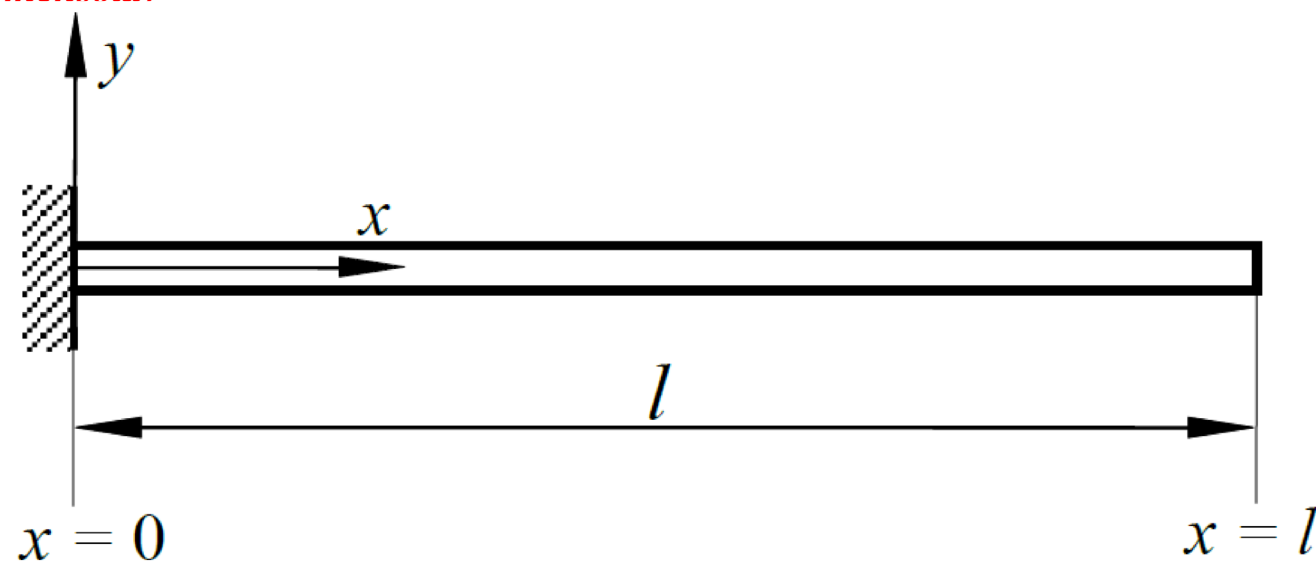


**Trave incastro - estremo libero
(mensola)**



$$Y(x) := A \cdot \cos(\beta \cdot x) + B \cdot \sin(\beta \cdot x) + C \cdot \cosh(\beta \cdot x) + D \cdot \sinh(\beta \cdot x)$$

$$Y'(x) := \frac{d}{dx} Y(x) \quad Y'(x) \text{ collect } ,\beta \rightarrow (B \cdot \cos(\beta \cdot x) - A \cdot \sin(\beta \cdot x) + C \cdot \sinh(\beta \cdot x) + D \cdot \cosh(\beta \cdot x)) \cdot \beta$$

$$Y''(x) := \frac{d^2}{dx^2} Y(x) \quad Y''(x) \text{ collect } ,\beta \rightarrow (C \cdot \cosh(\beta \cdot x) - B \cdot \sin(\beta \cdot x) - A \cdot \cos(\beta \cdot x) + D \cdot \sinh(\beta \cdot x)) \cdot \beta^2$$

$$Y'''(x) := \frac{d^3}{dx^3} Y(x) \quad Y'''(x) \text{ collect } ,\beta \rightarrow (A \cdot \sin(\beta \cdot x) - B \cdot \cos(\beta \cdot x) + C \cdot \sinh(\beta \cdot x) + D \cdot \cosh(\beta \cdot x)) \cdot \beta^3$$

