

## Vibrazioni torsionali (avviamento di un carico azionato da motore DC)

$$J_1 := 0.05$$

$$J_2 := 0.02$$

$$J_3 := 0.04$$

$$J_4 := 1$$

$$\tau := \frac{1}{4}$$

$$c_1 := 0.2$$

$$c_2 := 0.5$$

$$k_1 := \frac{G}{l_1} \cdot \left( \frac{\pi \cdot d_1^4}{32} \right) = 497.01 \quad \text{Nm/rad}$$

$$k_2 := \frac{G}{l_2} \cdot \left( \frac{\pi \cdot d_2^4}{32} \right) = 549.653 \quad \text{Nm/rad}$$

$$\underline{\underline{G}} := 80000 \cdot 10^6 = 8 \times 10^{10}$$

$$d_1 := 15 \cdot 10^{-3} = 0.015$$

$$d_2 := 18 \cdot 10^{-3} = 0.018$$

$$l_1 := 800 \cdot 10^{-3} = 0.8$$

$$l_2 := 1500 \cdot 10^{-3} = 1.5$$

$$SMORZ := 1$$

$$\mathbf{J} := \begin{bmatrix} J_1 & 0 & 0 \\ 0 & (J_2 + J_3 \cdot \tau^2) & 0 \\ 0 & 0 & J_4 \end{bmatrix} = \begin{pmatrix} 0.05 & 0 & 0 \\ 0 & 0.023 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{K} := \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & (k_1 + k_2 \cdot \tau^2) & -\tau \cdot k_2 \\ 0 & -\tau \cdot k_2 & k_2 \end{bmatrix} = \begin{pmatrix} 497.01 & -497.01 & 0 \\ -497.01 & 531.363 & -137.413 \\ 0 & -137.413 & 549.653 \end{pmatrix}$$

$$\mathbf{C} := \begin{bmatrix} c_1 & -c_1 & 0 \\ -c_1 & (c_1 + c_2 \cdot \tau^2) & -\tau \cdot c_2 \\ 0 & -\tau \cdot c_2 & c_2 \end{bmatrix} \cdot SMORZ = \begin{pmatrix} 0.2 & -0.2 & 0 \\ -0.2 & 0.231 & -0.125 \\ 0 & -0.125 & 0.5 \end{pmatrix}$$

$$|\mathbf{K}| = 0$$

$$\text{coefficienti} := \begin{pmatrix} 0 \\ 0 \\ -J_1 \cdot k_1 \cdot k_2 - J_2 \cdot k_1 \cdot k_2 - J_3 \cdot \tau^2 \cdot k_1 \cdot k_2 - J_4 \cdot \tau^2 \cdot k_1 \cdot k_2 \\ 0 \\ J_1 \cdot J_2 \cdot k_2 + J_1 \cdot J_4 \cdot k_1 + J_2 \cdot J_4 \cdot k_1 + J_1 \cdot J_3 \cdot \tau^2 \cdot k_2 + J_1 \cdot J_4 \cdot \tau^2 \cdot k_2 + J_3 \cdot J_4 \cdot \tau^2 \cdot k_1 \\ 0 \\ -J_1 \cdot J_3 \cdot J_4 \cdot \tau^2 - J_1 \cdot J_2 \cdot J_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -3.688 \times 10^4 \\ 38.369 \\ 0 \\ -1.125 \times 10^{-3} \end{pmatrix}$$

$$\text{polyroots}(\text{coefficienti}) = \begin{pmatrix} -181.978 \\ -31.463 \\ 0 \\ 0 \\ 31.463 \\ 181.978 \end{pmatrix}$$

$$\omega := \sqrt{\text{genvals}(\mathbf{K}, \mathbf{J})}$$

pulsazioni proprie [rad/s]

$$\omega = \begin{pmatrix} 181.978 \\ 31.463 \\ 0 \end{pmatrix}$$

$$\frac{\omega}{2 \cdot \pi} = \begin{pmatrix} 28.963 \\ 5.007 \\ 0 \end{pmatrix}$$

frequenze proprie [Hz]

Matrice modale

$$\Phi := \text{genvecs}(\mathbf{K}, \mathbf{J}) = \begin{pmatrix} 0.429 & -1 & -1 \\ -1 & -0.9 & -1 \\ 4.219 \times 10^{-3} & 0.281 & -0.25 \end{pmatrix}$$

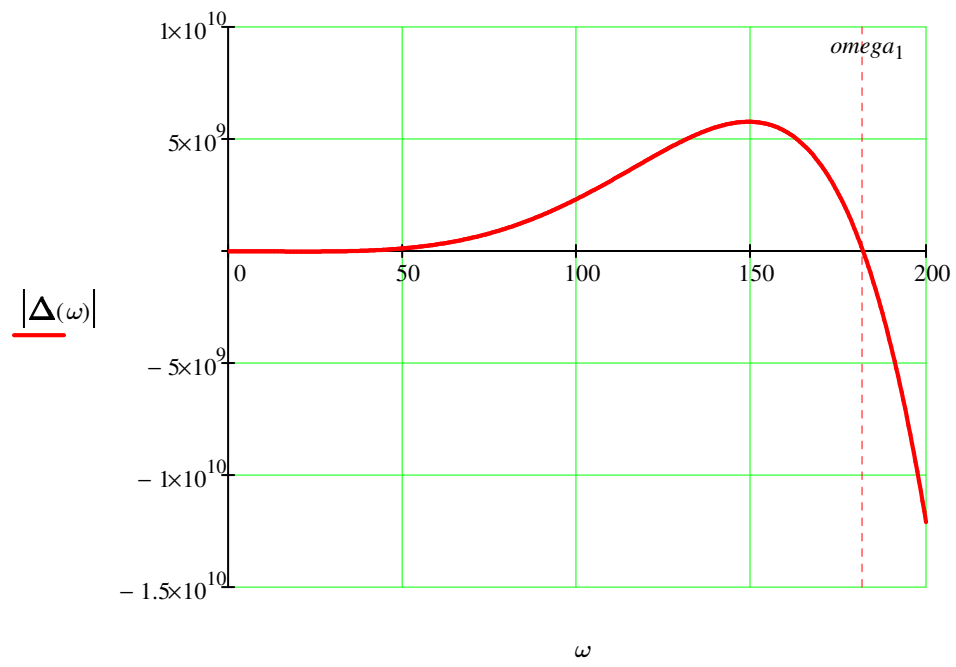
$$\omega = \begin{pmatrix} 181.978 \\ 31.463 \\ 0 \end{pmatrix}$$

