Vibrazioni torsionali (avviamento di un carico azionato da motore DC)

$J_1 := 0.05$

$J_2 := 0.02$

$J_3 := 0.04$

$J_4 := 1$

$\tau := \frac{1}{4}$

$c_1 := 0.2 \qquad c_2 := 0.5$

$G := 80000 \cdot 10^6 = 8 \times 10^{10}$

$d_1 := 15 \cdot 10^{-3} = 0.015$

$d_2 := 18 \cdot 10^{-3} = 0.018$

$l_1 := 800 \cdot 10^{-3} = 0.8$

$l_2 := 1500 \cdot 10^{-3} = 1.5$

$k_1 := \frac{G}{l_1} \cdot \left(\frac{\pi \cdot d_1^{\,4}}{32} \right) = 497.01$ Nm/rad

$k_2 := \frac{G}{l_2} \cdot \left(\frac{\pi \cdot d_2^{\,4}}{32} \right) = 549.653$ Nm/rad

$$\mathbf{J} := \begin{bmatrix} J_1 & 0 & 0 \\ 0 & \left(J_2 + J_3 \cdot \tau^2\right) & 0 \\ 0 & 0 & J_4 \end{bmatrix} = \begin{pmatrix} 0.05 & 0 & 0 \\ 0 & 0.023 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{K} := \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & \left(k_1 + k_2 \cdot \tau^2\right) & -\tau \cdot k_2 \\ 0 & -\tau \cdot k_2 & k_2 \end{bmatrix} = \begin{pmatrix} 497.01 & -497.01 & 0 \\ -497.01 & 531.363 & -137.413 \\ 0 & -137.413 & 549.653 \end{pmatrix}$$

$|\mathbf{K}| = 0 \qquad SMORZ := 1$

$$\mathbf{C} := \begin{bmatrix} c_1 & -c_1 & 0 \\ -c_1 & \left(c_1 + c_2 \cdot \tau^2\right) & -\tau \cdot c_2 \\ 0 & -\tau \cdot c_2 & c_2 \end{bmatrix} \cdot SMORZ = \begin{pmatrix} 0.2 & -0.2 & 0 \\ -0.2 & 0.231 & -0.125 \\ 0 & -0.125 & 0.5 \end{pmatrix}$$

$omega := \sqrt{genvals(\mathbf{K}, \mathbf{J})}$ *pulsazioni proprie [rad/s]* $\qquad omega = \begin{pmatrix} 181.978 \\ 31.463 \\ 0 \end{pmatrix}$

$\frac{omega}{2 \cdot \pi} = \begin{pmatrix} 28.963 \\ 5.007 \\ 0 \end{pmatrix}$ *frequenze proprie [Hz]*

Matrice modale

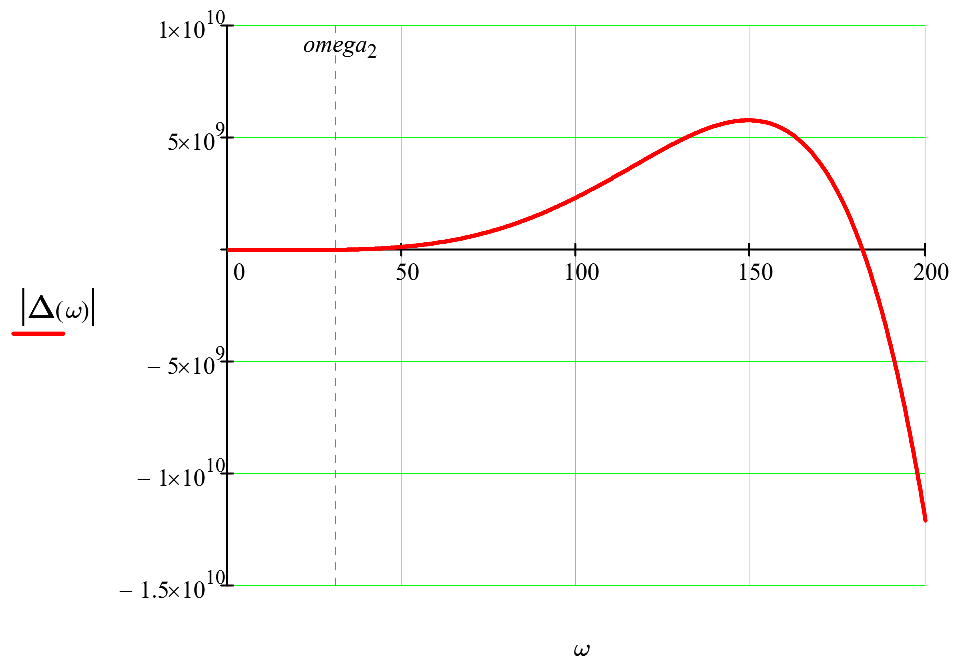
$$\Phi := genvecs(\mathbf{K},\mathbf{J}) = \begin{pmatrix} 0.429 & -1 & -1 \\ -1 & -0.9 & -1 \\ 4.219\times 10^{-3} & 0.281 & -0.25 \end{pmatrix}$$

Matrice modale normalizzata (l'ultima riga contiene elementi unitari)

$$\Phi_{\mathbf{norm}} := augment\left(\frac{\Phi^{\langle 1\rangle}}{\Phi_{3,1}}, \frac{\Phi^{\langle 2\rangle}}{\Phi_{3,2}}, \frac{\Phi^{\langle 3\rangle}}{\Phi_{3,3}}\right) = \begin{pmatrix} 101.648 & -3.558 & 4 \\ -236.996 & -3.204 & 4 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\Delta(\omega) := \mathbf{K} - \omega^2\cdot\mathbf{J}$$

