Vibrazioni torsionali (avviamento di un carico azionato da motore DC)

$J_1 := 0.05$

$J_2 := 0.02$

$J_3 := 0.04$

$J_4 := 1$

$\tau := \frac{1}{4}$

$G := 80000 \cdot 10^6 = 8 \times 10^{10}$

$d_1 := 15 \cdot 10^{-3} = 0.015$

$d_2 := 18 \cdot 10^{-3} = 0.018$

$l_1 := 800 \cdot 10^{-3} = 0.8$

$l_2 := 1500 \cdot 10^{-3} = 1.5$

$c_1 := 0.2 \qquad c_2 := 0.5$

$k_1 := \frac{G}{l_1} \cdot \left(\frac{\pi \cdot d_1^4}{32} \right) = 497.01$ Nm/rad

$k_2 := \frac{G}{l_2} \cdot \left(\frac{\pi \cdot d_2^4}{32} \right) = 549.653$ Nm/rad

$SMORZ := 1$

$$\mathbf{J} := \begin{bmatrix} J_1 & 0 & 0 \\ 0 & \left(J_2 + J_3 \cdot \tau^2\right) & 0 \\ 0 & 0 & J_4 \end{bmatrix} = \begin{pmatrix} 0.05 & 0 & 0 \\ 0 & 0.023 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{K} := \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & \left(k_1 + k_2 \cdot \tau^2\right) & -\tau \cdot k_2 \\ 0 & -\tau \cdot k_2 & k_2 \end{bmatrix} = \begin{pmatrix} 497.01 & -497.01 & 0 \\ -497.01 & 531.363 & -137.413 \\ 0 & -137.413 & 549.653 \end{pmatrix}$$

$$\mathbf{C} := \begin{bmatrix} c_1 & -c_1 & 0 \\ -c_1 & \left(c_1 + c_2 \cdot \tau^2\right) & -\tau \cdot c_2 \\ 0 & -\tau \cdot c_2 & c_2 \end{bmatrix} \cdot SMORZ = \begin{pmatrix} 0.2 & -0.2 & 0 \\ -0.2 & 0.231 & -0.125 \\ 0 & -0.125 & 0.5 \end{pmatrix}$$

$|\mathbf{K}| = 0$

$$coefficienti := \begin{pmatrix} 0 \\ 0 \\ -J_1 \cdot k_1 \cdot k_2 - J_2 \cdot k_1 \cdot k_2 - J_3 \cdot \tau^2 \cdot k_1 \cdot k_2 - J_4 \cdot \tau^2 \cdot k_1 \cdot k_2 \\ 0 \\ J_1 \cdot J_2 \cdot k_2 + J_1 \cdot J_4 \cdot k_1 + J_2 \cdot J_4 \cdot k_1 + J_1 \cdot J_3 \cdot \tau^2 \cdot k_2 + J_1 \cdot J_4 \cdot \tau^2 \cdot k_2 + J_3 \cdot J_4 \cdot \tau^2 \cdot k_1 \\ 0 \\ -J_1 \cdot J_3 \cdot J_4 \cdot \tau^2 - J_1 \cdot J_2 \cdot J_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -3.688 \times 10^4 \\ 0 \\ 38.369 \\ 0 \\ -1.125 \times 10^{-3} \end{pmatrix}$$

$$polyroots(coefficienti) = \begin{pmatrix} -181.978 \\ -31.463 \\ 0 \\ 0 \\ 31.463 \\ 181.978 \end{pmatrix}$$

$omega := \sqrt{genvals(\mathbf{K}, \mathbf{J})}$ *pulsazioni proprie [rad/s]*

$$omega = \begin{pmatrix} 181.978 \\ 31.463 \\ 0 \end{pmatrix}$$

$$\frac{omega}{2 \cdot \pi} = \begin{pmatrix} 28.963 \\ 5.007 \\ 0 \end{pmatrix}$$

frequenze proprie [Hz]

Matrice modale

$$\Phi := genvecs(\mathbf{K},\mathbf{J}) = \begin{pmatrix} 0.429 & -1 & -1 \\ -1 & -0.9 & -1 \\ 4.219\times 10^{-3} & 0.281 & -0.25 \end{pmatrix}$$