Matrice modale normalizzata (l'ultima riga contiene elementi unitari)

$$\Phi_{\text{norm}} := \text{augment} \left( \frac{\Phi^{(1)}}{\Phi_{3,1}}, \frac{\Phi^{(2)}}{\Phi_{3,2}}, \frac{\Phi^{(3)}}{\Phi_{3,3}} \right) = \begin{pmatrix} 101.648 & -3.558 & 4 \\ -236.996 & -3.204 & 4 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\Delta(\omega) := \mathbf{K} - \omega^2 \cdot \mathbf{J}$$

$$\omega := 0, 0.5..200$$

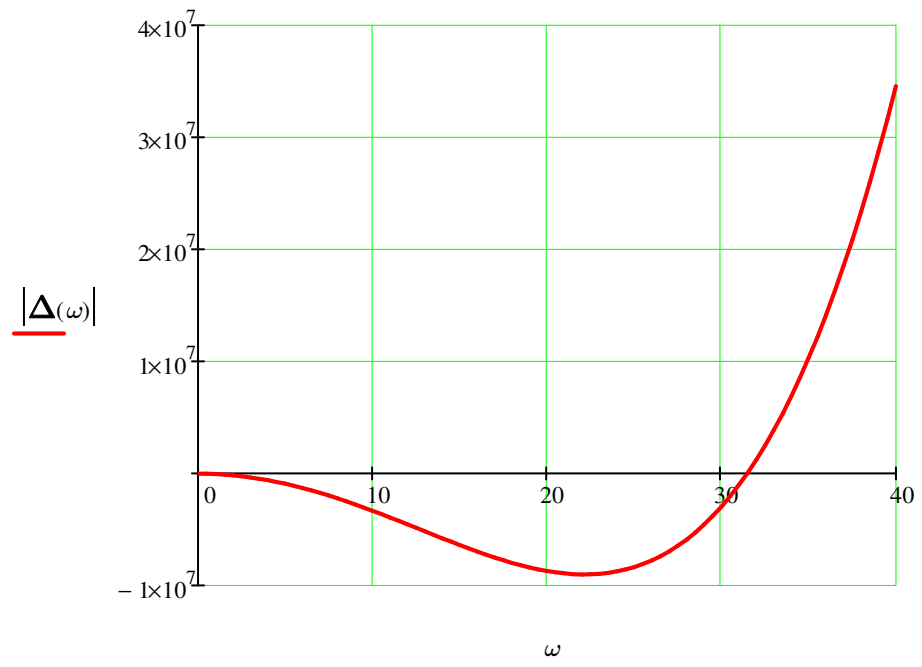


$$\omega_1 = 181.978$$

$\omega := 200$

$$\text{root}(|\Delta(\omega)|, \omega) = 181.978$$

$\omega := 0, 0.5..40$



$$\omega_1 = 181.978$$

Curve caratteristiche (motore e utilizzatore)

$$K_m := 0.07$$

$$V_m := 24$$

$$R := 0.06$$

$$A := \frac{V_m \cdot K_m}{R} = 28$$

$$B := \frac{K_m^2}{R} = 0.082$$

$$M_m(\omega_m) := A - B \cdot \omega_m$$

$$M_m(\omega_m) \rightarrow -0.08166666666666679 \cdot \omega_m + 28.000000000000004$$

