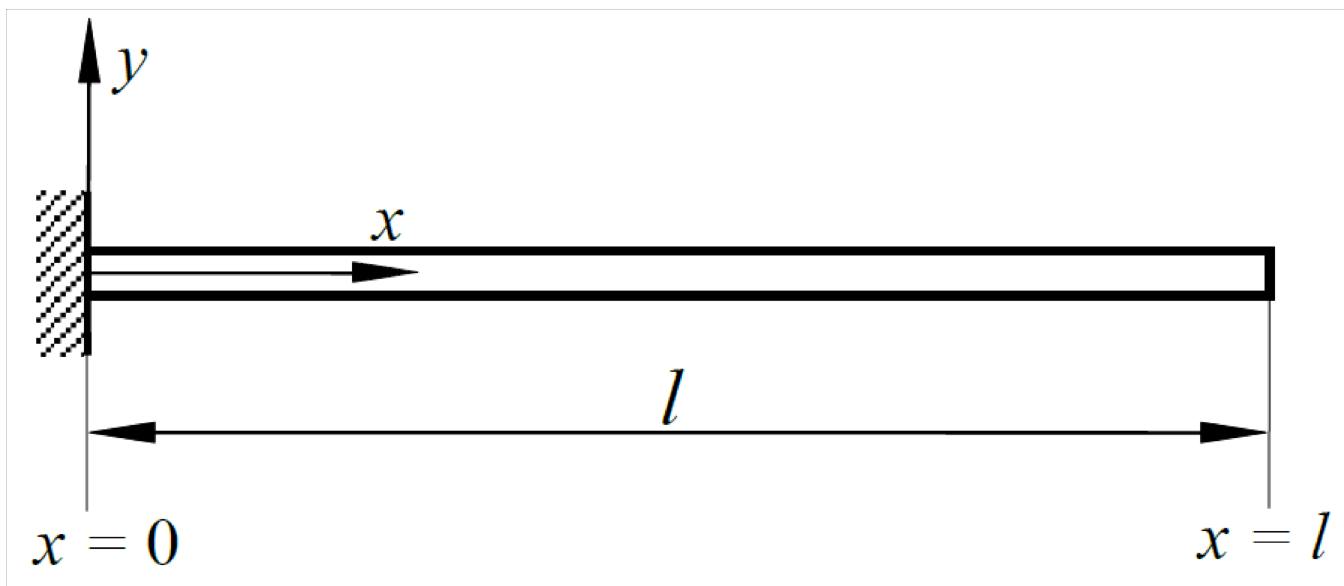
Trave incastro - estremo libero (mensola)



$Y(x) := A\cdot cos(\beta\cdot x) + B\cdot sin(\beta\cdot x) + C\cdot cosh(\beta\cdot x) + D\cdot sinh(\beta\cdot x)$

$Y'(x) := \frac{d}{dx}Y(x) \qquad Y'(x)\ collect, \beta \ \rightarrow (B\cdot cos(\beta\cdot x) - A\cdot sin(\beta\cdot x) + C\cdot sinh(\beta\cdot x) + D\cdot cosh(\beta\cdot x))\cdot\beta$

$Y''(x) := \frac{d^2}{dx^2}Y(x) \qquad Y''(x)\ collect, \beta \ \rightarrow (C\cdot cosh(\beta\cdot x) - B\cdot sin(\beta\cdot x) - A\cdot cos(\beta\cdot x) + D\cdot sinh(\beta\cdot x))\cdot\beta^2$

$Y'''(x) := \frac{d^3}{dx^3}Y(x) \qquad Y'''(x)\ collect, \beta \ \rightarrow (A\cdot sin(\beta\cdot x) - B\cdot cos(\beta\cdot x) + C\cdot sinh(\beta\cdot x) + D\cdot cosh(\beta\cdot x))\cdot\beta^3$

Condizioni al contorno:

in x = 0

$Y(0) \rightarrow A + C \qquad A + C = 0$

$Y'(0)\ collect, \beta \ \rightarrow (B + D)\cdot\beta \qquad B + D = 0$

in x = l

$Y''(l)\ collect, \beta \ \rightarrow (C\cdot cosh(\beta\cdot l) - B\cdot sin(\beta\cdot l) - A\cdot cos(\beta\cdot l) + D\cdot sinh(\beta\cdot l))\cdot\beta^2 \qquad C\cdot cosh(\beta\cdot l) - B\cdot sin(\beta\cdot l) - A\cdot cos(\beta\cdot l) + D\cdot sinh(\beta\cdot l) = 0$

