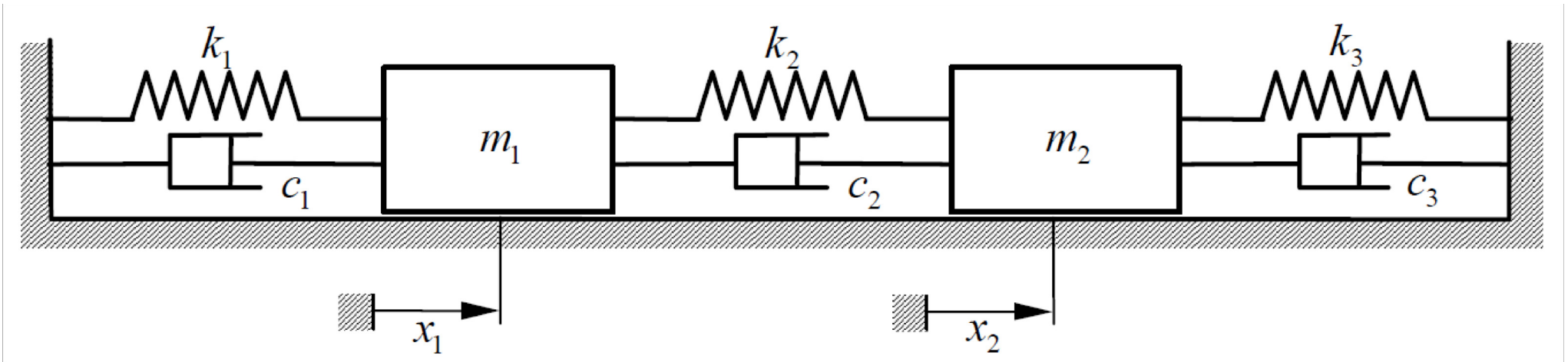


Calcolo dei coefficienti α e β (smorzamento di Rayleigh)



ORIGIN := 1

$m_1 := 5$

$m_2 := 6$

$k_1 := 5500$

$k_2 := 3500$

$k_3 := 1500$

$c_1 := 180$

$c_2 := 120$

$c_3 := 80$

$m_{11} := m_1 = 5$

$m_{12} := 0$

$m_{22} := m_2 = 6$

$$c_{11} := c_1 + c_2 = 300$$

$$c_{12} := -c_2 = -120$$

$$c_{22} := c_2 + c_3 = 200$$

$$k_{11} := k_1 + k_2 = 9000$$

$$k_{12} := -k_2 = -3500$$

$$k_{22} := k_2 + k_3 = 5000$$

$$\mathbf{M} := \begin{pmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{pmatrix} = \begin{pmatrix} 5 & 0 \\ 0 & 6 \end{pmatrix}$$

$$\mathbf{K} := \begin{pmatrix} k_{11} & k_{12} \\ k_{12} & k_{22} \end{pmatrix} = \begin{pmatrix} 9000 & -3500 \\ -3500 & 5000 \end{pmatrix}$$

$$\mathbf{C} := \begin{pmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{pmatrix} = \begin{pmatrix} 300 & -120 \\ -120 & 200 \end{pmatrix}$$

$$F_1(\alpha) := \frac{c_{11}}{k_{11}} - \frac{m_{11}}{k_{11}} \cdot \alpha$$

$$F_2(\alpha) := \frac{c_{12}}{k_{12}} - \frac{m_{12}}{k_{12}} \cdot \alpha$$

$$F_3(\alpha) := \frac{c_{22}}{k_{22}} - \frac{m_{22}}{k_{22}} \cdot \alpha$$

$$\mathbf{A} := \begin{pmatrix} m_{11} & k_{11} \\ m_{12} & k_{12} \\ m_{22} & k_{22} \end{pmatrix} = \begin{pmatrix} 5 & 9000 \\ 0 & -3500 \\ 6 & 5000 \end{pmatrix}$$