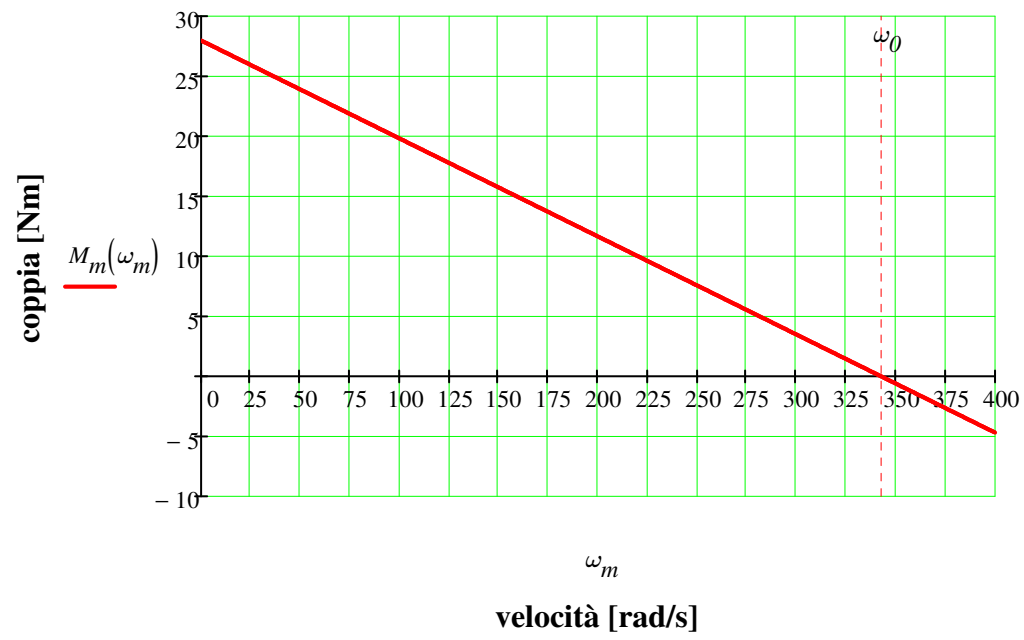
$$\omega_m := 0, 0.5.. 400$$

$$\omega_0 := \frac{A}{B} = 342.857$$

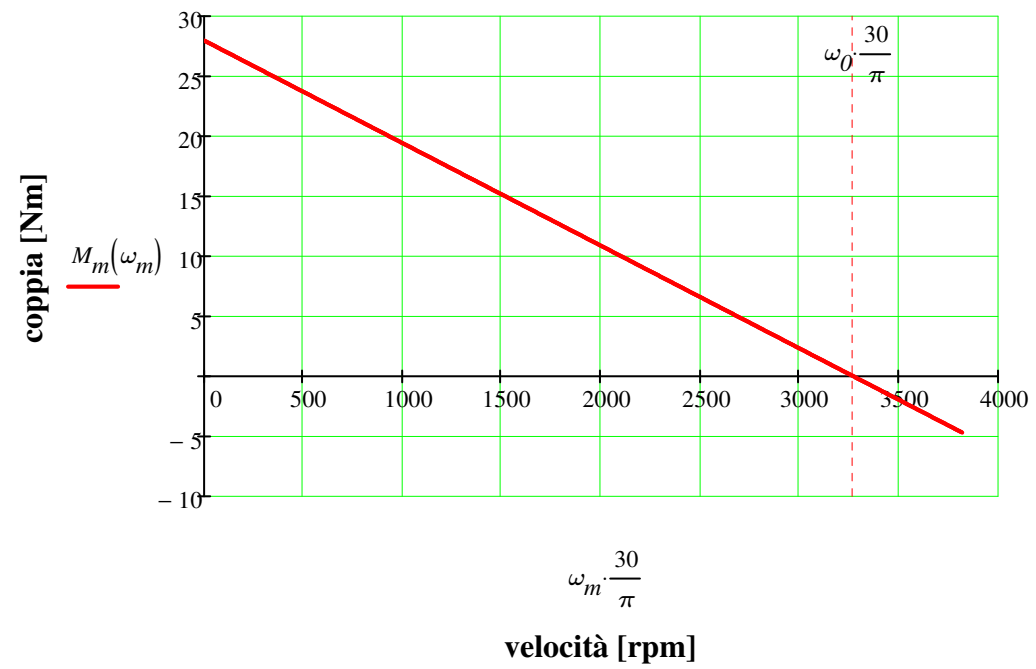
Velocità a vuoto

$$\omega_0 \cdot \frac{30}{\pi} = 3274.045$$

Coppia motrice



Coppia motrice



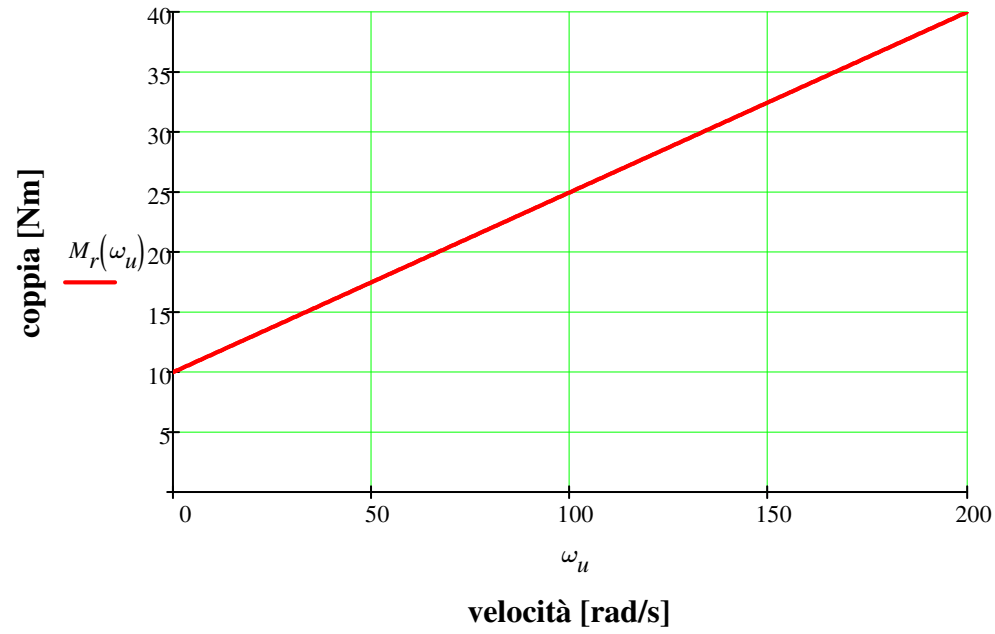
$$C := 10$$

$$D := 0.15$$

$$M_r(\omega_u) := C + D \cdot \omega_u$$

$$\omega_u := 0, 0.5.. 200$$

**Coppia resistente**



Metodo di integrazione numerica delle equazioni differenziali di moto

$$\mathbf{I} := \text{identity}(3) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{O} := 0 \cdot \mathbf{I} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{A}_{\text{sup}} := \text{augment}(\mathbf{O}, \mathbf{I}) = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{A}_{\text{inf}} := \text{augment}(-\mathbf{J}^{-1} \cdot \mathbf{K}, -\mathbf{J}^{-1} \cdot \mathbf{C}) = \begin{pmatrix} -9940.196 & 9940.196 & 0 & -4 & 4 & 0 \\ 22089.323 & -23616.137 & 6107.256 & 8.889 & -10.278 & 5.556 \\ 0 & 137.413 & -549.653 & 0 & 0.125 & -0.5 \end{pmatrix}$$

$$\mathbf{A} := \text{stack}(\mathbf{A}_{\text{sup}}, \mathbf{A}_{\text{inf}}) = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -9940.196 & 9940.196 & 0 & -4 & 4 & 0 \\ 22089.323 & -23616.137 & 6107.256 & 8.889 & -10.278 & 5.556 \\ 0 & 137.413 & -549.653 & 0 & 0.125 & -0.5 \end{pmatrix}$$