Condizioni al contorno:

in $x = 0$

$$Y(0) \rightarrow A + C \quad \mathbf{A + C = 0}$$

$$Y'(0) \text{ collect } ,\beta \rightarrow (B + D) \cdot \beta \quad \mathbf{B + D = 0}$$

in $x = l$

$$Y''(l) \text{ collect } ,\beta \rightarrow (C \cdot \cosh(\beta \cdot l) - B \cdot \sin(\beta \cdot l) - A \cdot \cos(\beta \cdot l) + D \cdot \sinh(\beta \cdot l)) \cdot \beta^2$$

$$C \cdot \cosh(\beta \cdot l) - B \cdot \sin(\beta \cdot l) - A \cdot \cos(\beta \cdot l) + D \cdot \sinh(\beta \cdot l) = 0$$

$$Y'''(l) \text{ collect } ,\beta \rightarrow (A \cdot \sin(\beta \cdot l) - B \cdot \cos(\beta \cdot l) + C \cdot \sinh(\beta \cdot l) + D \cdot \cosh(\beta \cdot l)) \cdot \beta^3$$

$$A \cdot \sin(\beta \cdot l) - B \cdot \cos(\beta \cdot l) + C \cdot \sinh(\beta \cdot l) + D \cdot \cosh(\beta \cdot l) = 0$$

Riassunto delle condizioni al contorno (che formano un sistema lineare)

$$A + C = 0$$

$$B + D = 0$$

$$C \cdot \cosh(\beta \cdot l) - B \cdot \sin(\beta \cdot l) - A \cdot \cos(\beta \cdot l) + D \cdot \sinh(\beta \cdot l) = 0$$

$$A \cdot \sin(\beta \cdot l) - B \cdot \cos(\beta \cdot l) + C \cdot \sinh(\beta \cdot l) + D \cdot \cosh(\beta \cdot l) = 0$$

Poniamo per semplicità: $\alpha = \beta l$ e otteniamo:

$$A + C = 0$$

$$B + D = 0$$

$$C \cdot \cosh(\alpha) - B \cdot \sin(\alpha) - A \cdot \cos(\alpha) + D \cdot \sinh(\alpha) = 0$$

$$A \cdot \sin(\alpha) - B \cdot \cos(\alpha) + C \cdot \sinh(\alpha) + D \cdot \cosh(\alpha) = 0$$

Le quattro condizioni al contorno si possono scrivere in forma matriciale, come segue

$$\Delta(\alpha) \cdot \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Delta(\alpha) := \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -\cos(\alpha) & -\sin(\alpha) & \cosh(\alpha) & \sinh(\alpha) \\ \sin(\alpha) & -\cos(\alpha) & \sinh(\alpha) & \cosh(\alpha) \end{pmatrix}$$

Infatti, il prodotto matriciale seguente, fornisce:

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -\cos(\alpha) & -\sin(\alpha) & \cosh(\alpha) & \sinh(\alpha) \\ \sin(\alpha) & -\cos(\alpha) & \sinh(\alpha) & \cosh(\alpha) \end{pmatrix} \cdot \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} \rightarrow \begin{pmatrix} A + C \\ B + D \\ C \cdot \cosh(\alpha) - B \cdot \sin(\alpha) - A \cdot \cos(\alpha) + D \cdot \sinh(\alpha) \\ A \cdot \sin(\alpha) - B \cdot \cos(\alpha) + C \cdot \sinh(\alpha) + D \cdot \cosh(\alpha) \end{pmatrix}$$

Si calcola ora il determinante della matrice $\Delta(\alpha)$ e lo si pone uguale a zero, per ricavare l'equazione caratteristica:

$$\det(\alpha) := |\Delta(\alpha)|$$

$$\det(\alpha) \text{ simplify} \rightarrow 2 \cdot \cosh(\alpha) \cdot \cos(\alpha) + 2$$

Rielaborando l'equazione, si ricava:

$$1 + \cosh(\alpha) \cdot \cos(\alpha) = 0 \quad \text{Equazione caratteristica (o equazione delle frequenze)}$$