$\omega := 0, 0.5 .. 200$

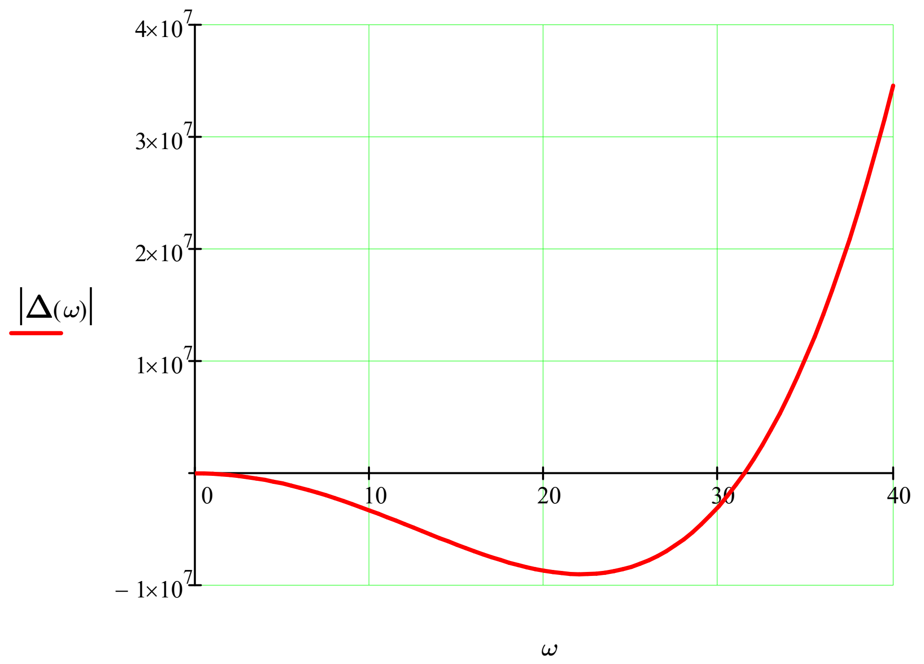


$omega_2 = 31.463$

$\omega := 200$

$root(|\Delta(\omega)|, \omega) = 181.978$

$\omega := 0, 0.5 .. 40$



$omega_1 = 181.978$

Curve caratteristiche (motore e utilizzatore)