Matrice pseudoinversa (o inversa generalizzata)

$$geninv(\mathbf{A}) = \begin{pmatrix} -0.052731 & 0.165276 & 0.210609 \\ 0.00011 & -0.000134 & -0.000091 \end{pmatrix}$$

$$\mathbf{A}_{pinv} := \left(\mathbf{A}^T \cdot \mathbf{A} \right)^{-1} \cdot \mathbf{A}^T = \begin{pmatrix} -0.052731 & 0.165276 & 0.210609 \\ 0.00011 & -0.000134 & -0.000091 \end{pmatrix}$$

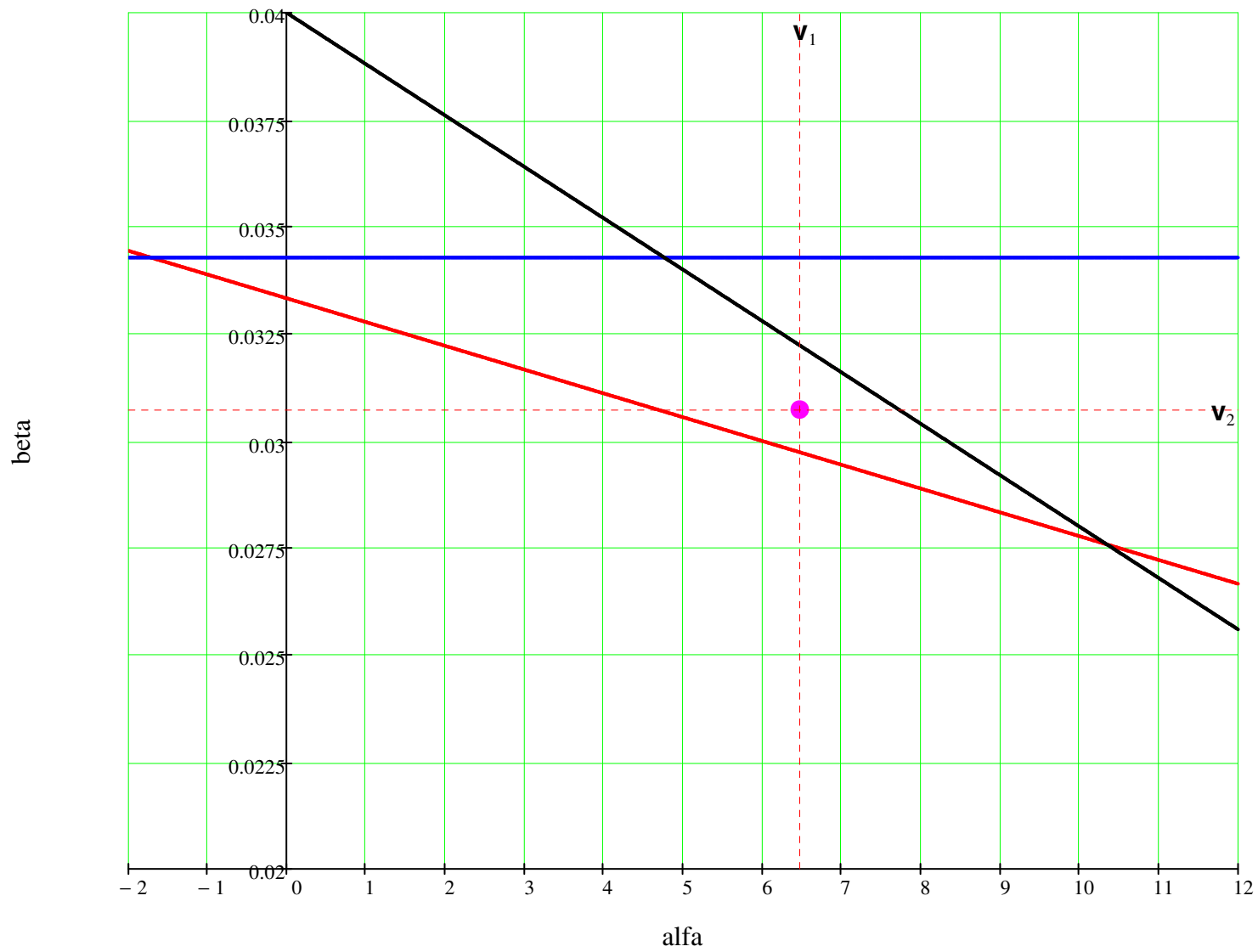
$$\mathbf{A}_{pinv} := \left(\mathbf{A}^T \cdot \mathbf{A} \right)^{-1} \cdot \mathbf{A}^T$$