Cerchiamo le soluzioni dell'equazione caratteristica per via grafica:

$$F(\alpha) := 1 + \cosh(\alpha) \cdot \cos(\alpha)$$

$$\alpha := 0, 0.01 \dots 5$$



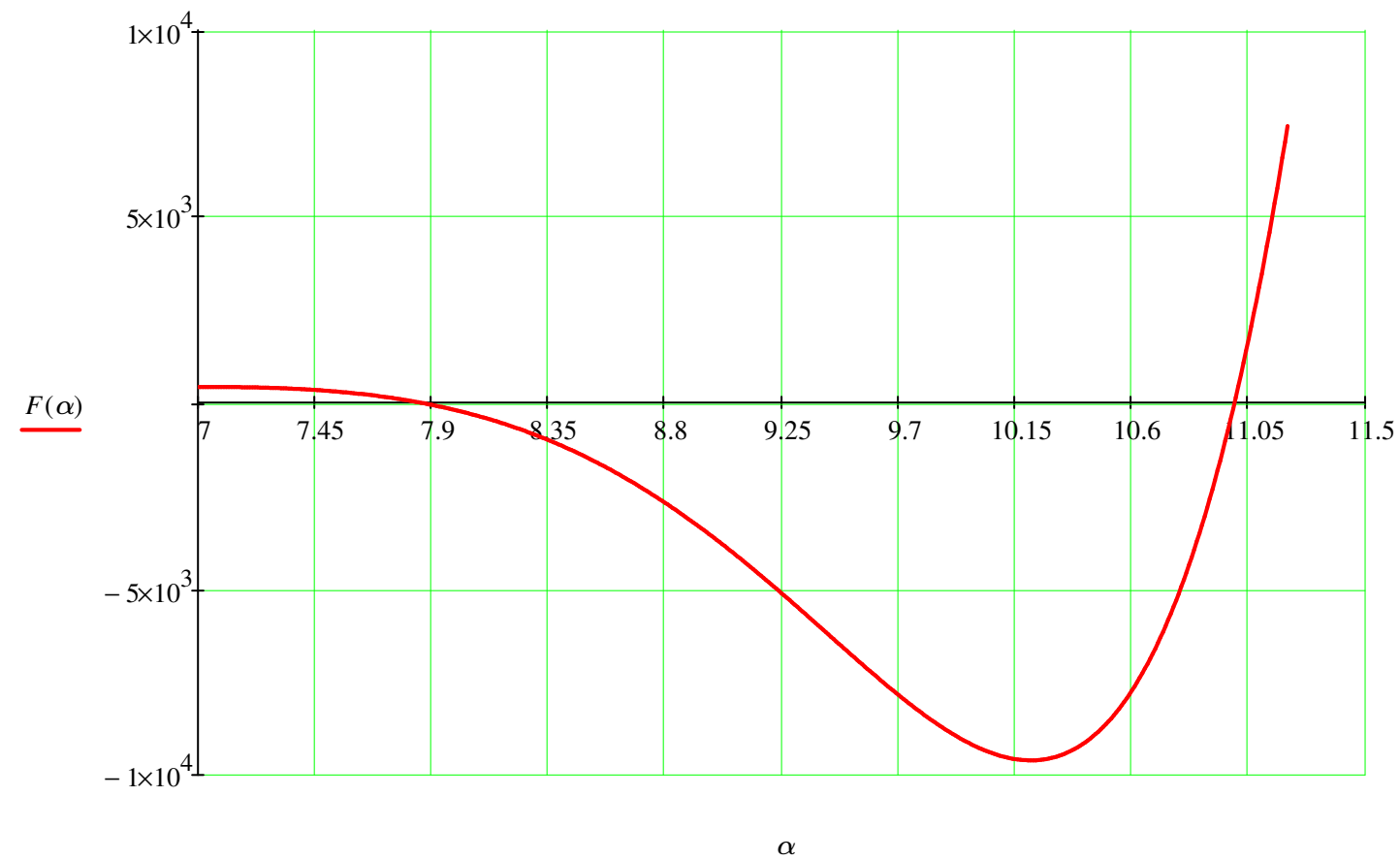
$$\alpha := 2$$

$$\alpha_1 := \text{root}(F(\alpha), \alpha) = 1.875104$$

$$\alpha := 4$$

$$\alpha_2 := \text{root}(F(\alpha), \alpha) = 4.694091$$

$$\alpha := 7, 7.01 \dots 11.2$$



$$\alpha := 8$$

$$\alpha_3 := \text{root}(F(\alpha), \alpha) = 7.854757$$

$$\alpha := 11$$

$$\alpha_4 := \text{root}(F(\alpha), \alpha) = 10.995541$$

Lunghezza

$$l_w := 1200 \cdot \text{mm} = 1.2 \text{ m}$$

Materiale: acciaio

$$\rho := 7800 \cdot \frac{\text{kg}}{\text{m}^3}$$

$$E := 206000 \cdot \text{MPa} = 2.06 \times 10^{11} \text{ Pa}$$

Sezione circolare di diametro d:

$$d := 15 \cdot \text{mm} = 0.015 \text{ m}$$

$$J_w := \frac{\pi \cdot d^4}{64} = 2.485 \times 10^{-9} \text{ m}^4$$

$$A_{sez} := \frac{\pi \cdot d^2}{4} = 1.767 \times 10^{-4} m^2$$

$$c := \sqrt{\frac{E \cdot J}{\rho \cdot A_{sez}}} = 19.272 \frac{m^2}{s}$$

$$\alpha = \beta \cdot l$$

$$\beta = \frac{\alpha}{l}$$

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} = \begin{pmatrix} 1.875104 \\ 4.694091 \\ 7.854757 \\ 10.995541 \end{pmatrix}$$

$$\begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix} := \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} \cdot \frac{1}{l} = \begin{pmatrix} 1.562587 \\ 3.911743 \\ 6.545631 \\ 9.162951 \end{pmatrix} \frac{1}{m}$$

$$\omega = \beta^2 \cdot c$$

$$\omega_1 := (\beta_1)^2 \cdot c = 47.055 \cdot \frac{rad}{s}$$

$$f_1 := \frac{\omega_1}{2 \cdot \pi} = 7.489 \cdot Hz$$

$$\omega_2 := (\beta_2)^2 \cdot c = 294.889 \cdot \frac{rad}{s}$$

$$f_2 := \frac{\omega_2}{2 \cdot \pi} = 46.933 \cdot Hz$$

$$\omega_3 := (\beta_3)^2 \cdot c = 825.697 \cdot \frac{rad}{s}$$

$$f_3 := \frac{\omega_3}{2 \cdot \pi} = 131.414 \cdot Hz$$

$$\omega_4 := (\beta_4)^2 \cdot c = 1618.036 \cdot \frac{rad}{s}$$

$$f_4 := \frac{\omega_4}{2 \cdot \pi} = 257.518 \cdot Hz$$

$$l := l$$

Given

$$\underline{A + C = 0}$$

$$B + D = 0$$

$$C \cdot \cosh(\beta \cdot l) - B \cdot \sin(\beta \cdot l) - A \cdot \cos(\beta \cdot l) + D \cdot \sinh(\beta \cdot l) = 0$$

$$\text{Find}(B, C, D) \rightarrow \begin{pmatrix} \frac{A \cdot \cos(\beta \cdot l) + A \cdot \cosh(\beta \cdot l)}{\sinh(\beta \cdot l) + \sin(\beta \cdot l)} \\ -A \\ \frac{A \cdot \cos(\beta \cdot l) + A \cdot \cosh(\beta \cdot l)}{\sinh(\beta \cdot l) + \sin(\beta \cdot l)} \end{pmatrix}$$

$$A := 1$$

$$B(\beta) := \frac{A \cdot \cos(\beta \cdot l) + A \cdot \cosh(\beta \cdot l)}{\sinh(\beta \cdot l) + \sin(\beta \cdot l)}$$

$$C := -A$$

$$D(\beta) := \frac{A \cdot \cos(\beta \cdot l) + A \cdot \cosh(\beta \cdot l)}{\sinh(\beta \cdot l) + \sin(\beta \cdot l)}$$