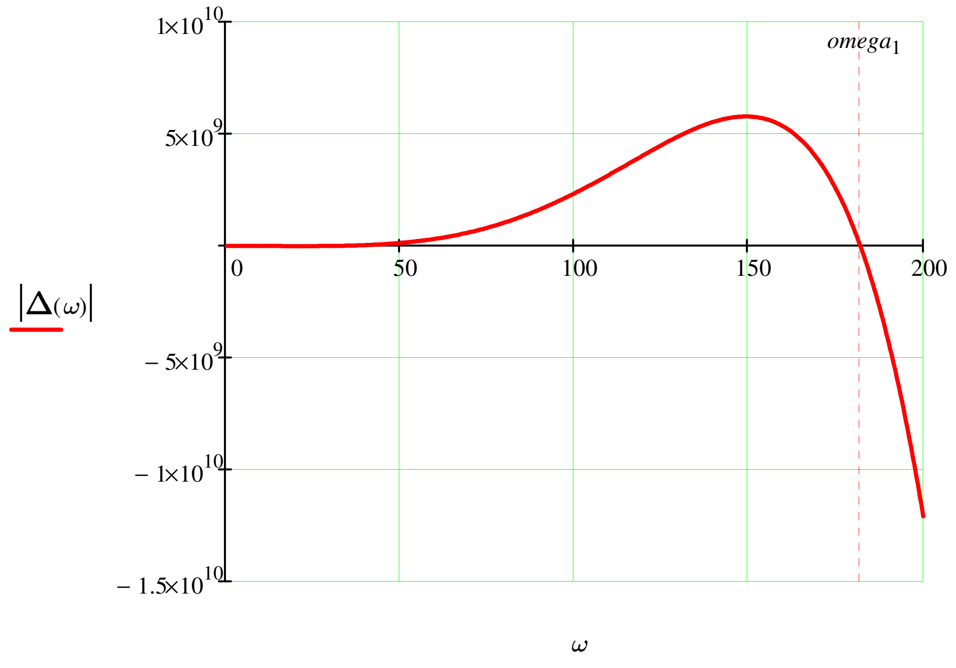
$$omega = \begin{pmatrix} 181.978 \\ 31.463 \\ 0 \end{pmatrix}$$

Matrice modale normalizzata (l'ultima riga contiene elementi unitari)

$$\Phi_{\mathbf{norm}} := augment\left(\frac{\Phi^{\langle 1\rangle}}{\Phi_{3,1}},\frac{\Phi^{\langle 2\rangle}}{\Phi_{3,2}},\frac{\Phi^{\langle 3\rangle}}{\Phi_{3,3}}\right) = \blacksquare$$

$$\Delta(\omega) := \mathbf{K} - \omega^2\cdot\mathbf{J}$$

$\omega := 0, 0.5 .. 200$

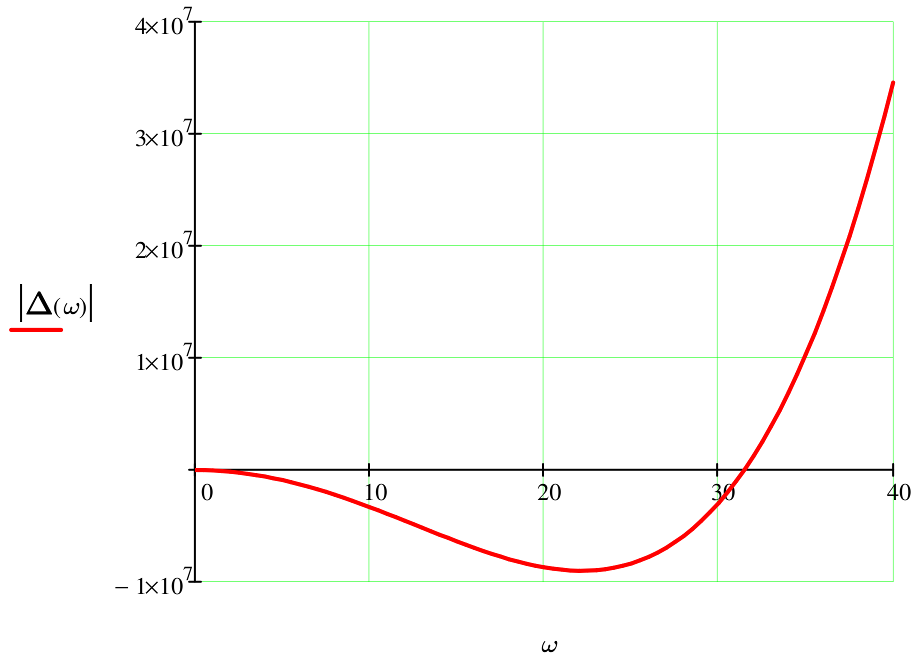


$omega_1 = 181.978$

$\omega := 200$

$root(|\Delta(\omega)|, \omega) = 181.978$

$\omega := 0, 0.5 .. 40$



$omega_1 = 181.978$

Curve caratteristiche (motore e utilizzatore)