$Y'''(l)\ collect, \beta \ \rightarrow (A\cdot sin(\beta\cdot l) - B\cdot cos(\beta\cdot l) + C\cdot sinh(\beta\cdot l) + D\cdot cosh(\beta\cdot l))\cdot\beta^3 \qquad A\cdot sin(\beta\cdot l) - B\cdot cos(\beta\cdot l) + C\cdot sinh(\beta\cdot l) + D\cdot cosh(\beta\cdot l) = 0$

Riassunto delle condizioni al contorno (che formano un sistema lineare)

$A + C = 0$

$B + D = 0$

$C\cdot cosh(\beta\cdot l) - B\cdot sin(\beta\cdot l) - A\cdot cos(\beta\cdot l) + D\cdot sinh(\beta\cdot l) = 0$

$A\cdot sin(\beta\cdot l) - B\cdot cos(\beta\cdot l) + C\cdot sinh(\beta\cdot l) + D\cdot cosh(\beta\cdot l) = 0$

Poniamo per semplicità: α = β l e otteniamo:

$A + C = 0$

$B + D = 0$

$C\cdot cosh(\alpha) - B\cdot sin(\alpha) - A\cdot cos(\alpha) + D\cdot sinh(\alpha) = 0$

$A\cdot sin(\alpha) - B\cdot cos(\alpha) + C\cdot sinh(\alpha) + D\cdot cosh(\alpha) = 0$

Le quattro condizioni al contorno si possono scrivere in forma matriciale, come segue

$$\Delta(\alpha)\cdot\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\Delta(\alpha) := \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -cos(\alpha) & -sin(\alpha) & cosh(\alpha) & sinh(\alpha) \\ sin(\alpha) & -cos(\alpha) & sinh(\alpha) & cosh(\alpha) \end{pmatrix}$$

Infatti, il prodotto matriciale seguente, fornisce:

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -cos(\alpha) & -sin(\alpha) & cosh(\alpha) & sinh(\alpha) \\ sin(\alpha) & -cos(\alpha) & sinh(\alpha) & cosh(\alpha) \end{pmatrix}\cdot\begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} \rightarrow \begin{pmatrix} A + C \\ B + D \\ C\cdot cosh(\alpha) - B\cdot sin(\alpha) - A\cdot cos(\alpha) + D\cdot sinh(\alpha) \\ A\cdot sin(\alpha) - B\cdot cos(\alpha) + C\cdot sinh(\alpha) + D\cdot cosh(\alpha) \end{pmatrix}$$

Si calcola ora il determinante della matrice $\mathbf{\Delta(\alpha)}$ *e lo si pone uguale a zero, per ricavare l'equazione caratteristica:*