$$\mathbf{O} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

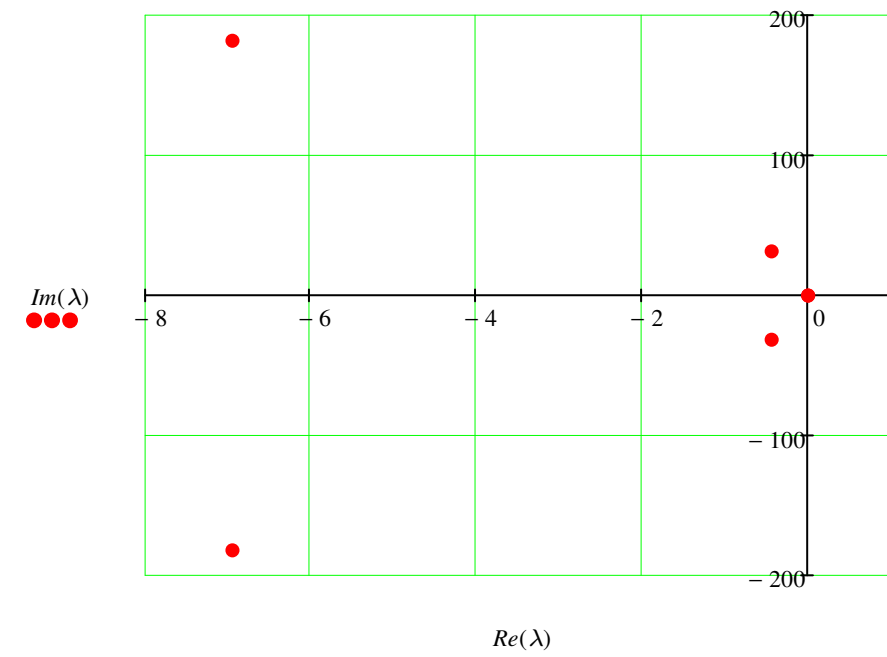
$$\mathbf{o} := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{Q}(\theta'_1, \theta'_4) := \begin{pmatrix} M_m(\theta'_1) \\ 0 \\ -M_r(\theta'_4) \end{pmatrix}$$

$$\mathbf{b}(\theta'_1, \theta'_4) := \text{stack}(\mathbf{o}, \mathbf{J}^{-1} \cdot \mathbf{Q}(\theta'_1, \theta'_4))$$

$$\mathbf{b}(\theta'_1, \theta'_4) \rightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1.63333333333333358 \cdot \theta'_1 + 560.000000000000008 \\ 0 \\ -0.15 \cdot \theta'_4 - 10 \end{pmatrix}$$

$$\lambda := \text{eigenvals}(\mathbf{A}) = \begin{pmatrix} -6.947 + 181.845i \\ -6.947 - 181.845i \\ -0.442 + 31.46i \\ -0.442 - 31.46i \\ 0 \\ 0 \end{pmatrix} \quad \text{omega} = \begin{pmatrix} 181.978 \\ 31.463 \\ 0 \end{pmatrix}$$



$$\mathbf{o} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{y} := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Condizioni iniziali

$\text{eigenvecs}(\mathbf{A}) =$

	1	2	3	4	5	6
1	$9.233 \cdot 10^{-5} + 2.164i \cdot 10^{-3}$	$9.233 \cdot 10^{-5} - 2.164i \cdot 10^{-3}$	$-3.245 \cdot 10^{-4} - 0.023i$	$-3.245 \cdot 10^{-4} + 0.023i$	-0.696	-0.696
2	$-1.928 \cdot 10^{-4} - 5.047i \cdot 10^{-3}$	$-1.928 \cdot 10^{-4} + 5.047i \cdot 10^{-3}$	$-3.276 \cdot 10^{-4} - 0.021i$	$-3.276 \cdot 10^{-4} + 0.021i$	-0.696	-0.696
3	$-1.114 \cdot 10^{-6} + 2.144i \cdot 10^{-5}$	$-1.114 \cdot 10^{-6} - 2.144i \cdot 10^{-5}$	$9.438 \cdot 10^{-5} + 6.494i \cdot 10^{-3}$	$9.438 \cdot 10^{-5} - 6.494i \cdot 10^{-3}$	-0.174	-0.174
4	$-0.394 + 1.757i \cdot 10^{-3}$	$-0.394 - 1.757i \cdot 10^{-3}$	0.727	0.727	$6.305 \cdot 10^{-7}$	$-6.305 \cdot 10^{-7}$
5	0.919	0.919	$0.655 - 1.117i \cdot 10^{-3}$	$0.655 + 1.117i \cdot 10^{-3}$	$6.305 \cdot 10^{-7}$	$-6.305 \cdot 10^{-7}$
6	$-3.891 \cdot 10^{-3} - 3.515i \cdot 10^{-4}$	$-3.891 \cdot 10^{-3} + 3.515i \cdot 10^{-4}$	$-0.204 + 1.005i \cdot 10^{-4}$	$-0.204 - 1.005i \cdot 10^{-4}$	$1.576 \cdot 10^{-7}$	$-1.576 \cdot 10^{-7}$
7						
8						
9						

$\lambda =$

	1
1	$-6.947 + 181.845i$
2	$-6.947 - 181.845i$
3	$-0.442 + 31.46i$
4	$-0.442 - 31.46i$
5	$-9.055 \cdot 10^{-7}$
6	$9.055 \cdot 10^{-7}$

$$T_{max} := 10$$

$$\omega_{max} := \max(\text{Re}(\text{omega})) = 181.978$$

$$\tau_{min} := \frac{2 \cdot \pi}{\omega_{max}} = 0.035$$

$$\Delta t_{cons} := \frac{1}{20} \cdot \tau_{min} = 1.726 \times 10^{-3}$$

$$\Delta t := 1 \cdot 10^{-3}$$

$$N := \text{ceil}\left(\frac{T_{max}}{\Delta t}\right) = 10000$$

$$\text{EQMOTO}(t, \mathbf{y}) := \mathbf{A} \cdot \mathbf{y} + \mathbf{b}(y_4, y_6)$$