$K_m := 0.07$

$V_m := 24$

$R := 0.06$

$A := \frac{V_m \cdot K_m}{R} = 28$

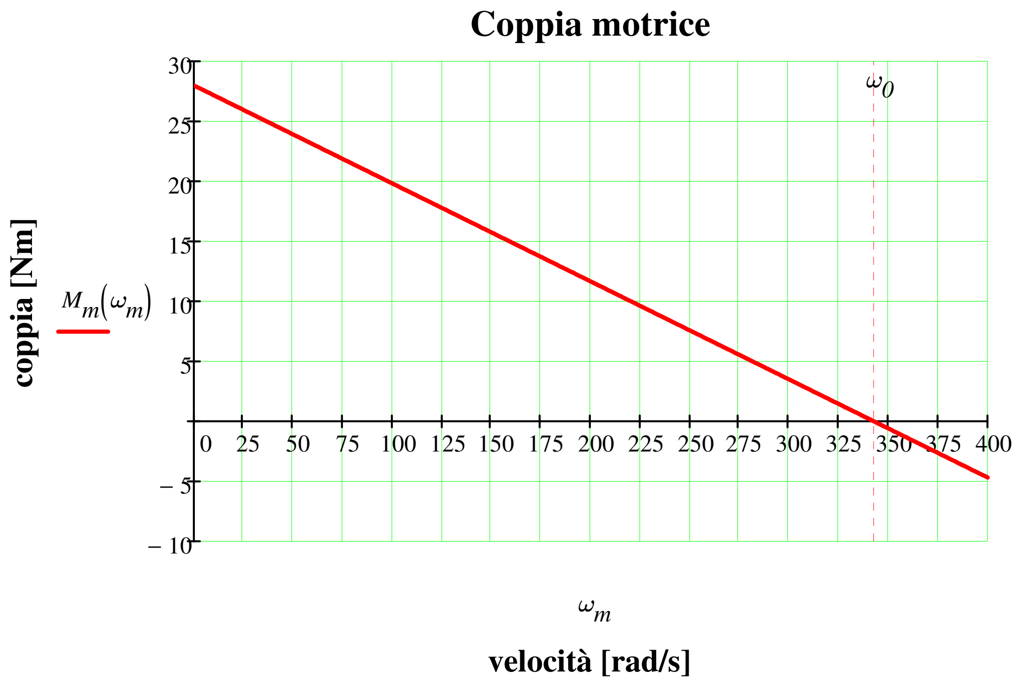
$B := \frac{K_m^{\,2}}{R} = 0.082$

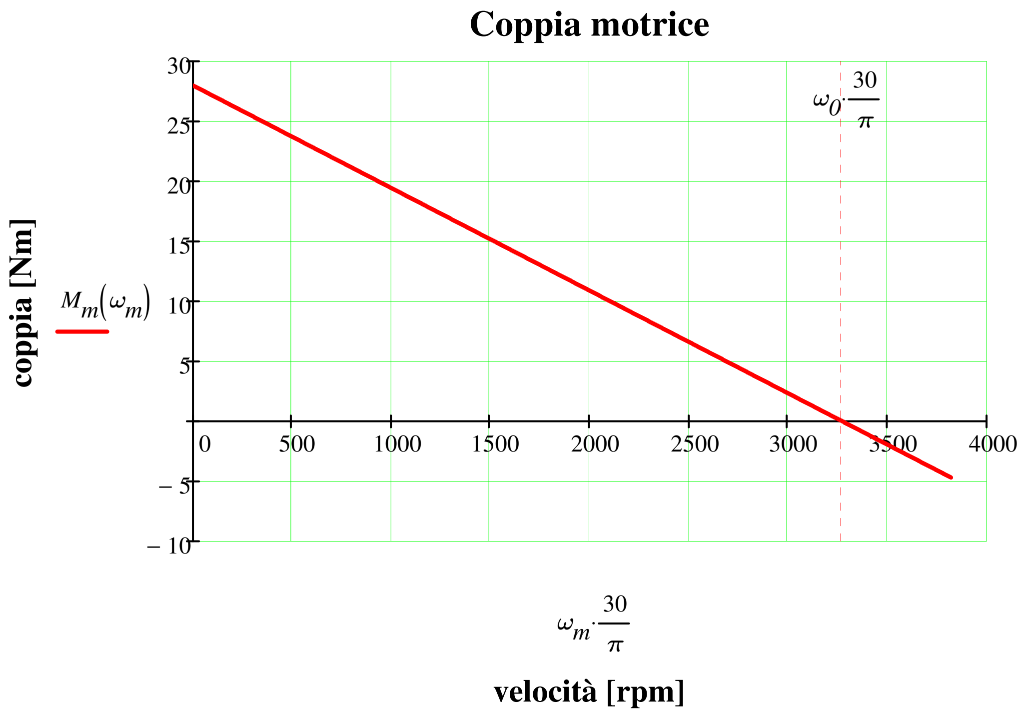
$M_m(\omega_m) := A - B \cdot \omega_m$

$M_m(\omega_m) \rightarrow -0.081666666666666679 \cdot \omega_m + 28.000000000000004$