$det(\alpha) := \left|\Delta(\alpha)\right|$

$det(\alpha)\ simplify\ \rightarrow 2\cdot cosh(\alpha)\cdot cos(\alpha) + 2$

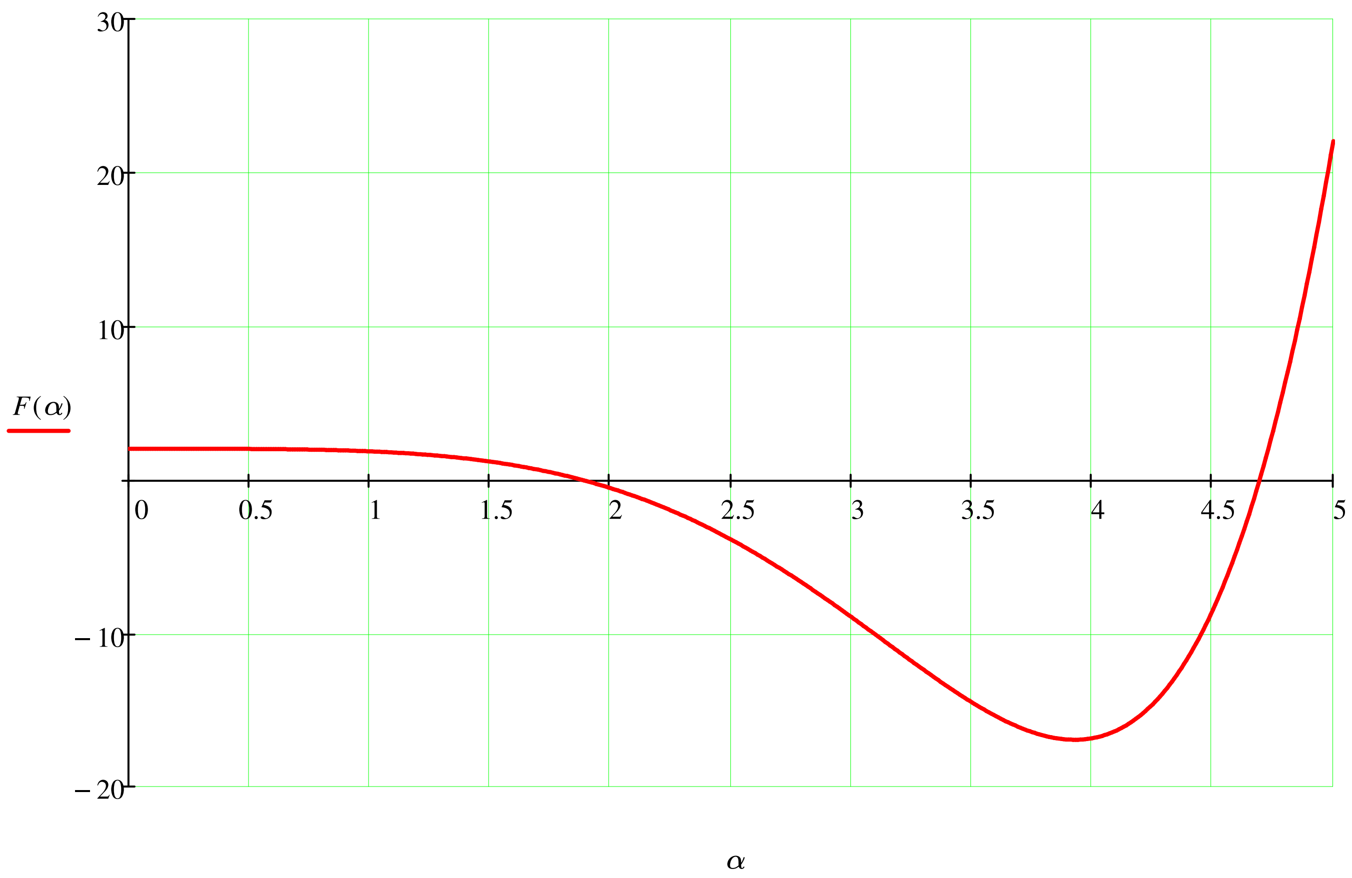
Rielaborando l'equazione, si ricava:

$1 + cosh(\alpha)\cdot cos(\alpha) = 0$ *Equazione caratteristica (o equazione delle frequenze)*

Cerchiamo le soluzioni dell'equazione caratteristica per via grafica:

$F(\alpha) := 1 + cosh(\alpha)\cdot cos(\alpha)$

$\alpha := 0, 0.01 .. 5$



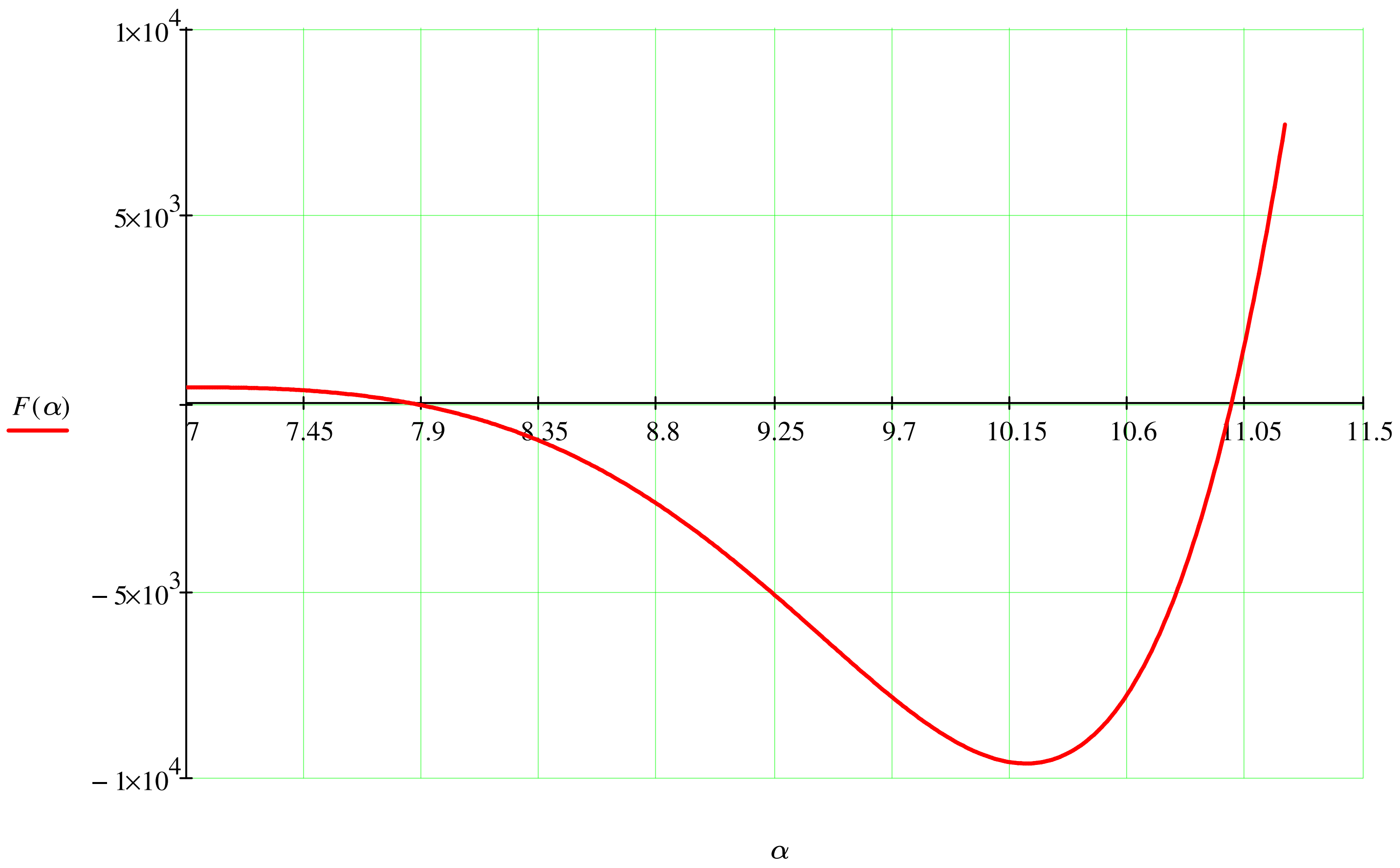
$\alpha := 2$

$\alpha_1 := root(F(\alpha), \alpha) = 1.875104$

$\alpha := 4$

$\alpha_2 := root(F(\alpha), \alpha) = 4.694091$

$\alpha := 7, 7.01 .. 11.2$



$\alpha := 8$

$\alpha_3 := root(F(\alpha), \alpha) = 7.854757$

$\alpha := 11$

$\alpha_4 := root(F(\alpha), \alpha) = 10.995541$

Lunghezza

$L := 1200\cdot mm = 1.2\,m$

Materiale: acciaio

$\rho := 7800\cdot \frac{kg}{m^3}$

$E := 206000\cdot MPa = 2.06 \times 10^{11}\,Pa$

Sezione circolare di diametro d:

$d := 15\cdot mm = 0.015\,m$

$J := \frac{\pi\cdot d^4}{64} = 2.485 \times 10^{-9}\,m^4$

$A_{sez} := \frac{\pi\cdot d^2}{4} = 1.767 \times 10^{-4}\,m^2$

$c := \sqrt{\frac{E\cdot J}{\rho\cdot A_{sez}}} = 19.272\,\frac{m^2}{s}$

$\alpha = \beta \cdot l$ $\quad\quad$ $\beta = \frac{\alpha}{l}$

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} = \begin{pmatrix} 1.875104 \\ 4.694091 \\ 7.854757 \\ 10.995541 \end{pmatrix} \quad\quad \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix} := \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} \cdot \frac{1}{l} = \begin{pmatrix} 1.562587 \\ 3.911743 \\ 6.545631 \\ 9.162951 \end{pmatrix} \frac{1}{m}$$