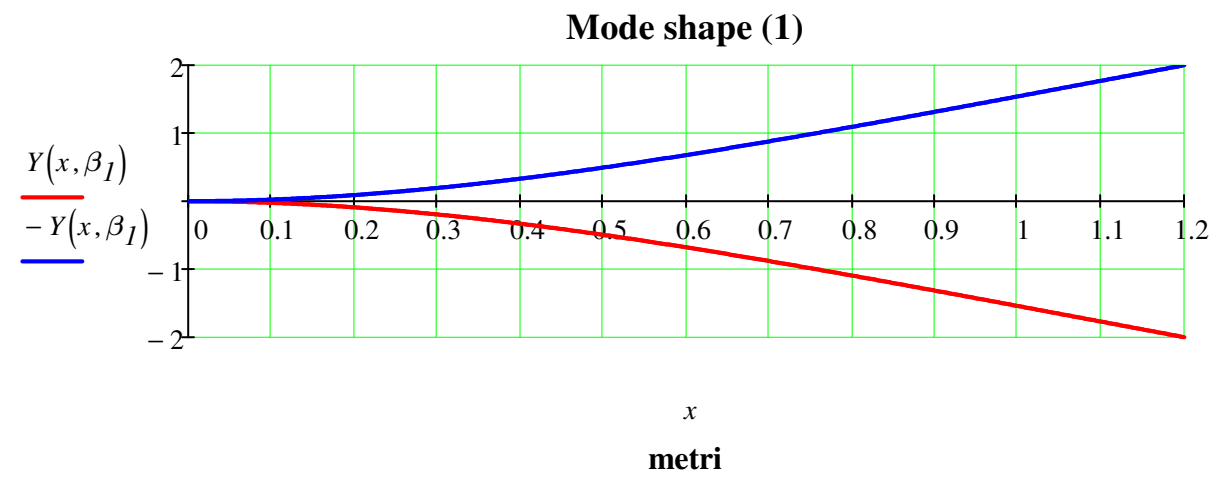
$$Y(x, \beta) := A \cdot \cos(\beta \cdot x) + B(\beta) \cdot \sin(\beta \cdot x) + C \cdot \cosh(\beta \cdot x) + D(\beta) \cdot \sinh(\beta \cdot x)$$