$\omega_m := 0, 0.5 .. 400$

$\omega_0 := \frac{A}{B} = 342.857$ Velocità a vuoto $\omega_0 \cdot \frac{30}{\pi} = 3274.045$



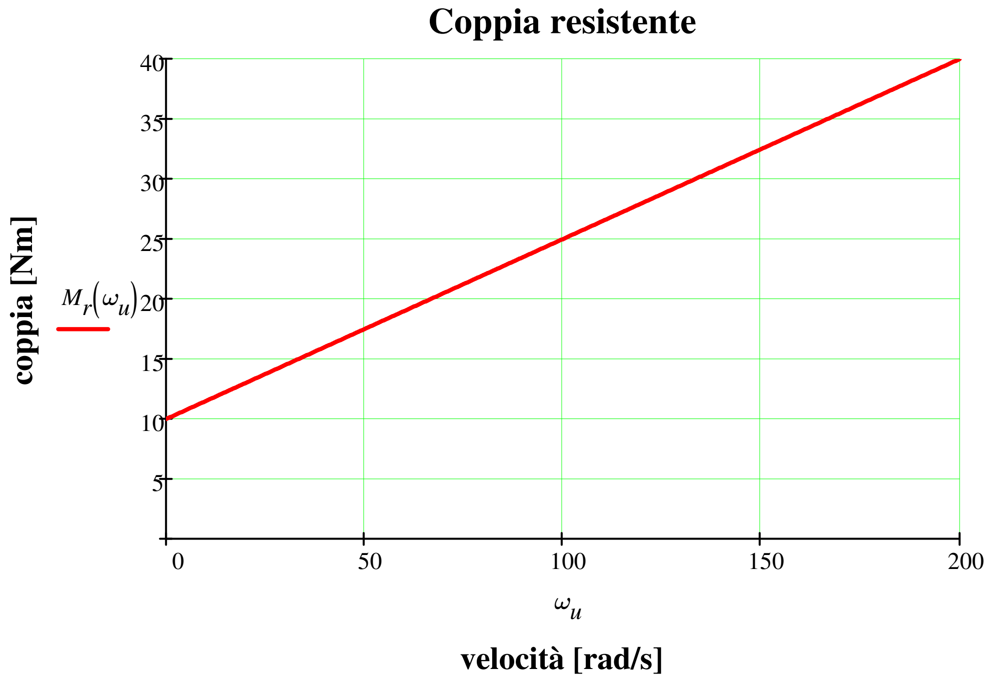


$C := 10$

$D := 0.15$

$M_r(\omega_u) := C + D \cdot \omega_u$

$\omega_u := 0, 0.5 .. 200$



Metodo di integrazione numerica delle equazioni differenziali di moto

$$\mathbf{I} := identity(3) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \mathbf{O} := 0 \cdot \mathbf{I} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{A_{sup}} := augment(\mathbf{O}, \mathbf{I}) = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{A_{inf}} := augment\left(-\mathbf{J}^{-1} \cdot \mathbf{K}, -\mathbf{J}^{-1} \cdot \mathbf{C}\right) = \begin{pmatrix} -9940.196 & 9940.196 & 0 & -4 & 4 & 0 \\ 22089.323 & -23616.137 & 6107.256 & 8.889 & -10.278 & 5.556 \\ 0 & 137.413 & -549.653 & 0 & 0.125 & -0.5 \end{pmatrix}$$

$$\mathbf{O} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{A} := stack(\mathbf{A_{sup}}, \mathbf{A_{inf}}) = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -9940.196 & 9940.196 & 0 & -4 & 4 & 0 \\ 22089.323 & -23616.137 & 6107.256 & 8.889 & -10.278 & 5.556 \\ 0 & 137.413 & -549.653 & 0 & 0.125 & -0.5 \end{pmatrix}$$