$K_m := 0.07$

$V_m := 24$

$R := 0.06$

$A := \frac{V_m \cdot K_m}{R} = 28$

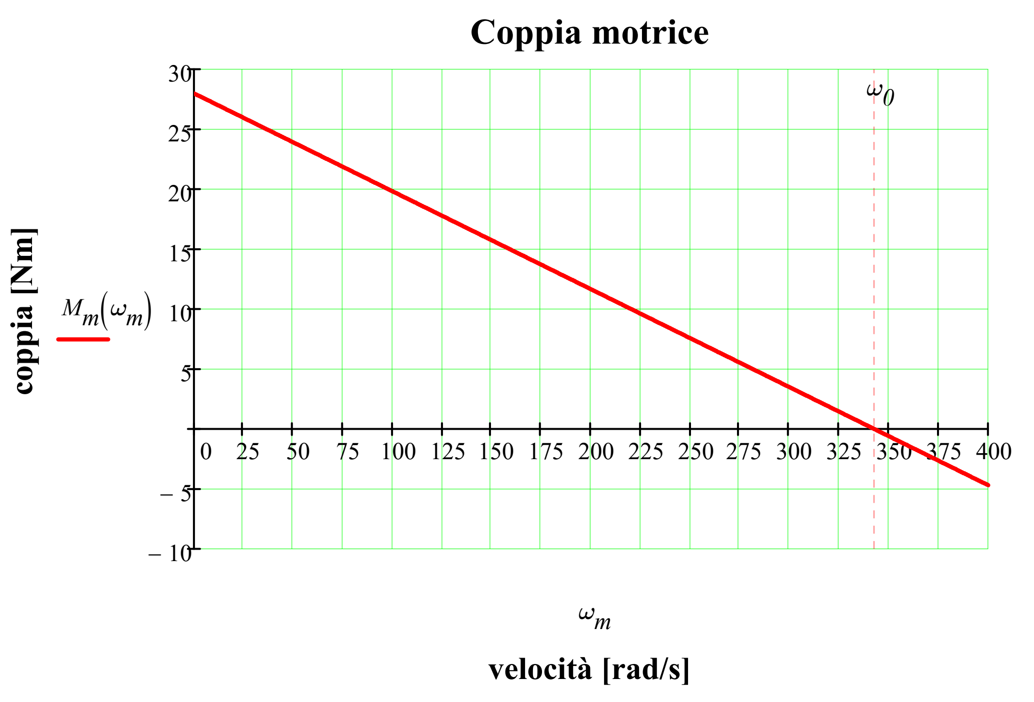
$B := \frac{K_m^2}{R} = 0.082$

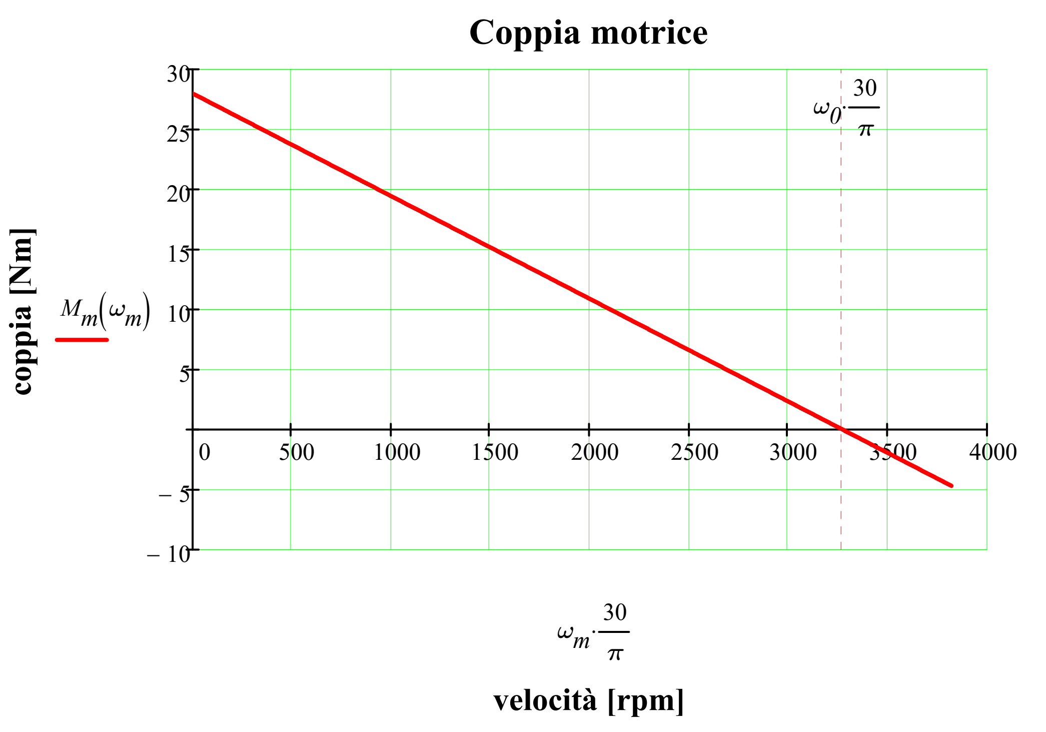
$M_m(\omega_m) := A - B \cdot \omega_m$

$M_m(\omega_m) \rightarrow -0.081666666666666679 \cdot \omega_m + 28.000000000000004$

$\omega_m := 0, 0.5 .. 400$

$\omega_0 := \frac{A}{B} = 342.857$ Velocità a vuoto $\omega_0 \cdot \frac{30}{\pi} = 3274.045$



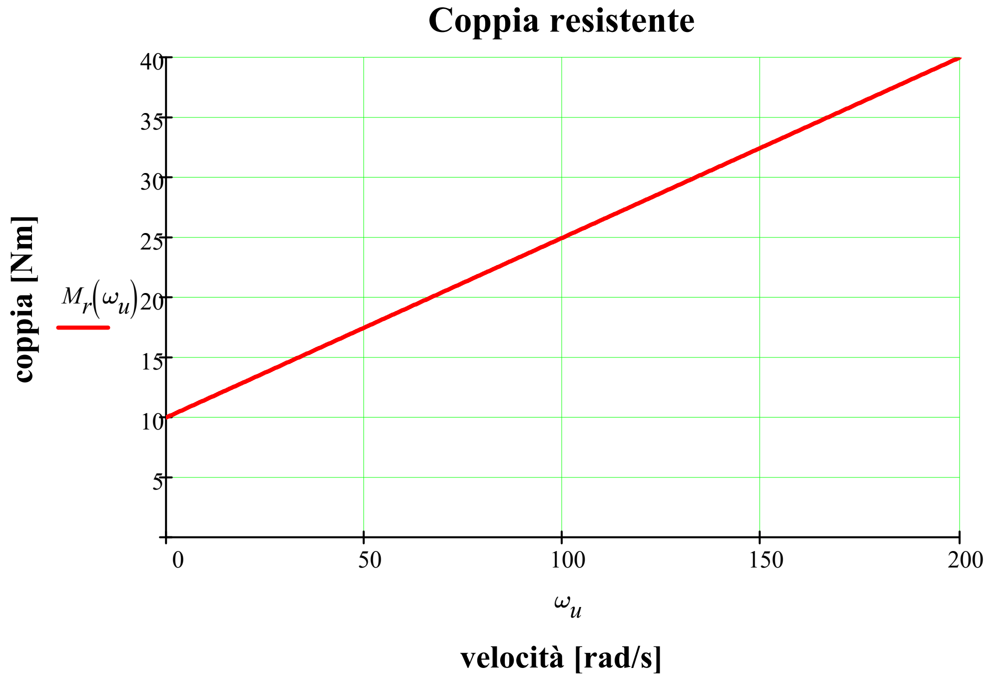


$C := 10$

$D := 0.15$

$M_r(\omega_u) := C + D \cdot \omega_u$

$\omega_u := 0, 0.5 .. 200$



Metodo di integrazione numerica delle equazioni differenziali di moto

$$\mathbf{I} := identity(3) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{O} := 0 \cdot \mathbf{I} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{A_{sup}} := augment(\mathbf{O}, \mathbf{I}) = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{A_{inf}} := augment\left(-\mathbf{J}^{-1} \cdot \mathbf{K}, -\mathbf{J}^{-1} \cdot \mathbf{C}\right) = \begin{pmatrix} -9940.196 & 9940.196 & 0 & -4 & 4 & 0 \\ 22089.323 & -23616.137 & 6107.256 & 8.889 & -10.278 & 5.556 \\ 0 & 137.413 & -549.653 & 0 & 0.125 & -0.5 \end{pmatrix}$$

$$\mathbf{A} := stack(\mathbf{A_{sup}}, \mathbf{A_{inf}}) = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -9940.196 & 9940.196 & 0 & -4 & 4 & 0 \\ 22089.323 & -23616.137 & 6107.256 & 8.889 & -10.278 & 5.556 \\ 0 & 137.413 & -549.653 & 0 & 0.125 & -0.5 \end{pmatrix}$$

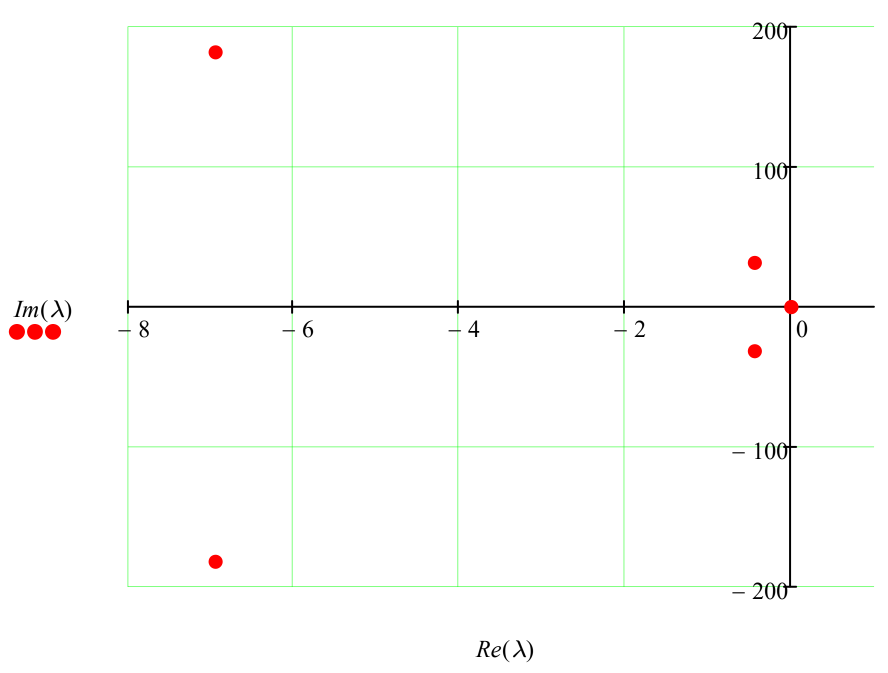
$$\mathbf{o} := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{Q}(\theta'_1, \theta'_4) := \begin{pmatrix} M_m(\theta'_1) \\ 0 \\ -M_r(\theta'_4) \end{pmatrix}$$

