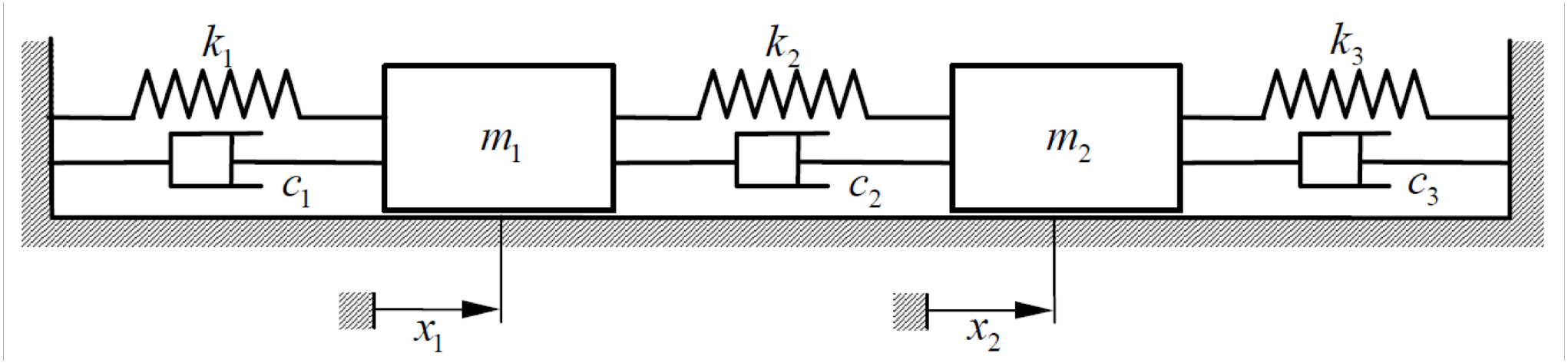
Calcolo dei coefficienti α e β (smorzamento di Rayleigh)



$ORIGIN := 1$

$m_1 := 5$ $m_2 := 6$

$k_1 := 5500$ $k_2 := 3500$ $k_3 := 1500$

$c_1 := 180$ $c_2 := 120$ $c_3 := 80$

$m_{11} := m_1 = 5$

$m_{12} := 0$

$m_{22} := m_2 = 6$

$c_{11} := c_1 + c_2 = 300$

$c_{12} := -c_2 = -120$

$c_{22} := c_2 + c_3 = 200$

$k_{11} := k_1 + k_2 = 9000$

$k_{12} := -k_2 = -3500$

$k_{22} := k_2 + k_3 = 5000$

$$\mathbf{M} := \begin{pmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 6 \end{pmatrix} \qquad \mathbf{K} := \begin{pmatrix} k_{11} & k_{12} \\ k_{12} & k_{22} \end{pmatrix} = \begin{pmatrix} 9000 & -3500 \\ -3500 & 5000 \end{pmatrix} \qquad \mathbf{C} := \begin{pmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{pmatrix} = \begin{pmatrix} 300 & -120 \\ -120 & 200 \end{pmatrix}$$

$$F_1(\alpha) := \frac{c_{11}}{k_{11}} - \frac{m_{11}}{k_{11}} \cdot \alpha$$

$$F_2(\alpha) := \frac{c_{12}}{k_{12}} - \frac{m_{12}}{k_{12}} \cdot \alpha$$

$$F_3(\alpha) := \frac{c_{22}}{k_{22}} - \frac{m_{22}}{k_{22}} \cdot \alpha$$

$$\mathbf{A} := \begin{pmatrix} m_{11} & k_{11} \\ m_{12} & k_{12} \\ m_{22} & k_{22} \end{pmatrix} = \begin{pmatrix} 5 & 9000 \\ 0 & -3500 \\ 6 & 5000 \end{pmatrix}$$

Matrice pseudoinversa (o inversa generalizzata)