$$\mathbf{o} := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

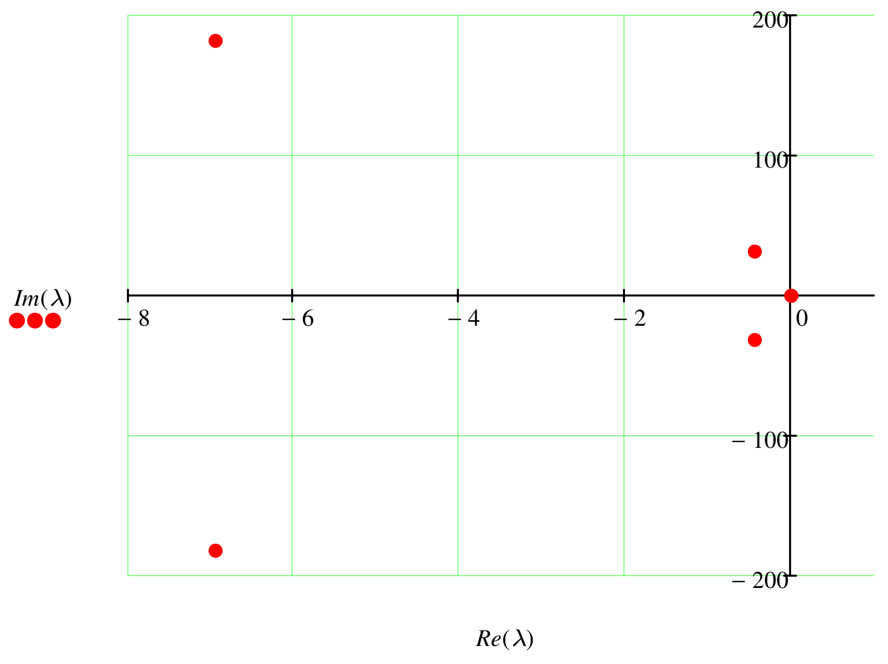
$$\mathbf{Q}(\theta'_1, \theta'_4) := \begin{pmatrix} M_m(\theta'_1) \\ 0 \\ -M_r(\theta'_4) \end{pmatrix}$$

$$\mathbf{o} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{b}(\theta'_1, \theta'_4) := stack\left(\mathbf{o}, \mathbf{J}^{-1} \cdot \mathbf{Q}(\theta'_1, \theta'_4)\right)$$

$$\mathbf{b}(\theta'_1, \theta'_4) \rightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1.6333333333333358 \cdot \theta'_1 + 560.00000000000008 \\ 0 \\ -0.15 \cdot \theta'_4 - 10 \end{pmatrix}$$

$$\lambda := eigenvals(\mathbf{A}) = \begin{pmatrix} -6.947 + 181.845i \\ -6.947 - 181.845i \\ -0.442 + 31.46i \\ -0.442 - 31.46i \\ 0 \\ 0 \end{pmatrix} \qquad omega = \begin{pmatrix} 181.978 \\ 31.463 \\ 0 \end{pmatrix}$$



$$\mathbf{y} := \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Condizioni iniziali

$eigenvecs(\mathbf{A}) =$

	1	2	3	4	5	6
1	$9.233\cdot10^{-5}+2.164i\cdot10^{-3}$	$9.233\cdot10^{-5}-2.164i\cdot10^{-3}$	$-3.245\cdot10^{-4}-0.023i$	$-3.245\cdot10^{-4}+0.023i$	-0.696	-0.696
2	$-1.928\cdot10^{-4}-5.047i\cdot10^{-3}$	$-1.928\cdot10^{-4}+5.047i\cdot10^{-3}$	$-3.276\cdot10^{-4}-0.021i$	$-3.276\cdot10^{-4}+0.021i$	-0.696	-0.696
3	$-1.114\cdot10^{-6}+2.144i\cdot10^{-5}$	$-1.114\cdot10^{-6}-2.144i\cdot10^{-5}$	$9.438\cdot10^{-5}+6.494i\cdot10^{-3}$	$9.438\cdot10^{-5}-6.494i\cdot10^{-3}$	-0.174	-0.174
4	$-0.394+1.757i\cdot10^{-3}$	$-0.394-1.757i\cdot10^{-3}$	0.727	0.727	$6.305\cdot10^{-7}$	$-6.305\cdot10^{-7}$
5	0.919	0.919	$0.655-1.117i\cdot10^{-3}$	$0.655+1.117i\cdot10^{-3}$	$6.305\cdot10^{-7}$	$-6.305\cdot10^{-7}$
6	$-3.891\cdot10^{-3}-3.515i\cdot10^{-4}$	$-3.891\cdot10^{-3}+3.515i\cdot10^{-4}$	$-0.204+1.005i\cdot10^{-4}$	$-0.204-1.005i\cdot10^{-4}$	$1.576\cdot10^{-7}$	$-1.576\cdot10^{-7}$
7						
8						
9						