Equazioni di moto nello spazio di stato

$$\text{TAB} := \text{rkfixed}(\mathbf{y}, 0, T_{max}, N, \text{EQMOTO})$$

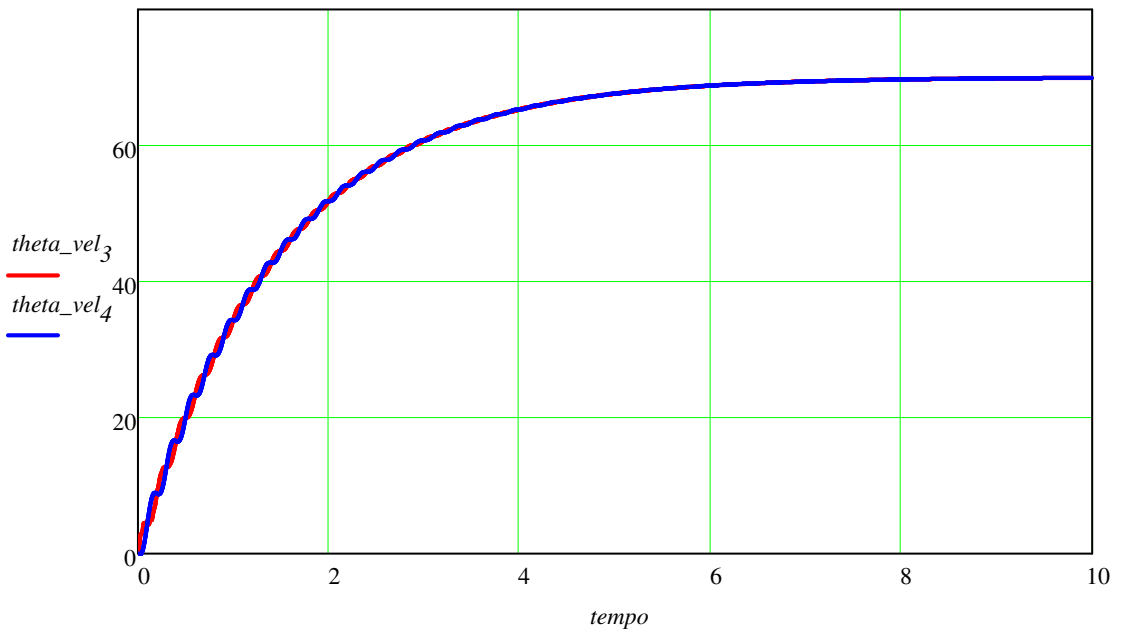
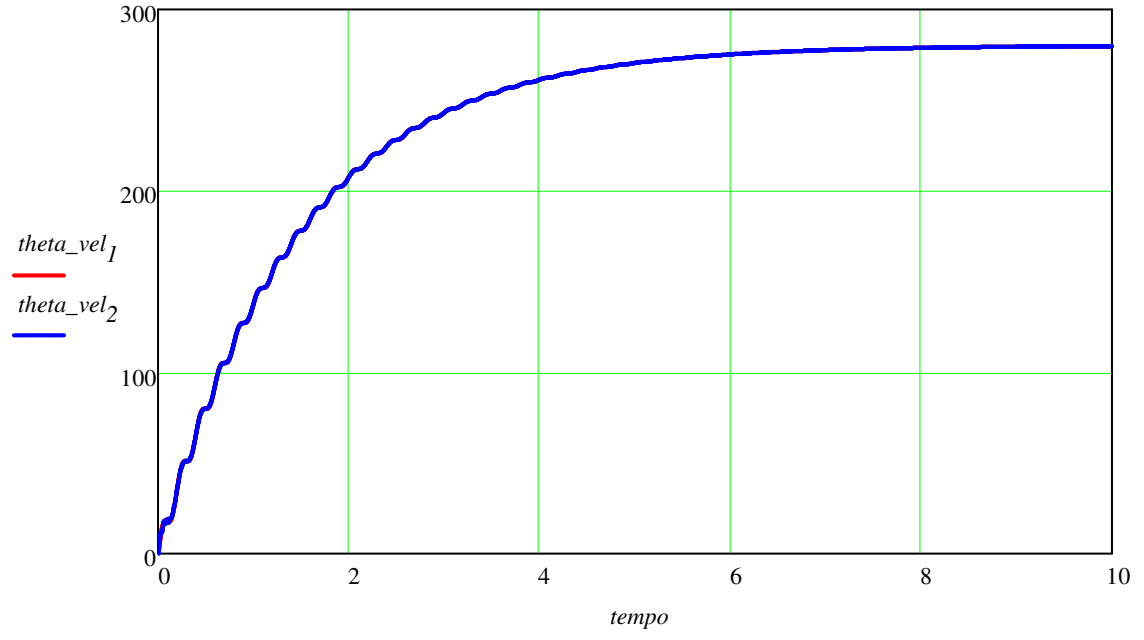
Soluzione per via numerica (Runge-Kutta)

	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	0	0	0					
2	$1 \cdot 10^{-3}$	$2.792 \cdot 10^{-4}$	$1.33 \cdot 10^{-6}$	$-4.999 \cdot 10^{-6}$	0.558	$4.484 \cdot 10^{-3}$	$-9.996 \cdot 10^{-3}$					