$\omega = \beta^2 \cdot c$

$\omega_1 := (\beta_1)^2 \cdot c = 47.055 \cdot \frac{rad}{s}$ $\quad\quad$ $f_1 := \frac{\omega_1}{2 \cdot \pi} = 7.489 \cdot Hz$

$\omega_2 := (\beta_2)^2 \cdot c = 294.889 \cdot \frac{rad}{s}$ $\quad\quad$ $f_2 := \frac{\omega_2}{2 \cdot \pi} = 46.933 \cdot Hz$

$\omega_3 := (\beta_3)^2 \cdot c = 825.697 \cdot \frac{rad}{s}$ $\quad\quad$ $f_3 := \frac{\omega_3}{2 \cdot \pi} = 131.414 \cdot Hz$

$\omega_4 := (\beta_4)^2 \cdot c = 1618.036 \cdot \frac{rad}{s}$ $\quad\quad$ $f_4 := \frac{\omega_4}{2 \cdot \pi} = 257.518 \cdot Hz$

$l := l$

Given

$A + C = 0$

$B + D = 0$

$C \cdot cosh(\beta \cdot l) - B \cdot sin(\beta \cdot l) - A \cdot cos(\beta \cdot l) + D \cdot sinh(\beta \cdot l) = 0$

$$Find(B, C, D) \rightarrow \begin{pmatrix} -\frac{A \cdot cos(\beta \cdot l) + A \cdot cosh(\beta \cdot l)}{sinh(\beta \cdot l) + sin(\beta \cdot l)} \\ -A \\ \frac{A \cdot cos(\beta \cdot l) + A \cdot cosh(\beta \cdot l)}{sinh(\beta \cdot l) + sin(\beta \cdot l)} \end{pmatrix}$$

$A := 1$

$B(\beta) := -\frac{A \cdot cos(\beta \cdot l) + A \cdot cosh(\beta \cdot l)}{sinh(\beta \cdot l) + sin(\beta \cdot l)}$

$C := -A$

$D(\beta) := \frac{A \cdot cos(\beta \cdot l) + A \cdot cosh(\beta \cdot l)}{sinh(\beta \cdot l) + sin(\beta \cdot l)}$

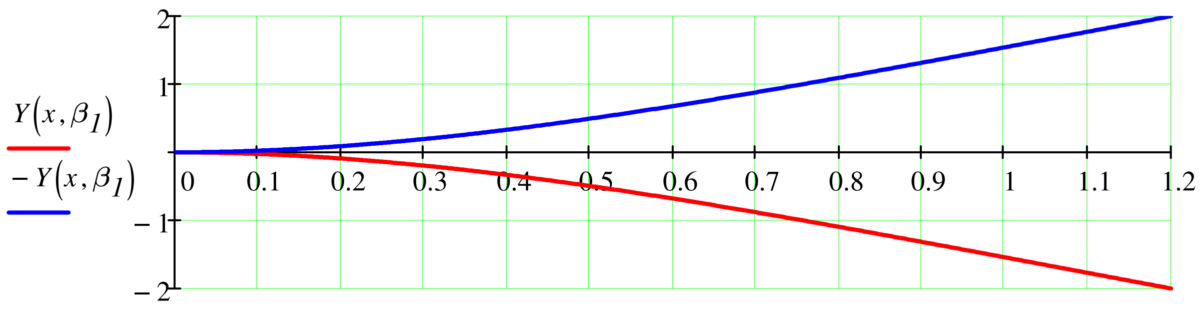
$Y(x, \beta) := A \cdot cos(\beta \cdot x) + B(\beta) \cdot sin(\beta \cdot x) + C \cdot cosh(\beta \cdot x) + D(\beta) \cdot sinh(\beta \cdot x)$

$x := 0, \frac{l}{500} .. l$

$FS := 2$ *Fondo scala per i grafici*

$f_1 = 7.489 \cdot Hz$

Mode shape (1)