$\lambda =$

	1
1	-6.947+181.845i
2	-6.947-181.845i
3	-0.442+31.46i
4	-0.442-31.46i
5	$-9.055\cdot10^{-7}$
6	$9.055\cdot10^{-7}$

$T_{max} := 10$

$\omega_{max} := max(Re(omega)) = 181.978$

$\tau_{min} := \frac{2\cdot\pi}{\omega_{max}} = 0.035$

$\Delta t_{cons} := \frac{1}{20}\cdot\tau_{min} = 1.726\times 10^{-3}$

$\Delta t := 1\cdot 10^{-3}$

$N := ceil\left(\frac{T_{max}}{\Delta t}\right) = 10000$

$EQMOTO(t, \mathbf{y}) := \mathbf{A}\cdot\mathbf{y} + \mathbf{b}(\mathbf{y}_4, \mathbf{y}_6)$

Equazioni di moto nello spazio di stato

$TAB := rkfixed(\mathbf{y}, 0, T_{max}, N, EQMOTO)$

Soluzione per via numerica (Runge-Kutta)

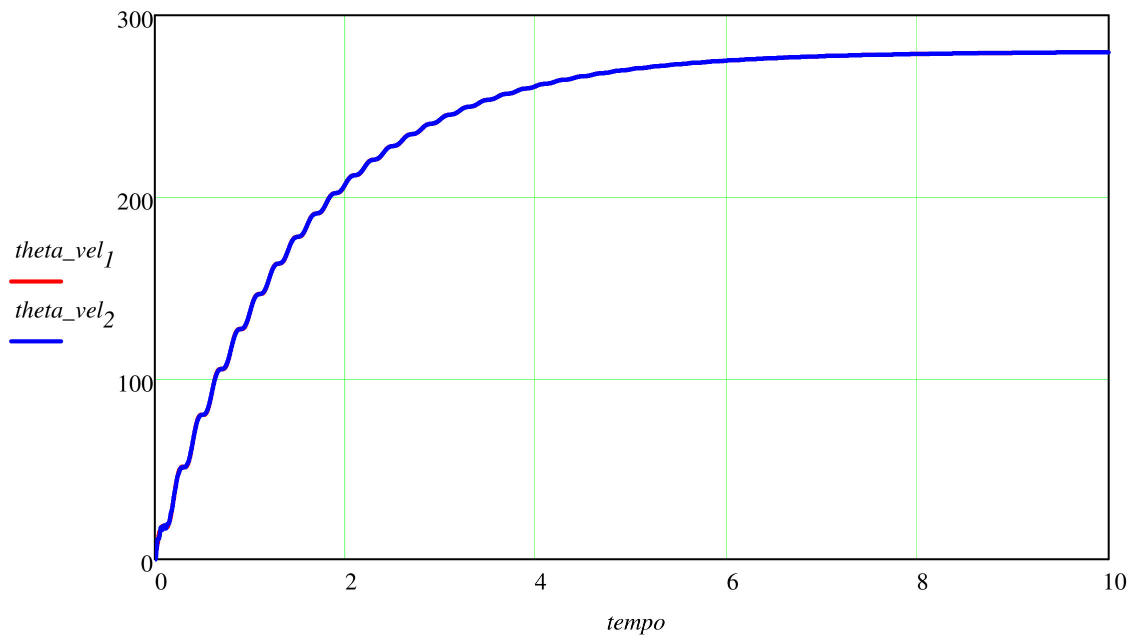
$TAB =$

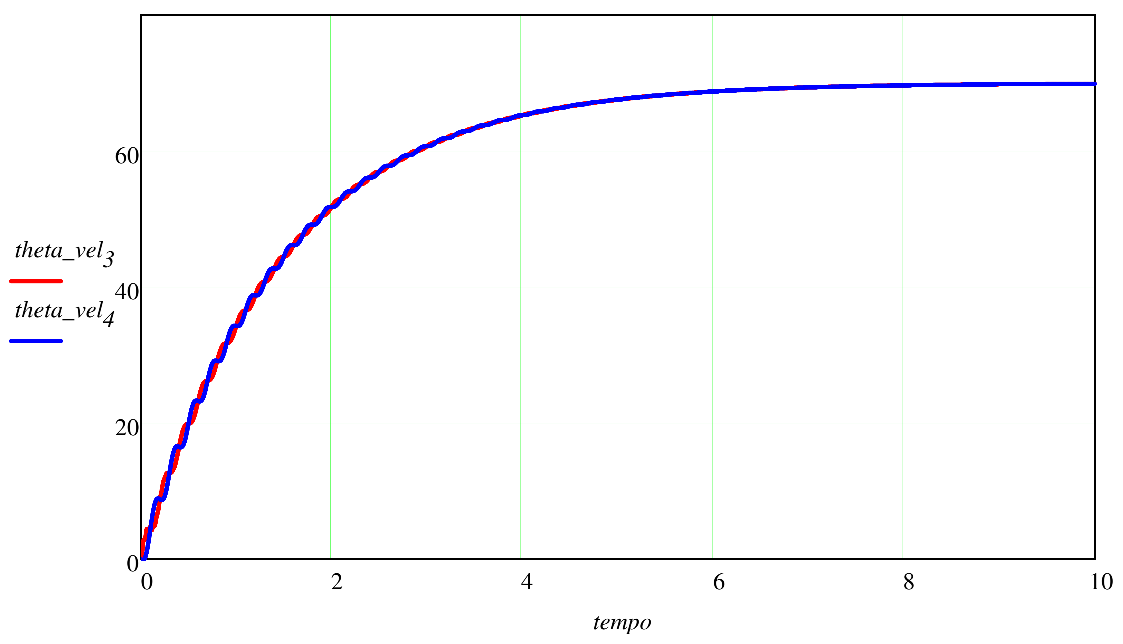
	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	0	0	0					
2	$1\cdot10^{-3}$	$2.792\cdot10^{-4}$	$1.33\cdot10^{-6}$	$-4.999\cdot10^{-6}$	0.558	$4.484\cdot10^{-3}$	$-9.996\cdot10^{-3}$					
3	$2\cdot10^{-3}$	$1.112\cdot10^{-3}$	$1.46\cdot10^{-5}$	$-1.999\cdot10^{-5}$	1.106	0.026	-0.02					
