Calcolo dei coefficienti α e β (smorzamento di Rayleigh)

$ORIGIN := 1$

$c_{11} := 300$ $m_{11} := 5$ $k_{11} := 9000$

$c_{12} := -120$ $m_{12} := 2$ $k_{12} := -3500$

$c_{22} := 200$ $m_{22} := 6$ $k_{22} := 5000$

$$\mathbf{M} := \begin{pmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{pmatrix} = \begin{pmatrix} 5 & 2 \\ 2 & 6 \end{pmatrix} \qquad \mathbf{K} := \begin{pmatrix} k_{11} & k_{12} \\ k_{12} & k_{22} \end{pmatrix} = \begin{pmatrix} 9000 & -3500 \\ -3500 & 5000 \end{pmatrix} \qquad \mathbf{C} := \begin{pmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{pmatrix} = \begin{pmatrix} 300 & -120 \\ -120 & 200 \end{pmatrix}$$

$$F_1(\alpha) := \frac{c_{11}}{k_{11}} - \frac{m_{11}}{k_{11}} \cdot \alpha$$

$$F_2(\alpha) := \frac{c_{12}}{k_{12}} - \frac{m_{12}}{k_{12}} \cdot \alpha$$

$$F_3(\alpha) := \frac{c_{22}}{k_{22}} - \frac{m_{22}}{k_{22}} \cdot \alpha$$

$$\mathbf{A} := \begin{pmatrix} m_{11} & k_{11} \\ m_{12} & k_{12} \\ m_{22} & k_{22} \end{pmatrix} = \begin{pmatrix} 5 & 9000 \\ 2 & -3500 \\ 6 & 5000 \end{pmatrix}$$

Matrice pseudoinversa (o inversa generalizzata)

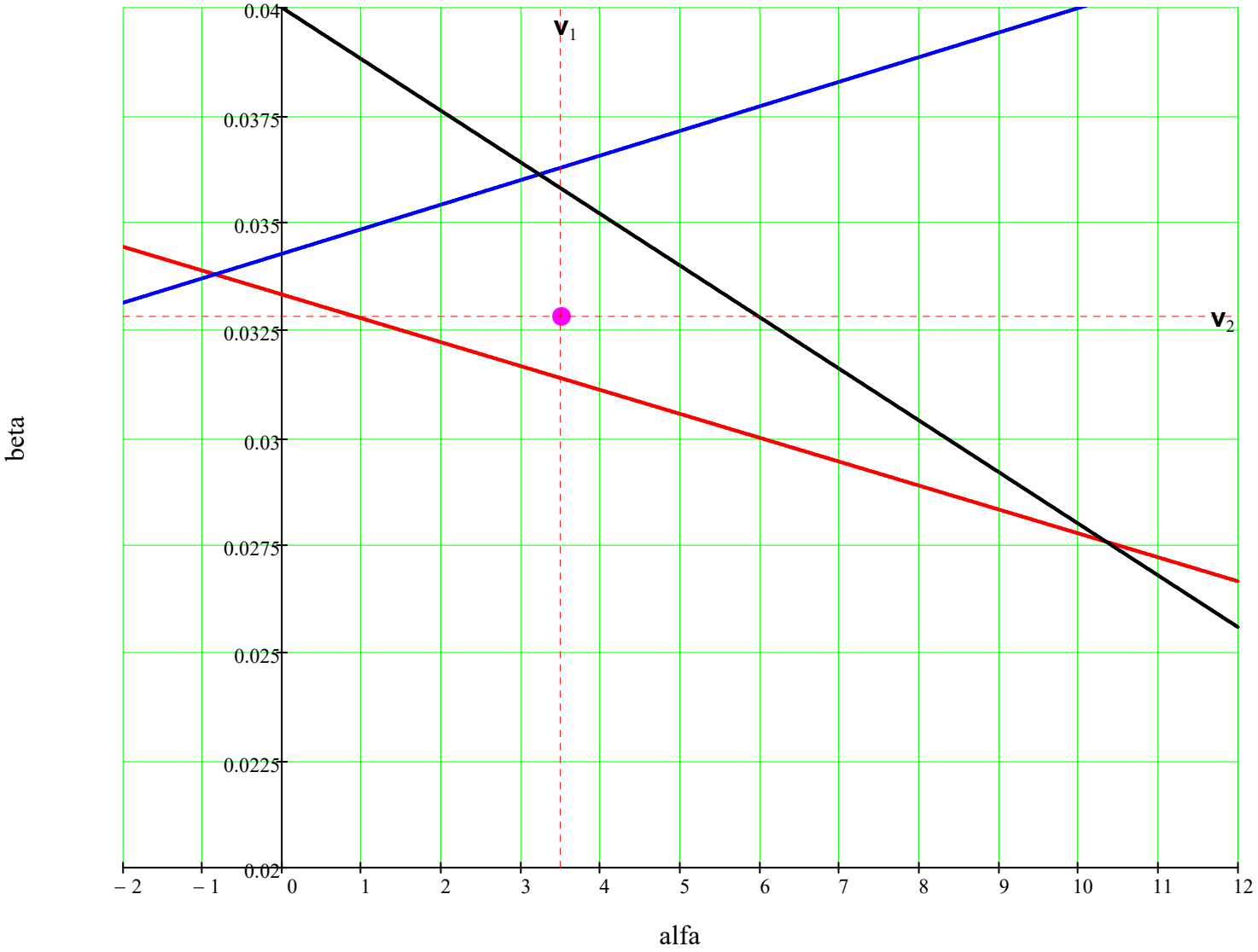
$$\mathbf{A}_{\mathbf{pinv}} := \left(\mathbf{A}^{T} \cdot \mathbf{A}\right)^{-1} \cdot \mathbf{A}^{T}$$