$$\mathbf{b}(\theta'_1, \theta'_4) := stack\left(\mathbf{o}, \mathbf{J}^{-1} \cdot \mathbf{Q}(\theta'_1, \theta'_4)\right)$$

$$\mathbf{b}(\theta'_1, \theta'_4) \rightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1.6333333333333333358 \cdot \theta'_1 + 560.00000000000008 \\ 0 \\ -0.15 \cdot \theta'_4 - 10 \end{pmatrix}$$

$$\lambda := eigenvals(\mathbf{A}) = \begin{pmatrix} -6.947 + 181.845i \\ -6.947 - 181.845i \\ -0.442 + 31.46i \\ -0.442 - 31.46i \\ 0 \\ 0 \end{pmatrix} \qquad omega = \begin{pmatrix} 181.978 \\ 31.463 \\ 0 \end{pmatrix}$$



$\mathbf{y} := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ Condizioni iniziali

$T_{max} := 10$

$\omega_{max} := max(Re(omega)) = 181.978$

$\tau_{min} := \frac{2 \cdot \pi}{\omega_{max}} = 0.035$

$\Delta t_{cons} := \frac{1}{20} \cdot \tau_{min} = 1.726 \times 10^{-3}$

$\Delta t := 1 \cdot 10^{-3}$

$N := ceil\left(\frac{T_{max}}{\Delta t}\right) = 10000$

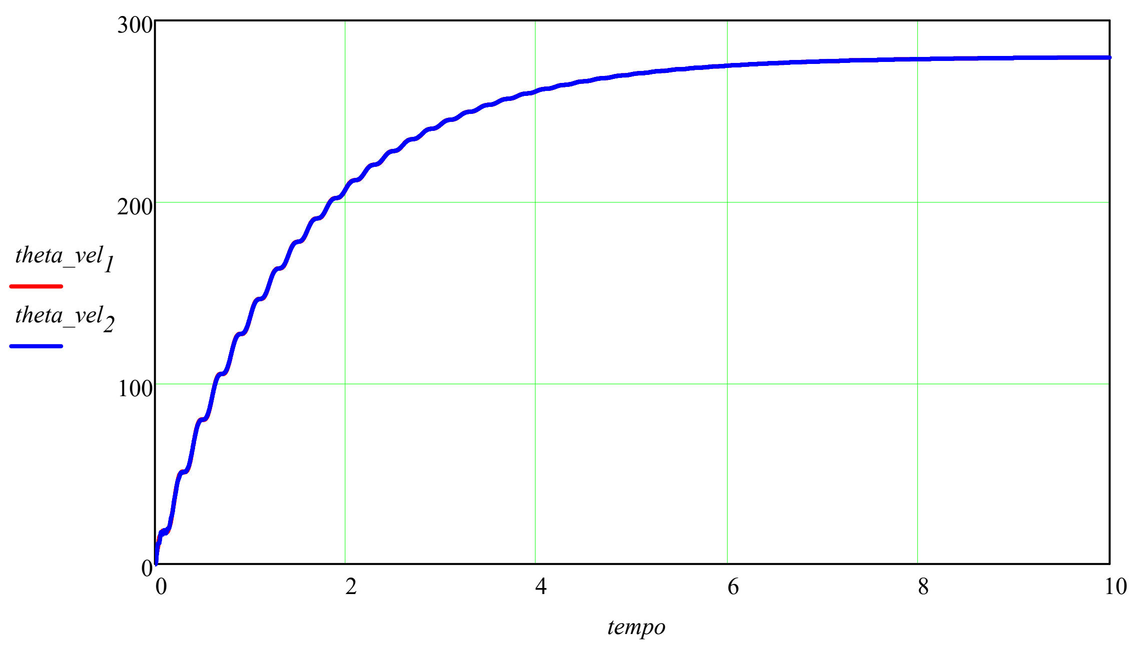
$EQMOTO(t, \mathbf{y}) := \mathbf{A} \cdot \mathbf{y} + \mathbf{b}(\mathbf{y}_4, \mathbf{y}_6)$ Equazioni di moto nello spazio di stato