4	$3\cdot10^{-3}$	$2.488\cdot10^{-3}$	$6.233\cdot10^{-5}$	$-4.494\cdot10^{-5}$	1.642	0.075	-0.03					
5	$4\cdot10^{-3}$	$4.39\cdot10^{-3}$	$1.777\cdot10^{-4}$	$-7.984\cdot10^{-5}$	2.159	0.163	-0.04					
6	$5\cdot10^{-3}$	$6.798\cdot10^{-3}$	$4.033\cdot10^{-4}$	$-1.246\cdot10^{-4}$	2.654	0.297	-0.05					
7	$6\cdot10^{-3}$	$9.689\cdot10^{-3}$	$7.896\cdot10^{-4}$	$-1.792\cdot10^{-4}$	3.123	0.485	-0.059					
8	$7\cdot10^{-3}$	0.013	$1.393\cdot10^{-3}$	$-2.435\cdot10^{-4}$	3.565	0.732	-0.069					
9	$8\cdot10^{-3}$	0.017	$2.274\cdot10^{-3}$	$-3.173\cdot10^{-4}$	3.977	1.04	-0.079					
10	$9\cdot10^{-3}$	0.021	$3.494\cdot10^{-3}$	$-4.004\cdot10^{-4}$	4.359	1.411	-0.088					
11	0.01	0.026	$5.117\cdot10^{-3}$	$-4.926\cdot10^{-4}$	4.712	1.844	-0.097					
12	0.011	0.03	$7.202\cdot10^{-3}$	$-5.935\cdot10^{-4}$	5.036	2.334	-0.105					
13	0.012	0.036	$9.804\cdot10^{-3}$	$-7.028\cdot10^{-4}$	5.334	2.878	-0.113					
14	0.013	0.041	0.013	$-8.198\cdot10^{-4}$	5.608	3.467	-0.121					
15	0.014	0.047	0.017	$-9.441\cdot10^{-4}$	5.861	4.093	...					