$$\mathbf{A}_{pinv} = \begin{pmatrix} -0.05273 & 0.16528 & 0.21061 \\ 1.09555 \times 10^{-4} & -1.34425 \times 10^{-4} & -9.12955 \times 10^{-5} \end{pmatrix}$$

$$geninv(\mathbf{A}) = \begin{pmatrix} -0.05273 & 0.16528 & 0.21061 \\ 1.09555 \times 10^{-4} & -1.34425 \times 10^{-4} & -9.12955 \times 10^{-5} \end{pmatrix}$$

$$\mathbf{c} := \begin{pmatrix} c_{11} \\ c_{12} \\ c_{22} \end{pmatrix} = \begin{pmatrix} 300 \\ -120 \\ 200 \end{pmatrix}$$

$$\mathbf{v} := \mathbf{A}_{pinv} \cdot \mathbf{c} = \begin{pmatrix} 6.46938 \\ 0.03074 \end{pmatrix}$$



$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} := \mathbf{v} \quad \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 6.46938 \\ 0.03074 \end{pmatrix}$$

Matrice C "originale"

$$\mathbf{C} = \begin{pmatrix} 300 & -120 \\ -120 & 200 \end{pmatrix} \quad \begin{bmatrix} (c_1 + c_2) & -c_2 \\ -c_2 & (c_2 + c_3) \end{bmatrix} = \begin{pmatrix} 300 & -120 \\ -120 & 200 \end{pmatrix}$$

Matrice C ottenuta con l'ipotesi di Rayleigh

$$\mathbf{M} = \begin{pmatrix} 5 & 0 \\ 0 & 6 \end{pmatrix} \quad \mathbf{K} = \begin{pmatrix} 9000 & -3500 \\ -3500 & 5000 \end{pmatrix} \quad \alpha = 6.46938 \quad \beta = 0.03074$$

$$\mathbf{C}_{\text{ray}} := \alpha \cdot \mathbf{M} + \beta \cdot \mathbf{K} = \begin{pmatrix} 308.991 & -107.5838 \\ -107.5838 & 192.5075 \end{pmatrix}$$

Differenza fra le due matrici (per valutare l'errore commesso con l'ipotesi di Rayleigh)

$$\mathbf{C} - \mathbf{C}_{\text{ray}} = \begin{pmatrix} -8.99103 & -12.41618 \\ -12.41618 & 7.49252 \end{pmatrix}$$

$$\omega := \sqrt{\text{genvals}(\mathbf{K}, \mathbf{M})} = \begin{pmatrix} 46.02044 \\ 22.70357 \end{pmatrix} \quad \text{Pulsazioni proprie}$$

$$\alpha = 6.46938 \quad \beta = 0.03074$$

$$\xi(\omega) := \frac{\alpha + \omega^2 \cdot \beta}{2 \cdot \omega}$$

$$\omega_1 = 46.02044 \quad \xi_1 := \xi(\omega_1) = 0.77758 \quad \xi_1 = 77.75817\%$$

$$\omega_2 = 22.70357 \quad \xi_2 := \xi(\omega_2) = 0.49141 \quad \xi_2 = 49.14089\%$$

Fattori di smorzamento modali

$\omega := 0,0.01..80$

Fattore di smorzamento modale al variare della pulsazione