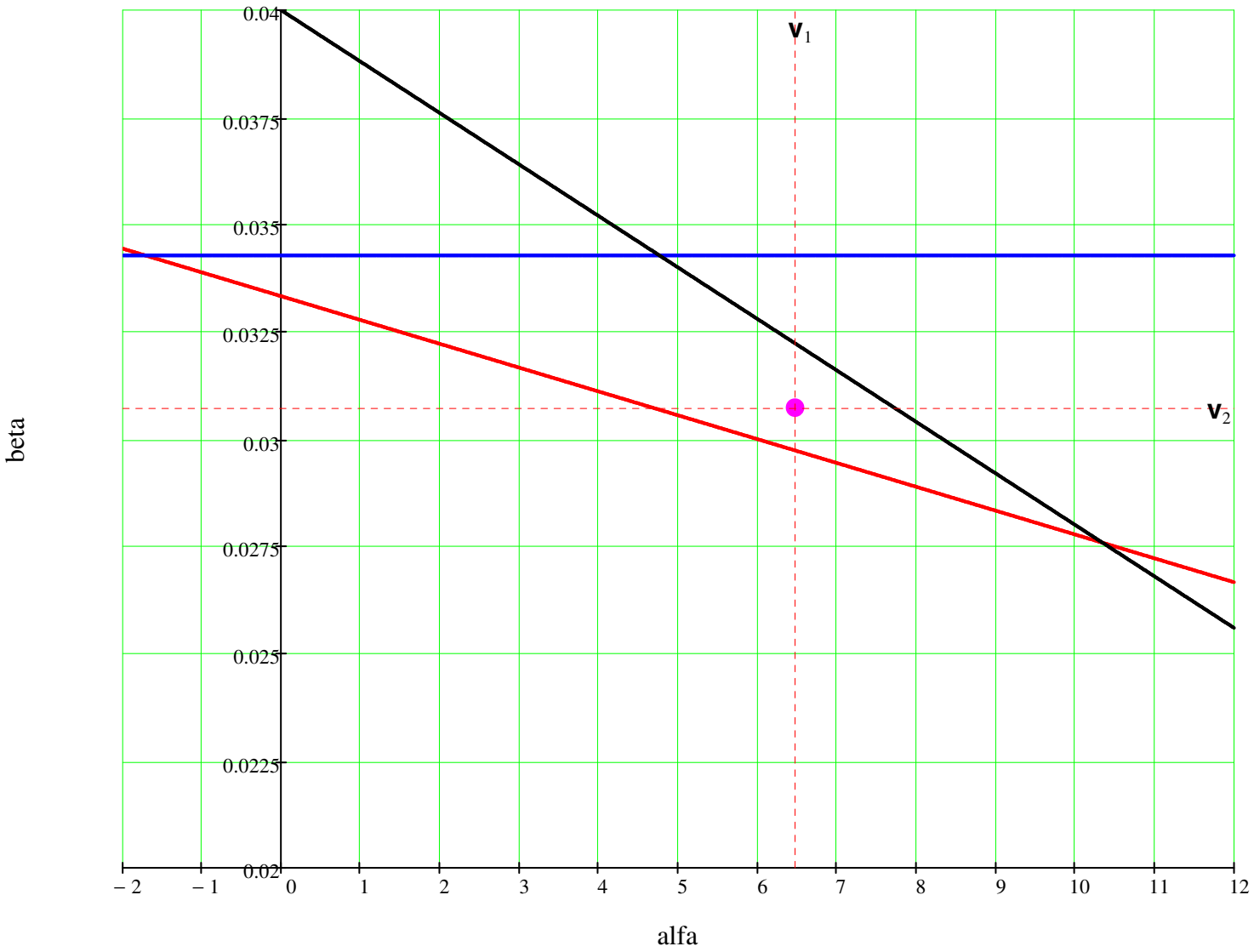
$$geninv(\mathbf{A}) = \begin{pmatrix} -0.052731 & 0.165276 & 0.210609 \\ 0.00011 & -0.000134 & -0.000091 \end{pmatrix}$$