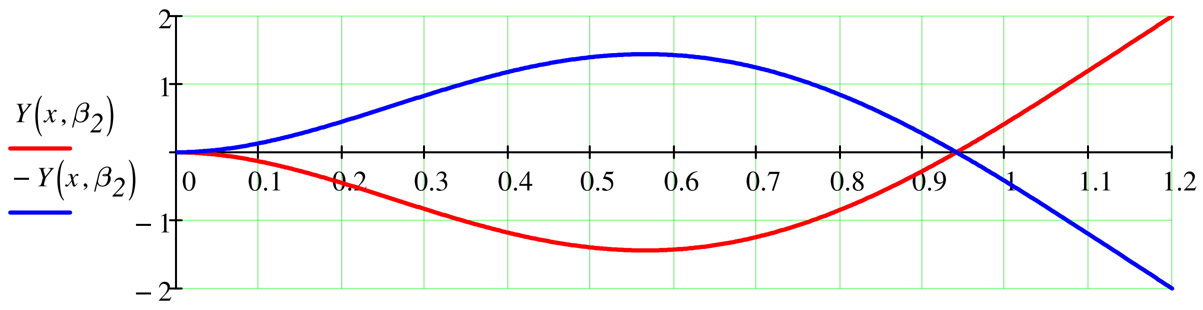


x

metri

$f_2 = 46.933 \cdot Hz$

Mode shape (2)

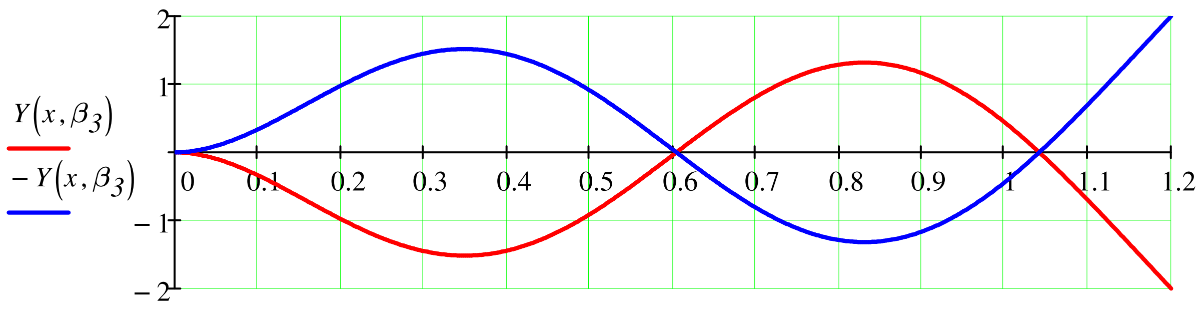


x

metri

$f_3 = 131.414 \cdot Hz$

Mode shape (3)

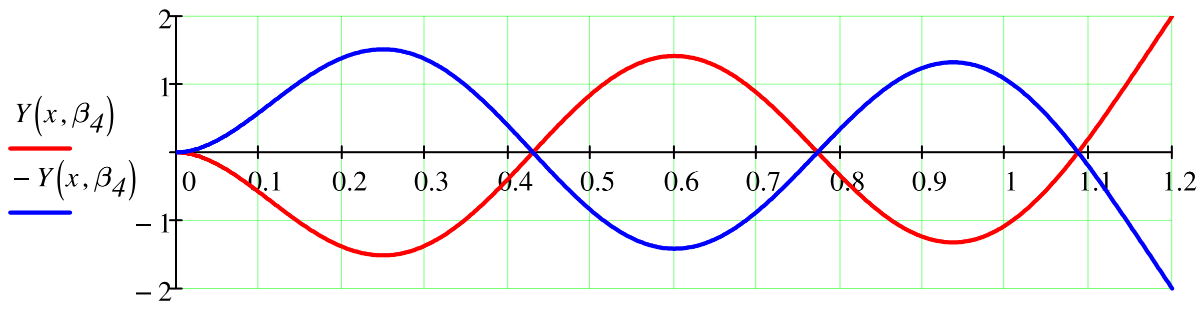


x

metri

$f_4 = 257.518 \cdot Hz$

Mode shape (4)