$$x := 0, \frac{l}{500} .. l$$

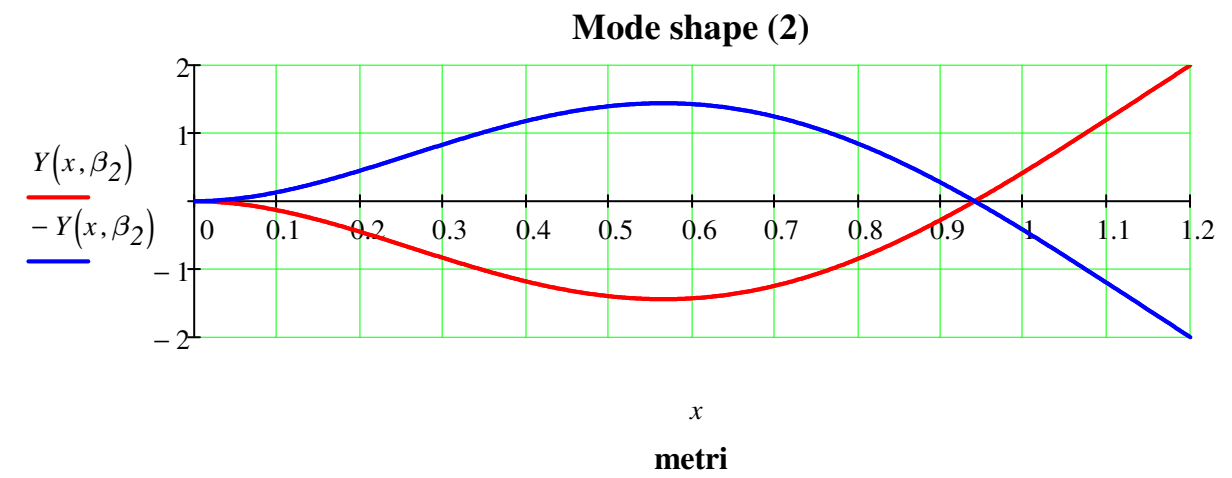
$FS := 2$

Fondo scala per i grafici

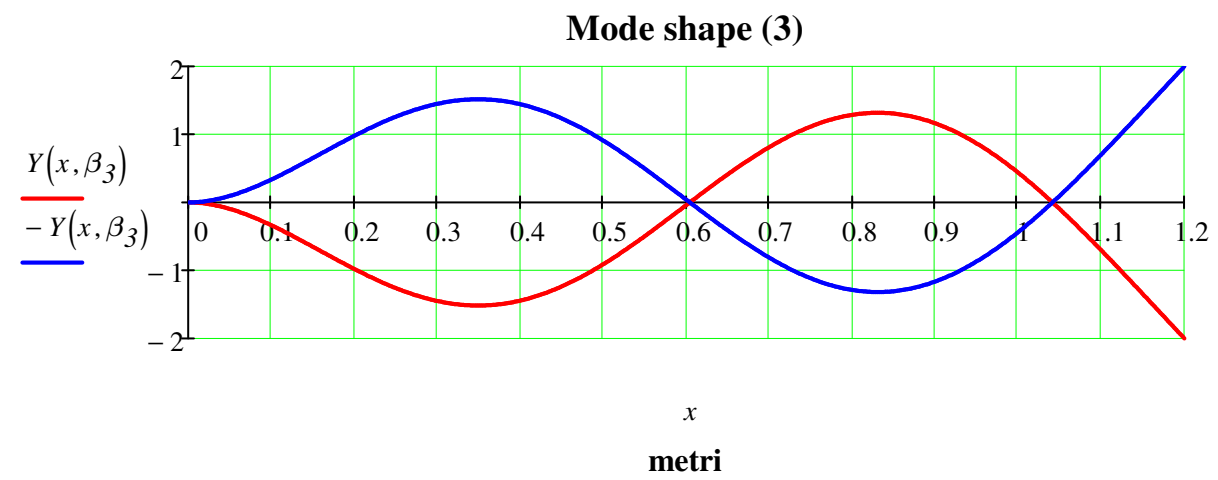
$f_1 = 7.489 \cdot Hz$



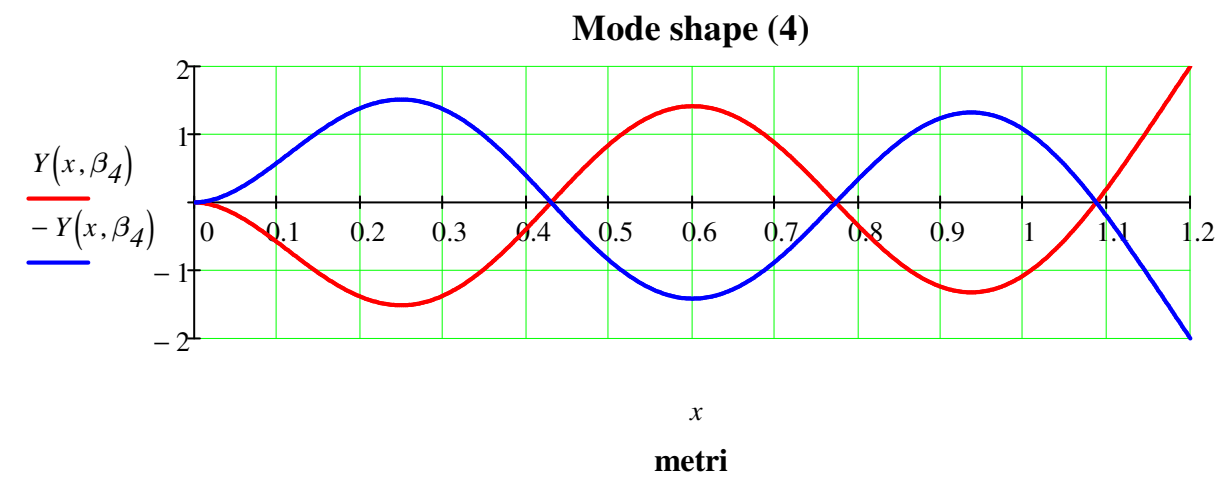
$f_2 = 46.933 \cdot Hz$



$f_3 = 131.414 \cdot Hz$

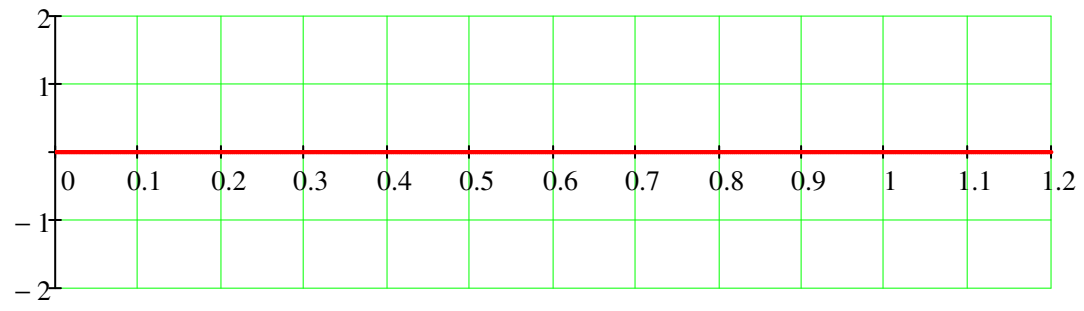