$$\mathbf{A_{pinv}} := \left(\mathbf{A}^T \cdot \mathbf{A}\right)^{-1} \cdot \mathbf{A}^T = \begin{pmatrix} -0.052731 & 0.165276 & 0.210609 \\ 0.00011 & -0.000134 & -0.000091 \end{pmatrix}$$

$$\mathbf{A_{pinv}} := \left(\mathbf{A}^T \cdot \mathbf{A}\right)^{-1} \cdot \mathbf{A}^T \qquad \mathbf{A_{pinv}} = \begin{pmatrix} -0.05273 & 0.16528 & 0.21061 \\ 1.09555 \times 10^{-4} & -1.34425 \times 10^{-4} & -9.12955 \times 10^{-5} \end{pmatrix}$$

$$geninv(\mathbf{A}) = \begin{pmatrix} -0.05273 & 0.16528 & 0.21061 \\ 1.09555 \times 10^{-4} & -1.34425 \times 10^{-4} & -9.12955 \times 10^{-5} \end{pmatrix}$$

$$\mathbf{c} := \begin{pmatrix} c_{11} \\ c_{12} \\ c_{22} \end{pmatrix} = \begin{pmatrix} 300 \\ -120 \\ 200 \end{pmatrix} \qquad \mathbf{v} := \mathbf{A_{pinv}} \cdot \mathbf{c} = \begin{pmatrix} 6.46938 \\ 0.03074 \end{pmatrix}$$


0.04
0.0375
0.035
0.0325
0.03
0.0275
0.025
0.0225
0.02
beta
v1
v2
– 2
– 1
0
1
2
3
4
5
6
7
8
9
10
11
12
alfa

$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} := \mathbf{v}$ $\quad$ $\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \begin{pmatrix}6.46938\\ 0.03074\end{pmatrix}$

Matrice C "originale"

$\mathbf{C} = \begin{pmatrix}300 & -120\\ -120 & 200\end{pmatrix}$ $\quad$ $\begin{bmatrix}(c_1 + c_2) & -c_2\\ -c_2 & (c_2 + c_3)\end{bmatrix} = \begin{pmatrix}300 & -120\\ -120 & 200\end{pmatrix}$

Matrice C ottenuta con l'ipotesi di Rayleigh

$\mathbf{M} = \begin{pmatrix}5 & 0\\ 0 & 6\end{pmatrix}$ $\quad$ $\mathbf{K} = \begin{pmatrix}9000 & -3500\\ -3500 & 5000\end{pmatrix}$ $\quad$ $\alpha = 6.46938$ $\quad$ $\beta = 0.03074$

$\mathbf{C_{ray}} := \alpha \cdot \mathbf{M} + \beta \cdot \mathbf{K} = \begin{pmatrix}308.991 & -107.5838\\ -107.5838 & 192.5075\end{pmatrix}$

Differenza fra le due matrici (per valutare l'errore commesso con l'ipotesi di Rayleigh

$\mathbf{C} - \mathbf{C_{ray}} = \begin{pmatrix}-8.99103 & -12.41618\\ -12.41618 & 7.49252\end{pmatrix}$

$\omega := \sqrt{genvals(\mathbf{K}, \mathbf{M})} = \begin{pmatrix}46.02044\\ 22.70357\end{pmatrix}$ $\quad$ **Pulsazioni proprie**

$\alpha = 6.46938$ $\quad$ $\beta = 0.03074$

$\xi(\omega) := \dfrac{\alpha + \omega^2 \cdot \beta}{2 \cdot \omega}$

$\omega_1 = 46.02044$ $\quad$ $\xi_1 := \xi(\omega_1) = 0.77758$ $\quad$ $\xi_1 = 77.75817 \cdot \%$

Fattori di smorzamento modali

$\omega_2 = 22.70357$ $\quad$ $\xi_2 := \xi(\omega_2) = 0.49141$ $\quad$ $\xi_2 = 49.14089 \cdot \%$

$\omega := 0, 0.01 .. 80$

Fattore di smorzamento modale al variare della pulsazione