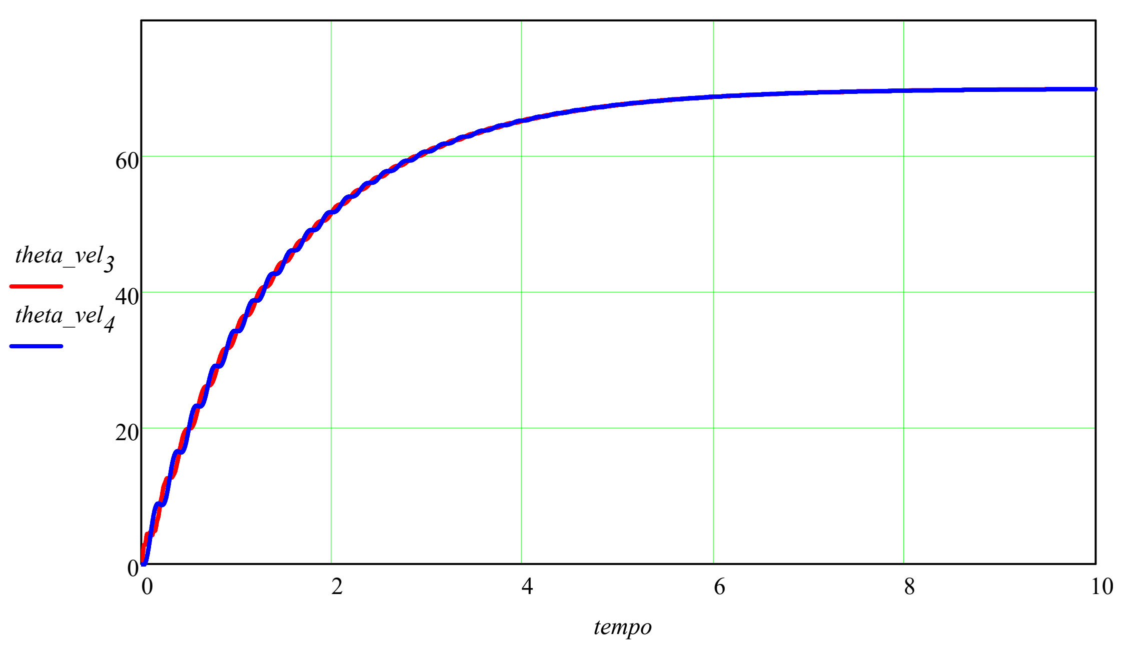
$TAB := rkfixed(\mathbf{y}, 0, T_{max}, N, EQMOTO)$ Soluzione per via numerica (Runge-Kutta)

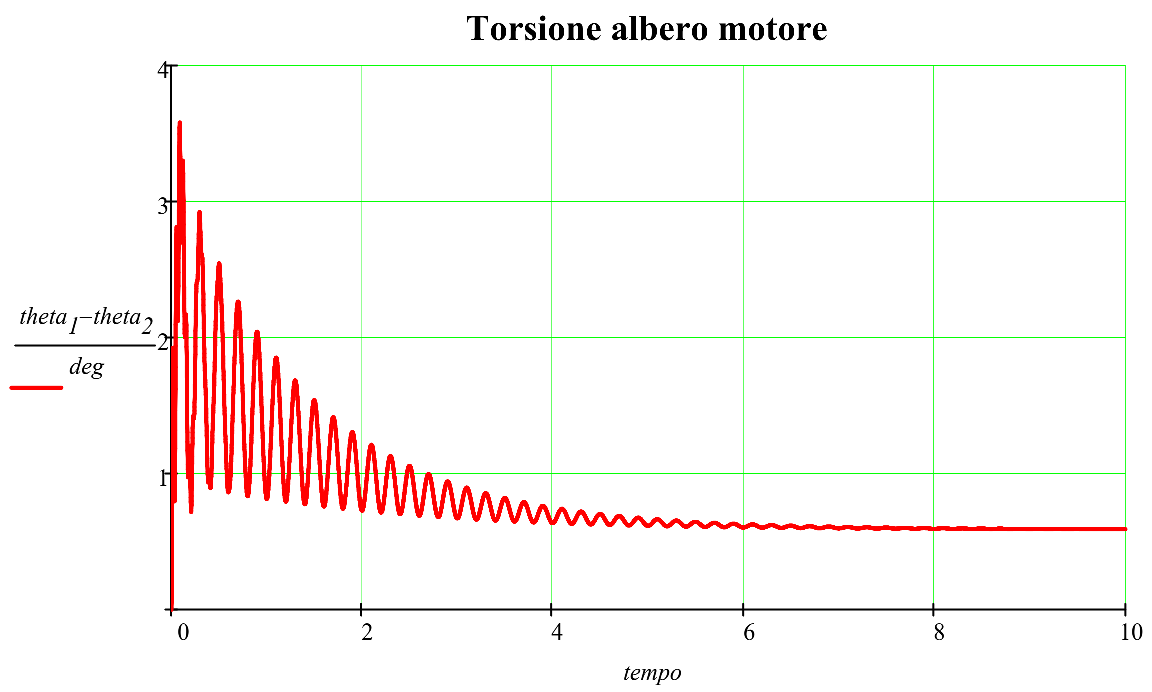
$tempo := TAB^{\langle 1 \rangle}$

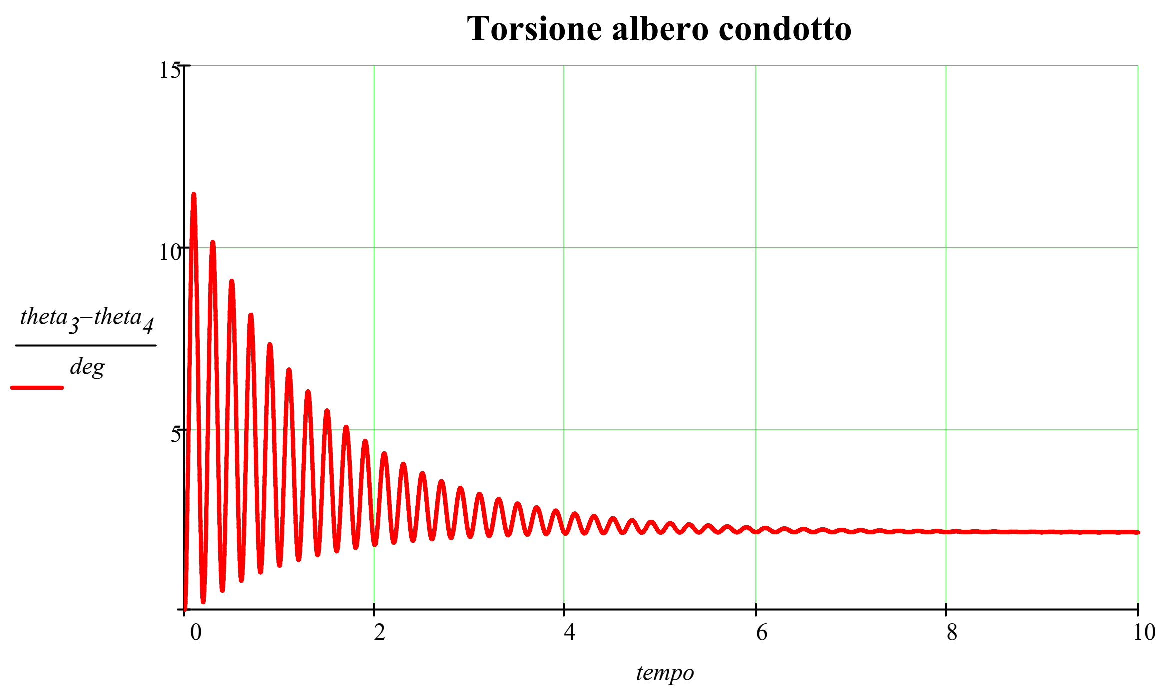
$theta_1 := TAB^{\langle 2 \rangle}$ $theta_2 := TAB^{\langle 3 \rangle}$ $theta_4 := TAB^{\langle 4 \rangle}$ $theta_3 := \tau \cdot theta_2$

