = 4.888$ $\beta_3 = 7.604$ $\beta_4 = 10.459$

$\beta := 0, 0.01 .. 12$