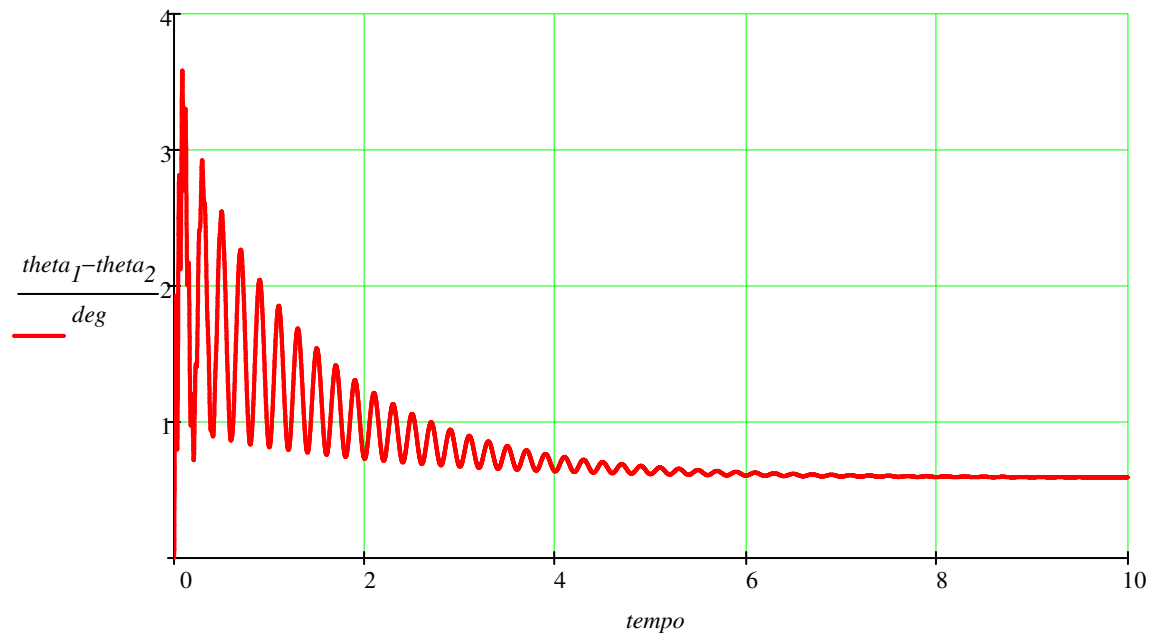
$TAB =$

3	$2 \cdot 10^{-3}$	$1.112 \cdot 10^{-3}$	$1.46 \cdot 10^{-5}$	$-1.999 \cdot 10^{-5}$	1.106	0.026	-0.02					
4	$3 \cdot 10^{-3}$	$2.488 \cdot 10^{-3}$	$6.233 \cdot 10^{-5}$	$-4.494 \cdot 10^{-5}$	1.642	0.075	-0.03					
5	$4 \cdot 10^{-3}$	$4.39 \cdot 10^{-3}$	$1.777 \cdot 10^{-4}$	$-7.984 \cdot 10^{-5}$	2.159	0.163	-0.04					
6	$5 \cdot 10^{-3}$	$6.798 \cdot 10^{-3}$	$4.033 \cdot 10^{-4}$	$-1.246 \cdot 10^{-4}$	2.654	0.297	-0.05					
7	$6 \cdot 10^{-3}$	$9.689 \cdot 10^{-3}$	$7.896 \cdot 10^{-4}$	$-1.792 \cdot 10^{-4}$	3.123	0.485	-0.059					
8	$7 \cdot 10^{-3}$	0.013	$1.393 \cdot 10^{-3}$	$-2.435 \cdot 10^{-4}$	3.565	0.732	-0.069					
9	$8 \cdot 10^{-3}$	0.017	$2.274 \cdot 10^{-3}$	$-3.173 \cdot 10^{-4}$	3.977	1.04	-0.079					
10	$9 \cdot 10^{-3}$	0.021	$3.494 \cdot 10^{-3}$	$-4.004 \cdot 10^{-4}$	4.359	1.411	-0.088					
11	0.01	0.026	$5.117 \cdot 10^{-3}$	$-4.926 \cdot 10^{-4}$	4.712	1.844	-0.097					
12	0.011	0.03	$7.202 \cdot 10^{-3}$	$-5.935 \cdot 10^{-4}$	5.036	2.334	-0.105					
13	0.012	0.036	$9.804 \cdot 10^{-3}$	$-7.028 \cdot 10^{-4}$	5.334	2.878	-0.113					
14	0.013	0.041	0.013	$-8.198 \cdot 10^{-4}$	5.608	3.467	-0.121					
15	0.014	0.047	0.017	$-9.441 \cdot 10^{-4}$	5.861	4.093	...					

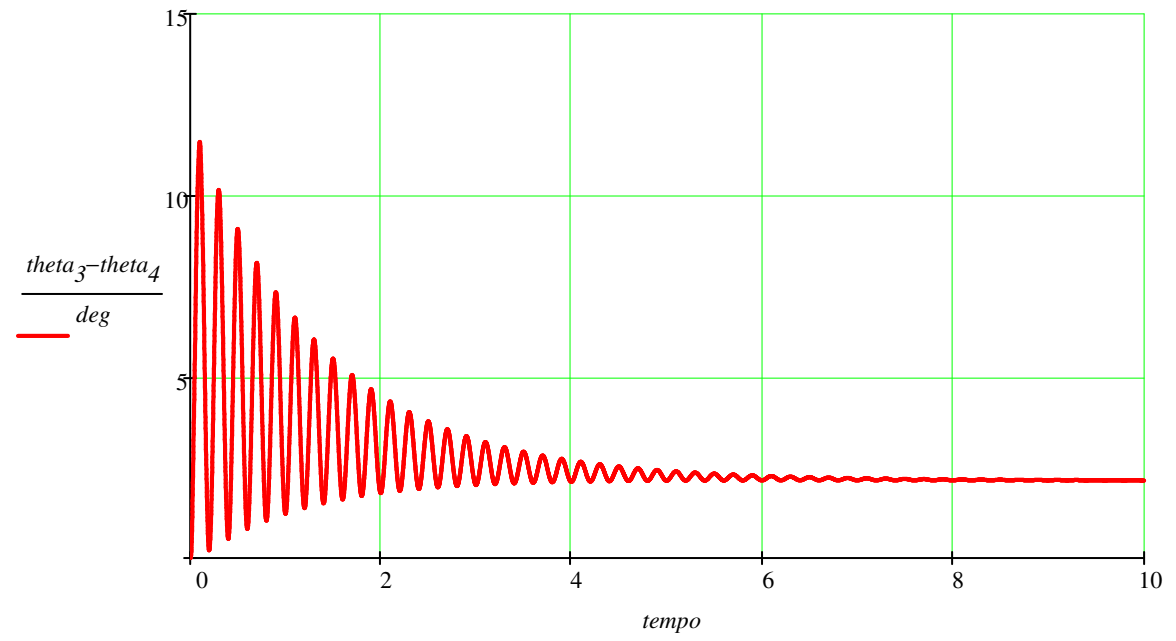
$tempo := TAB^{(1)}$   
 $theta_1 := TAB^{(2)}$        $theta_2 := TAB^{(3)}$        $theta_4 := TAB^{(4)}$        $theta_3 := \tau \cdot theta_2$   
 $theta\_vel_1 := TAB^{(5)}$        $theta\_vel_2 := TAB^{(6)}$        $theta\_vel_4 := TAB^{(7)}$        $theta\_vel_3 := \tau \cdot theta\_vel_2$