$theta\_vel_1 := TAB^{\langle 5 \rangle}$ $theta\_vel_2 := TAB^{\langle 6 \rangle}$ $theta\_vel_4 := TAB^{\langle 7 \rangle}$ $theta\_vel_3 := \tau \cdot theta\_vel_2$









Caso rigido

Momento d'inerzia equivalente ridotto all'asse del motore

$J_{eq} := (J_1 + J_2) + \tau^2 \cdot (J_3 + J_4) = 0.135$

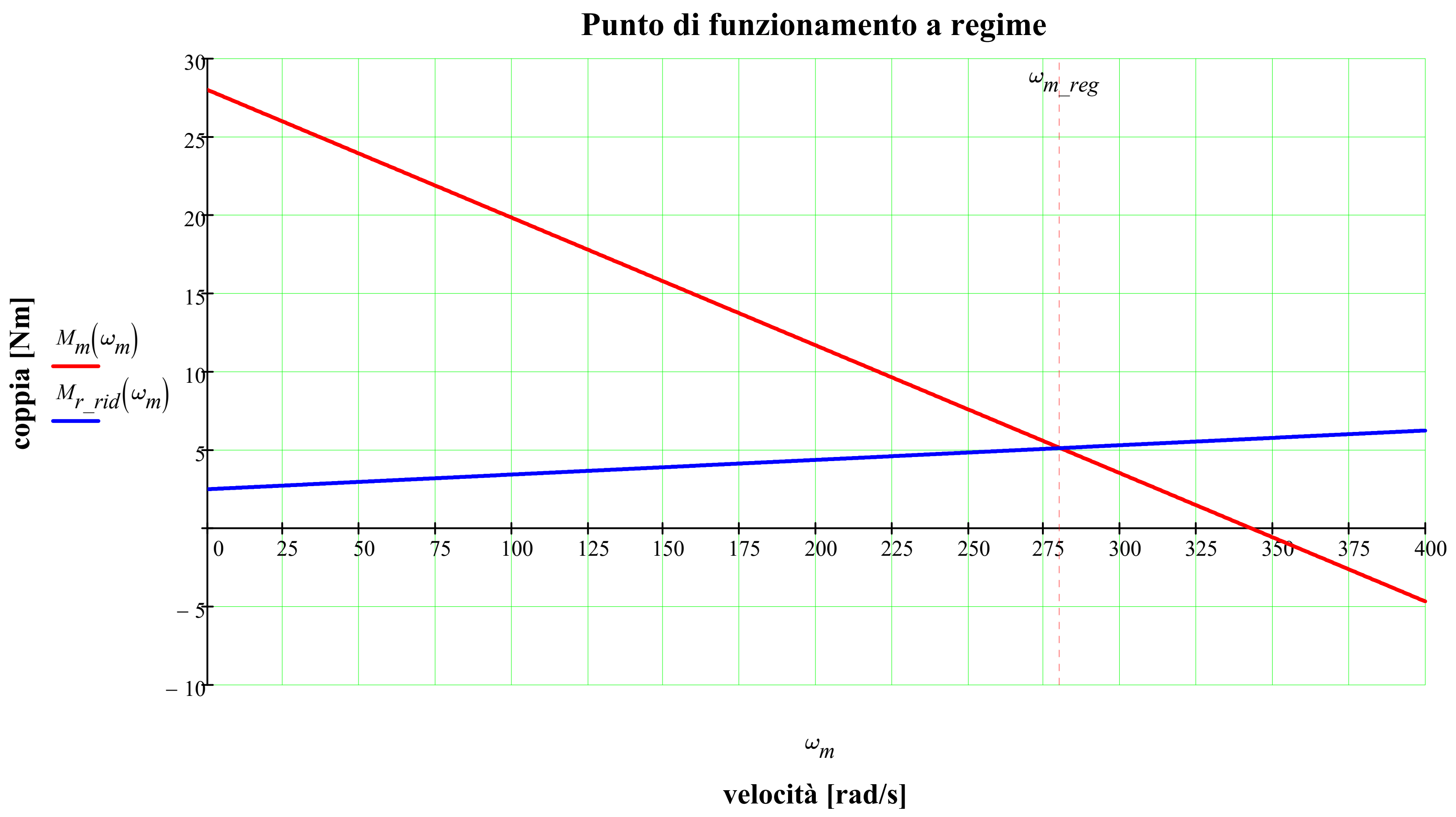
Velocità del motore a regime

$\omega_{m\_reg} := \frac{A - \tau \cdot C}{B + \tau^2 \cdot D} = 280.092$

Momento resistente ridotto all'asse del motore

$M_{r\_rid}(\omega_m) := \tau \cdot (C + D \cdot \tau \cdot \omega_m)$

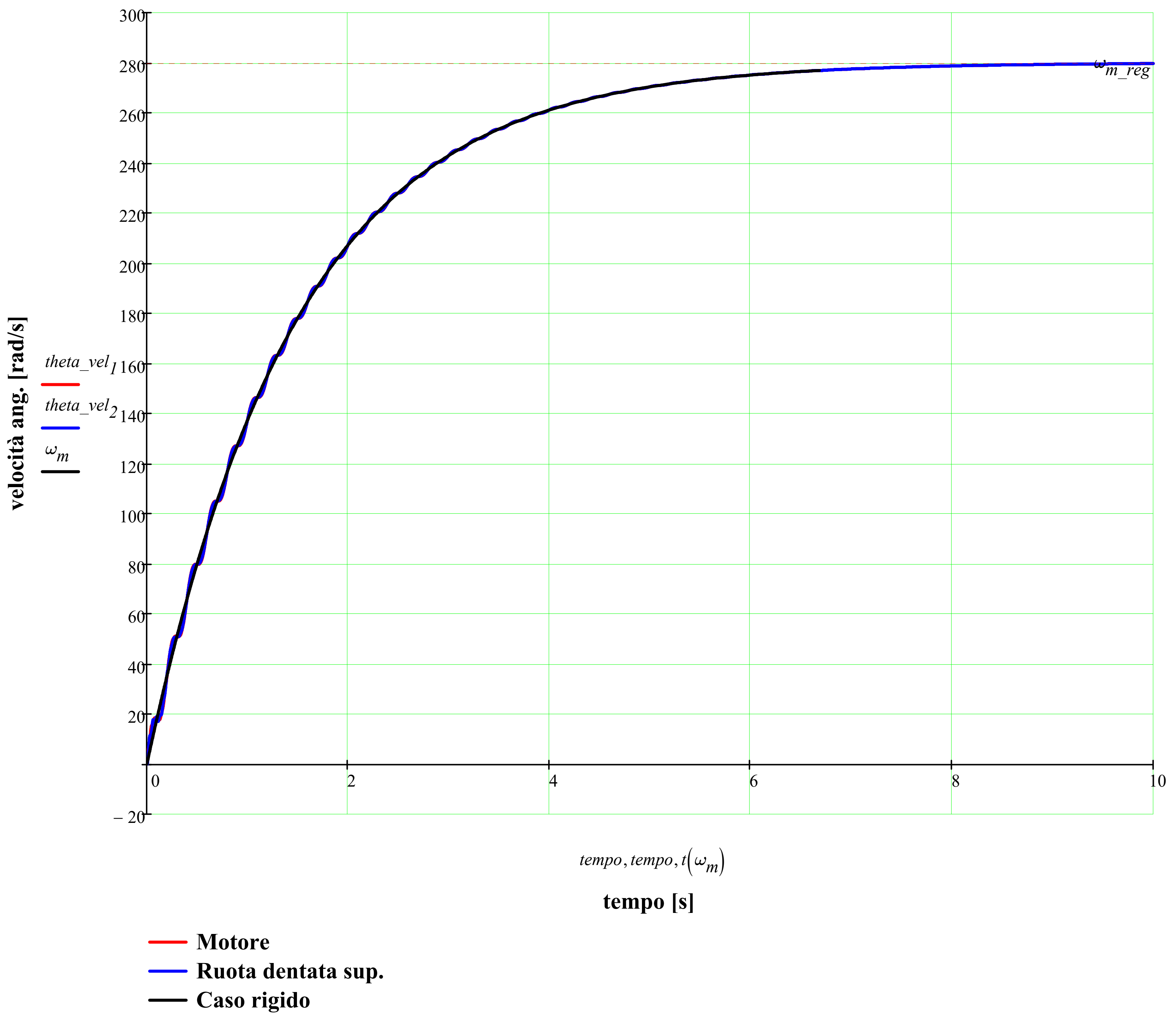
$M_m(\omega_{m\_reg}) = 5.126$ $M_{r\_rid}(\omega_{m\_reg}) = 5.126$