$f_4 = 257.518 \cdot Hz$



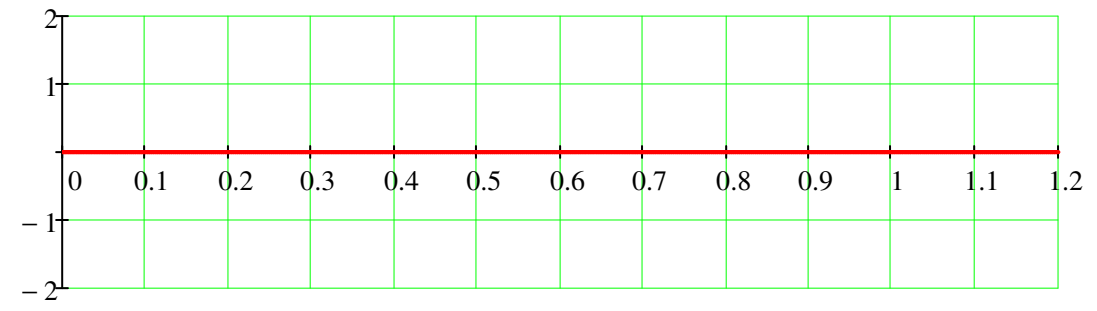
Animazioni delle deformate (Usare il menù: Strumenti - Animazione - Registra)

Mode shape (1)



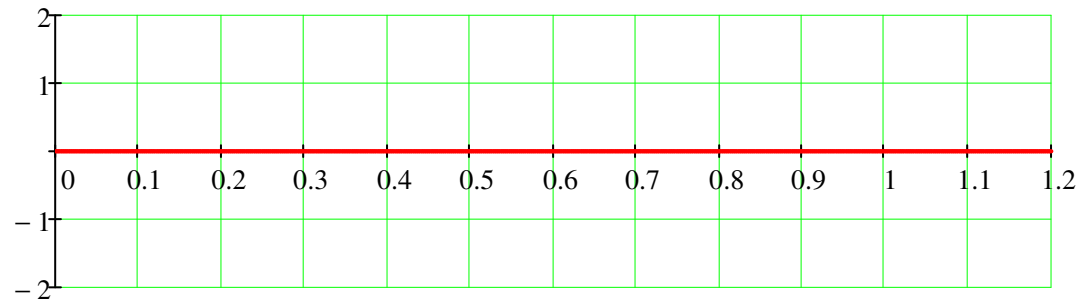
metri

Mode shape (2)



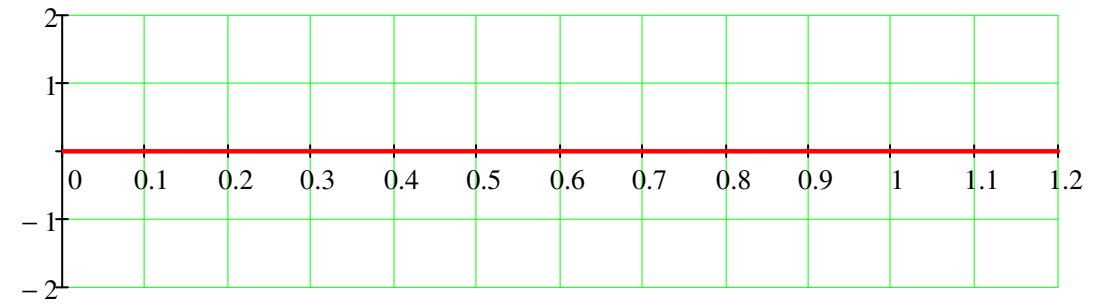
metri

Mode shape (3)



metri

Mode shape (4)



metri