$$\mathbf{A}_{\mathbf{pinv}} = \begin{pmatrix} -6.776 \times 10^{-3} & 154.951 \times 10^{-3} & 120.663 \times 10^{-3} \\ 80.007 \times 10^{-6} & -118.704 \times 10^{-6} & -27.104 \times 10^{-6} \end{pmatrix}$$

$$geninv(\mathbf{A}) = \begin{pmatrix} -6.776 \times 10^{-3} & 154.951 \times 10^{-3} & 120.663 \times 10^{-3} \\ 80.007 \times 10^{-6} & -118.704 \times 10^{-6} & -27.104 \times 10^{-6} \end{pmatrix}$$

$$\mathbf{c} := \begin{pmatrix} c_{11} \\ c_{12} \\ c_{22} \end{pmatrix} = \begin{pmatrix} 300 \\ -120 \\ 200 \end{pmatrix}$$

$$\mathbf{v} := \mathbf{A}_{\mathbf{pinv}} \cdot \mathbf{c} = \begin{pmatrix} 3.506 \\ 0.033 \end{pmatrix}$$



$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} := \mathbf{v}$

$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 3.506 \\ 0.033 \end{pmatrix}$

Matrice C "originale"

$$\mathbf{C} = \begin{pmatrix} 300 & -120 \\ -120 & 200 \end{pmatrix}$$

Matrice C ottenuta con l'ipotesi di Rayleigh

$$\mathbf{C_{ray}} := \alpha \cdot \mathbf{M} + \beta \cdot \mathbf{K} = \begin{pmatrix} 312.958 & -107.878 \\ -107.878 & 185.161 \end{pmatrix}$$

Differenza fra le due matrici (per valutare l'errore commesso con l'ipotesi di Rayleigh

$$\mathbf{C} - \mathbf{C_{ray}} = \begin{pmatrix} -12.958 & -12.122 \\ -12.122 & 14.839 \end{pmatrix}$$

$$\omega := \sqrt{genvals(\mathbf{K}, \mathbf{M})} = \begin{pmatrix} 19.899 \\ 56.4 \end{pmatrix}$$

Pulsazioni proprie

$\alpha = 3.506$ $\beta = 0.033$

$$\xi(\omega) := \frac{\alpha + \omega^2 \cdot \beta}{2 \cdot \omega}$$

$\omega_1 = 19.899$ $\xi_1 := \xi(\omega_1) = 0.415$ $\xi_1 = 41.469 \cdot \%$

$\omega_2 = 56.4$ $\xi_2 := \xi(\omega_2) = 0.957$ $\xi_2 = 95.675 \cdot \%$

Fattori di smorzamento modali

$\omega := 0, 0.01 .. 80$

Fattore di smorzamento modale al variare della pulsazione