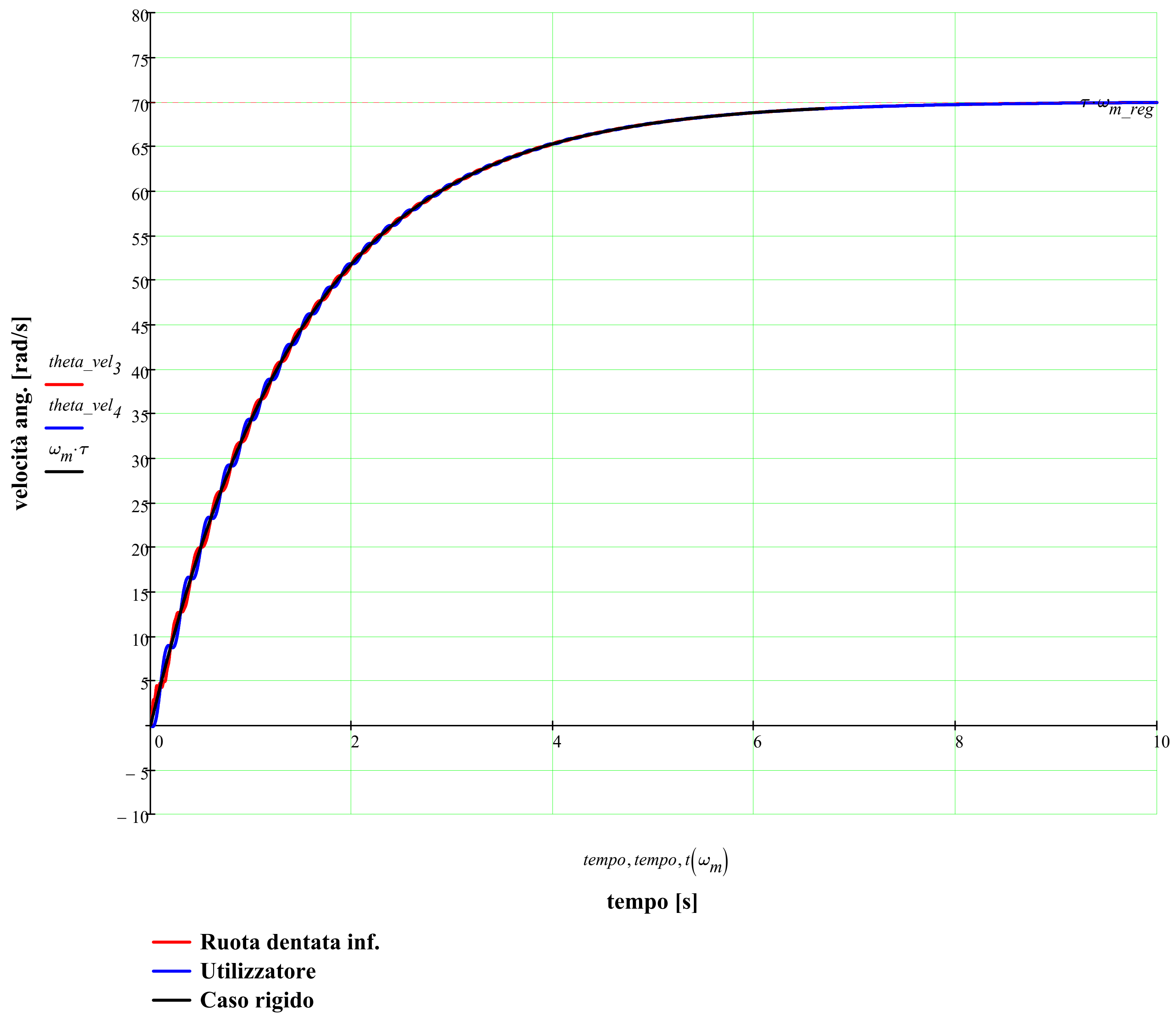


Studio del transitorio (caso rigido)

$t(\omega_m) := J_{eq} \cdot \int_0^{\omega_m} \frac{1}{M_m(\omega_m) - M_{r\_rid}(\omega_m)} \, d\omega_m$

$\omega_m := 0, 0.5 .. \omega_{m\_reg} \cdot 0.99$





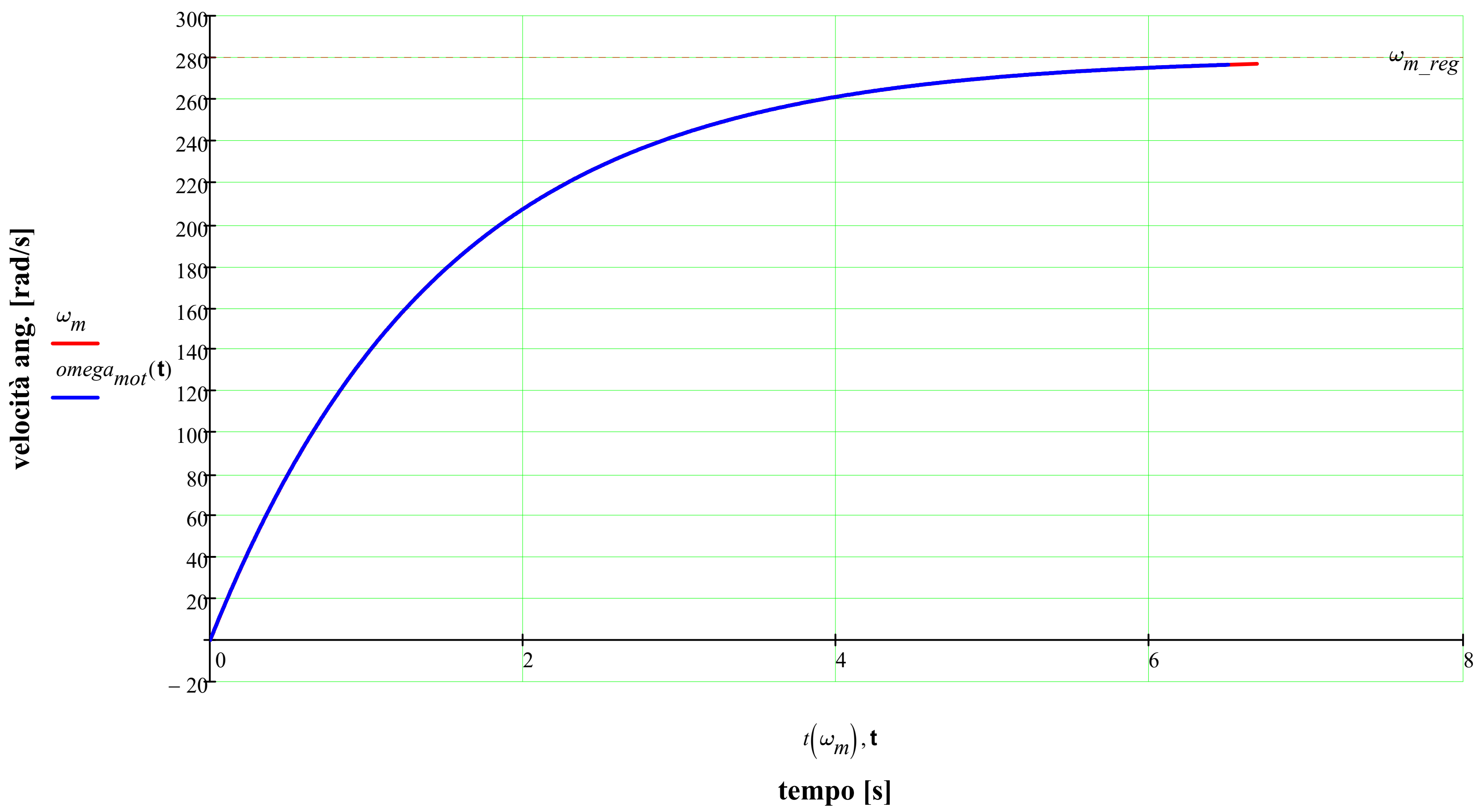
$P := A - \tau \cdot C = 25.5$ $\quad Q := B + \tau^2 \cdot D = 0.091$ $\quad \frac{P}{Q} = 280.092$

$\omega_{m\_reg} = 280.092$

$\gamma := \frac{J_{eq}}{Q} = 1.483$ Costante di tempo [s]

$omega_{mot}(t) := \frac{P}{Q} \cdot \left(1 - e^{\frac{-t}{\gamma}}\right)$

$\mathbf{t} := 0, 0.01 .. 6.5$



velocità ang. [rad/s]
300
280
260
240
220
200
180
160
140
120
100
80
60
40
20
− 20
0
2
4
6
8
ω_m
$omega_{mot}(\mathbf{t})$
ω_{m_reg}
$t(\omega_m), \mathbf{t}$
tempo [s]