**Torsione albero motore**



**Torsione albero condotto**



Caso rigido

Momento d'inerzia equivalente ridotto all'asse del motore

$$J_{eq} := (J_1 + J_2) + \tau^2 \cdot (J_3 + J_4) = 0.135$$

Velocità del motore a regime

$$\omega_{m\_reg} := \frac{A - \tau \cdot C}{B + \tau^2 \cdot D} = 280.092$$

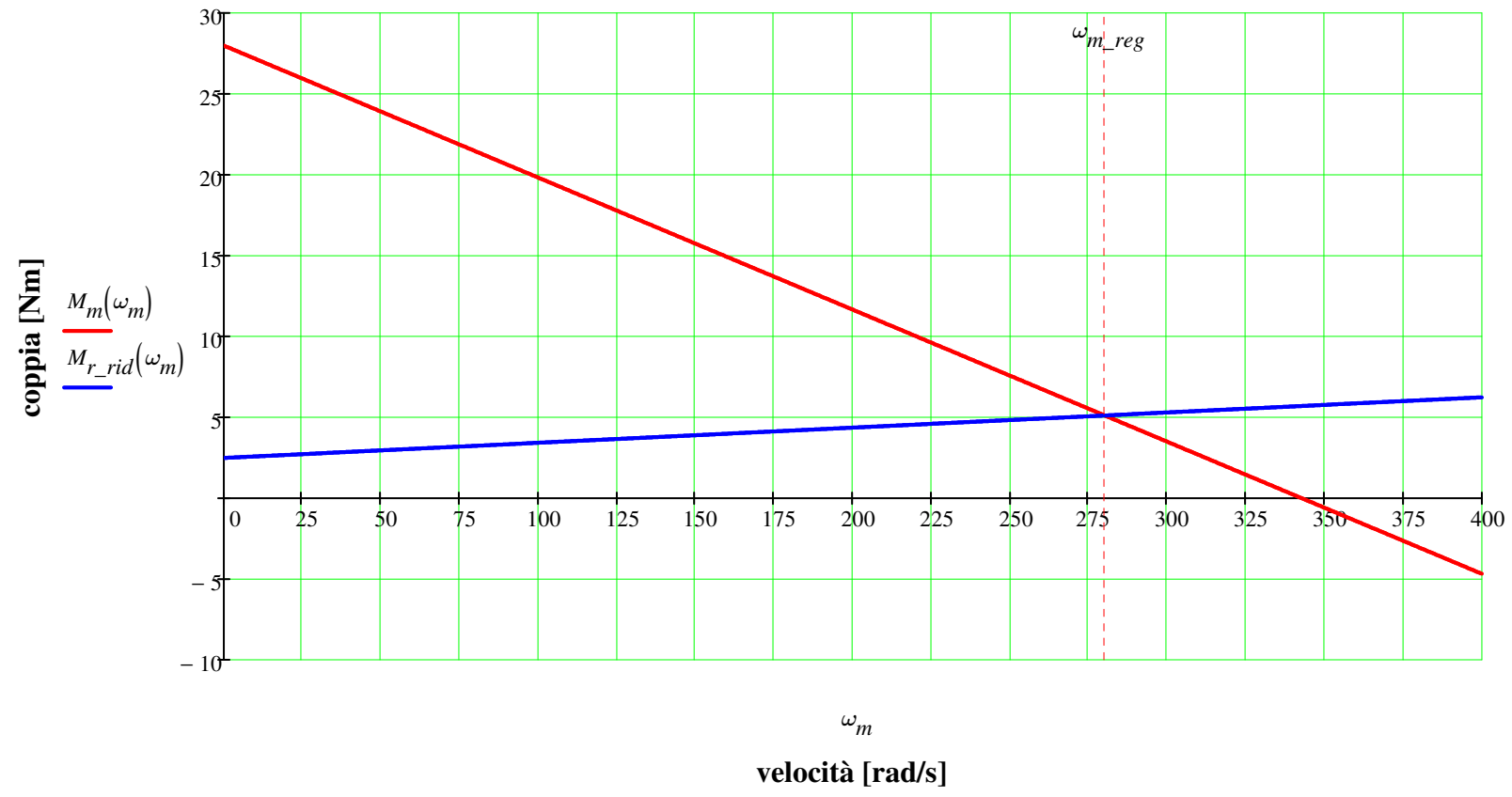
Momento resistente ridotto all'asse del motore