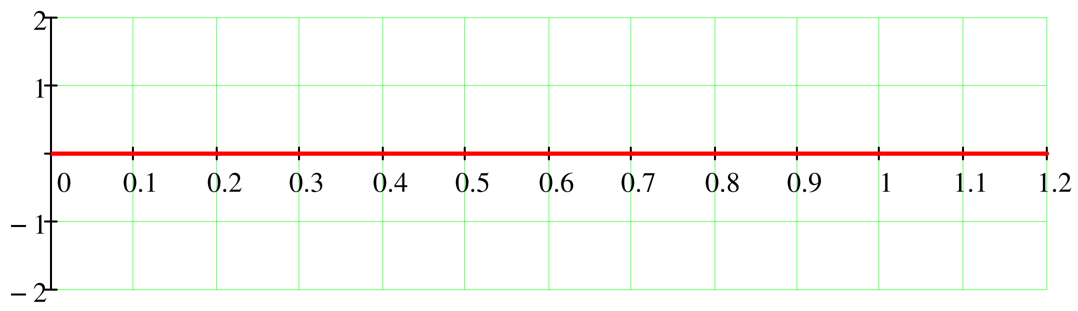


x

metri

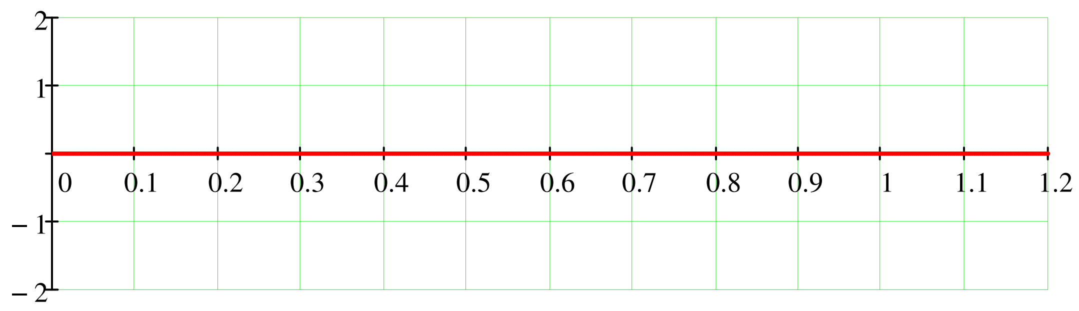
Animazioni delle deformate (Usare il menù: Strumenti - Aminazione - Registra)

Mode shape (1)



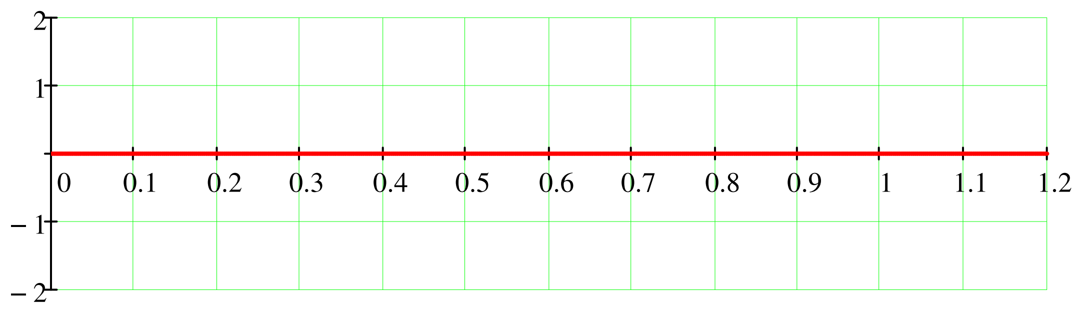
metri

Mode shape (2)



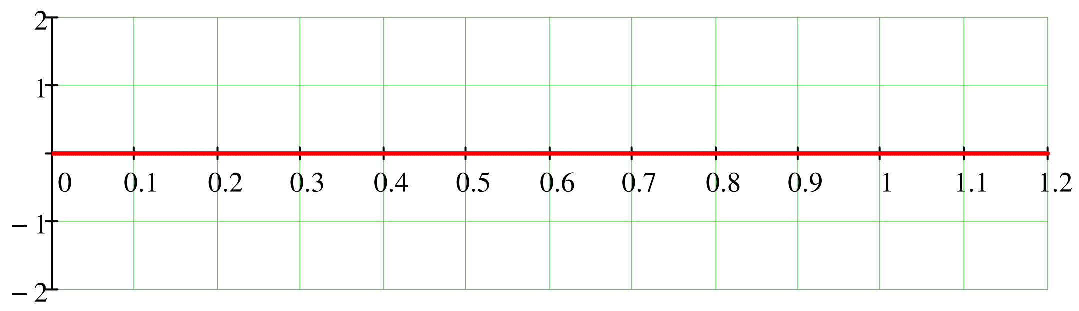
metri

Mode shape (3)



metri

Mode shape (4)



metri