$$M_{r\_rid}(\omega_m) := \tau \cdot (C + D \cdot \tau \cdot \omega_m)$$

$$M_m(\omega_{m\_reg}) = 5.126$$

$$M_{r\_rid}(\omega_{m\_reg}) = 5.126$$

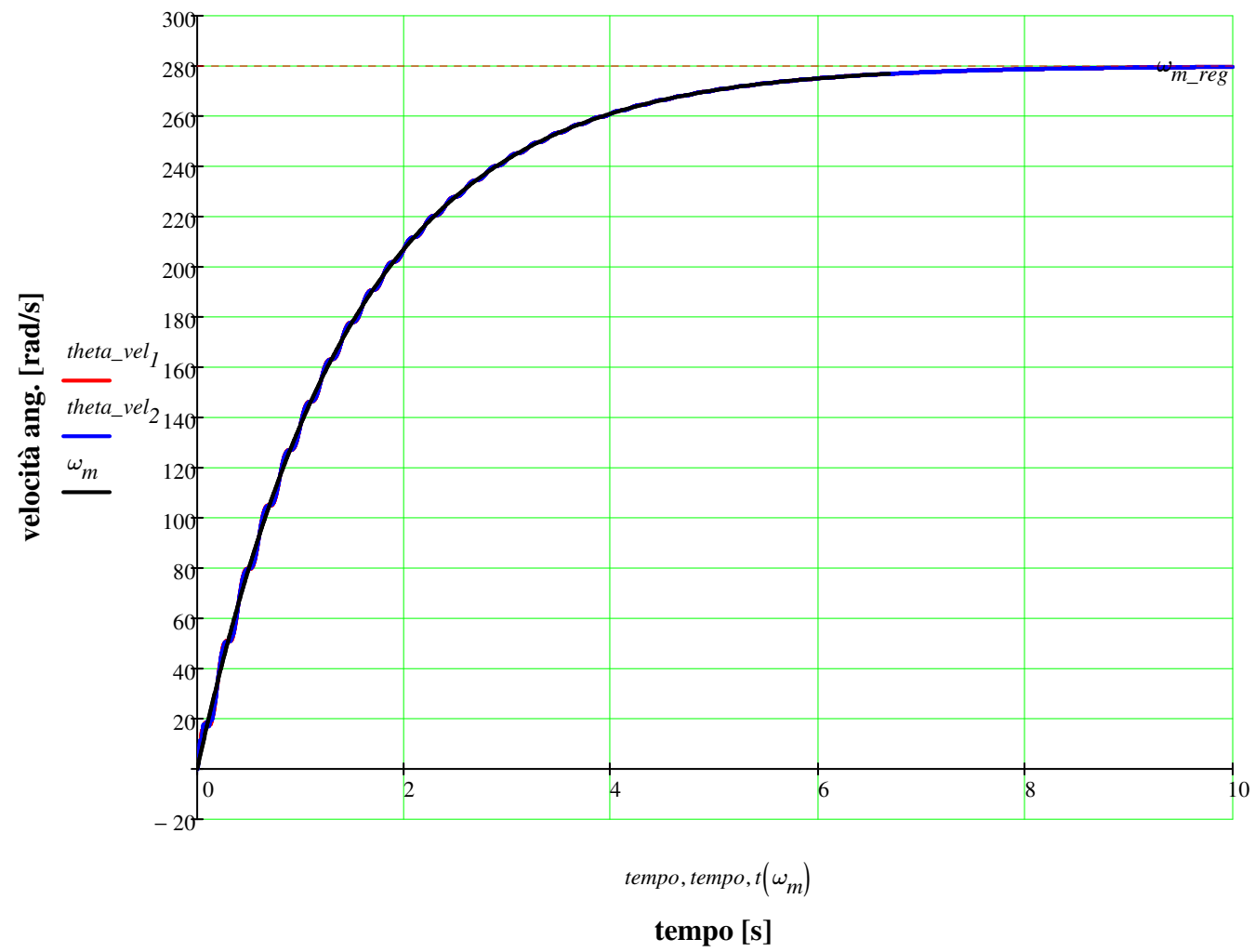
### Punto di funzionamento a regime



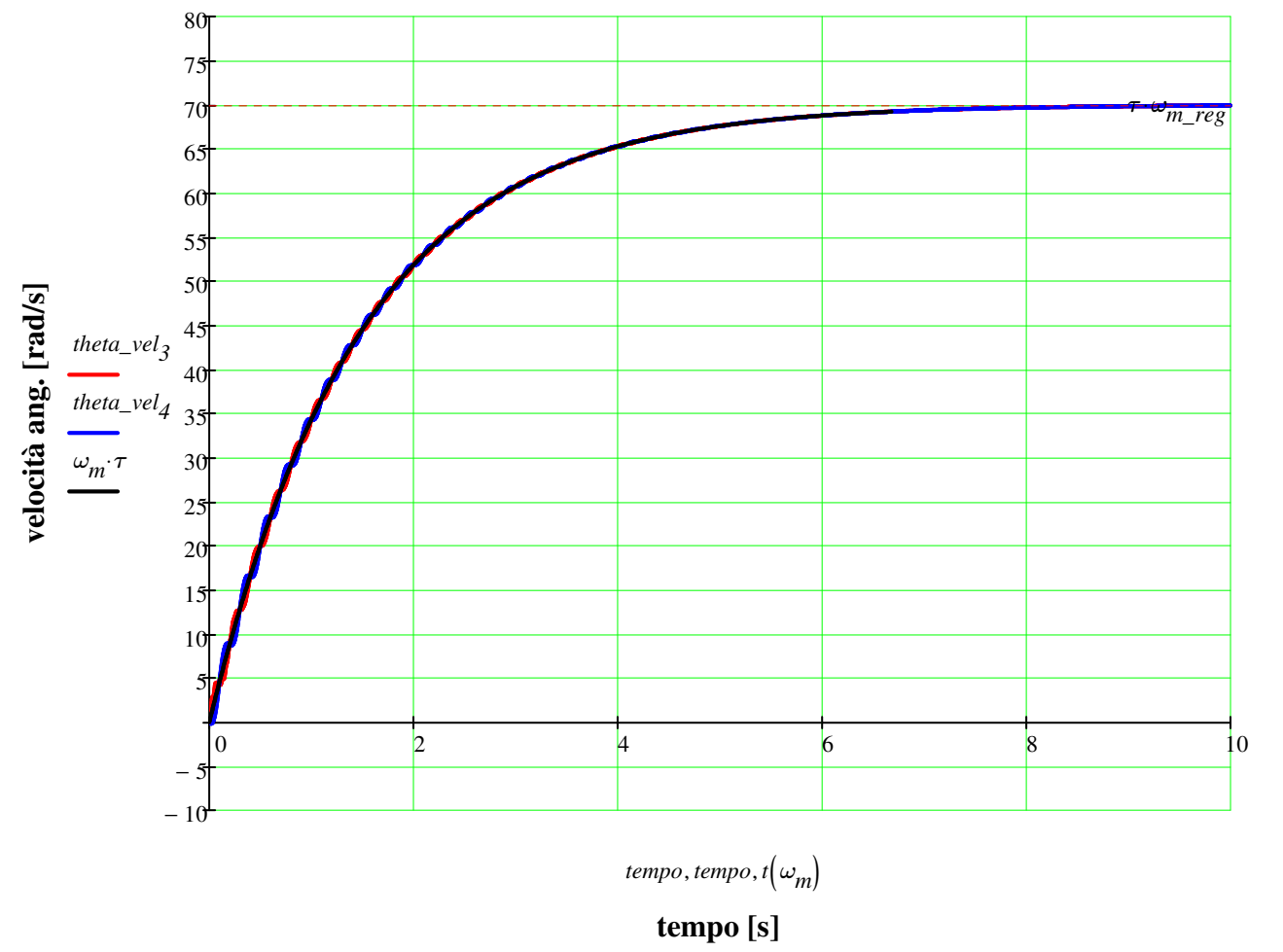
### Studio del transitorio (caso rigido)

$$t(\omega_m) := J_{eq} \int_0^{\omega_m} \frac{1}{M_m(\omega_m) - M_{r\_rid}(\omega_m)} d\omega_m$$

$$\omega_m := 0, 0.5 \dots \omega_{m\_reg} \cdot 0.99$$



- Motore
- Ruota dentata sup.
- Caso rigido



- Ruota dentata inf.
- Utilizzatore
- Caso rigido

$$P := A - \tau \cdot C = 25.5$$

$$Q := B + \tau^2 \cdot D = 0.091$$

$$\frac{P}{Q} = 280.092$$

$$\omega_{m\_reg} = 280.092$$

$$\gamma := \frac{J_{eq}}{Q} = 1.483$$

Costante di tempo [s]

$$\omega_{mot}(t) := \frac{P}{Q} \left( 1 - e^{-\frac{t}{\gamma}} \right)$$

t := 0,001..6.5