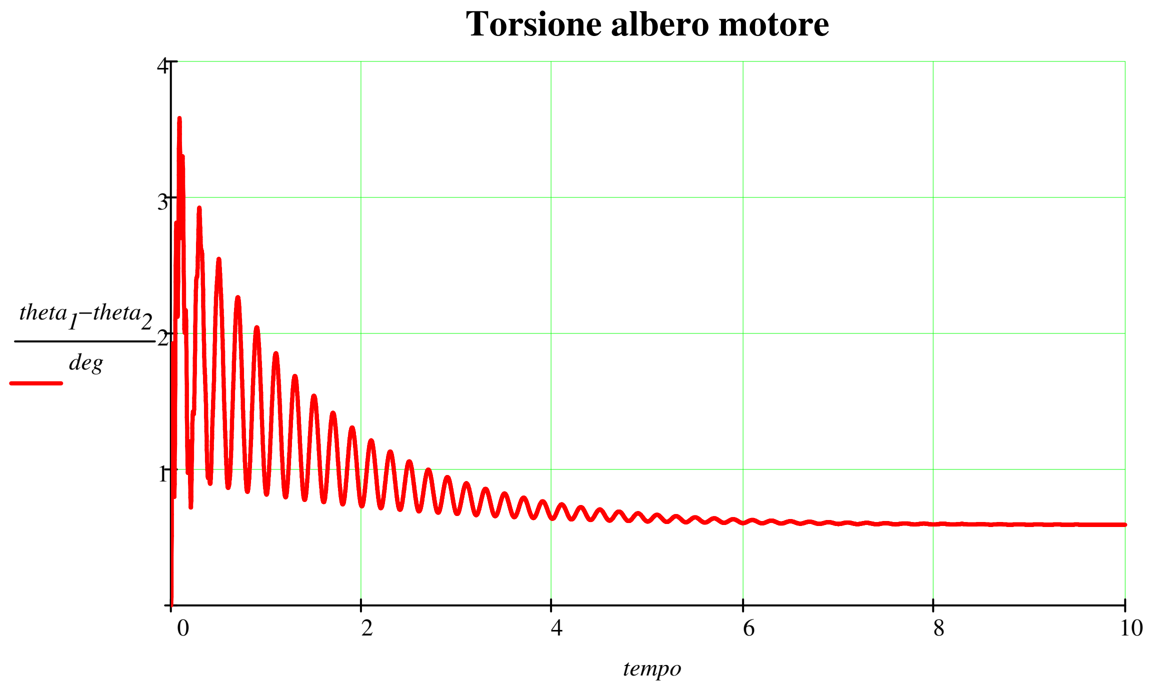
$tempo := TAB^{\langle 1 \rangle}$

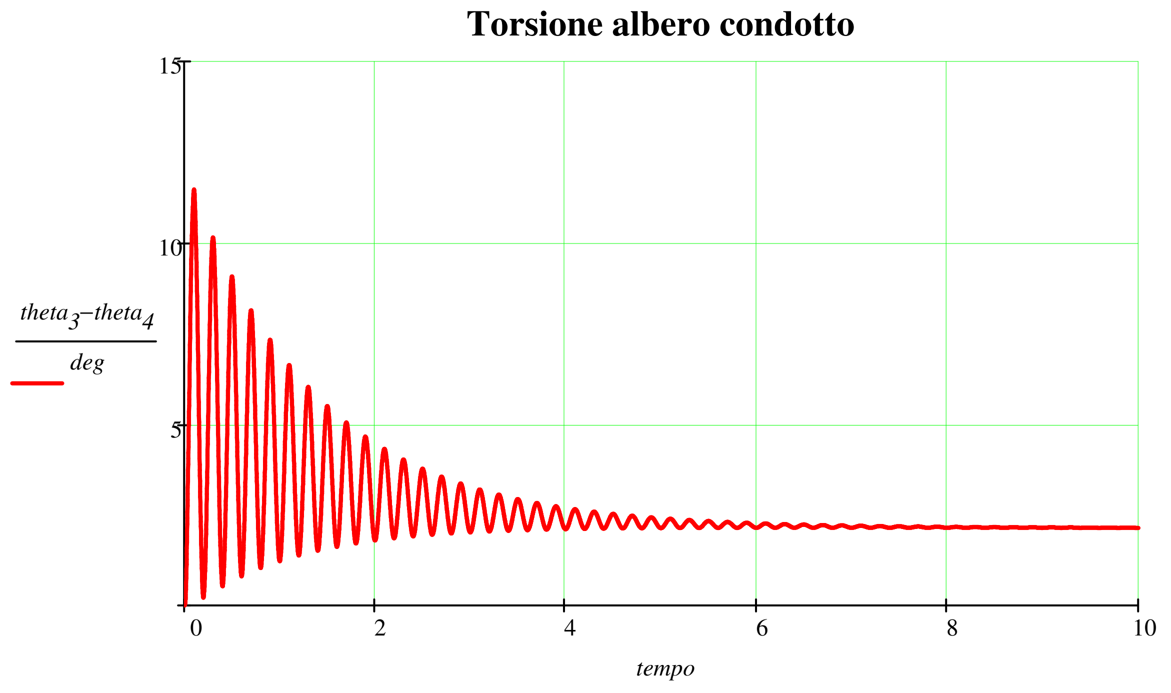
$theta_1 := TAB^{\langle 2 \rangle}$ $theta_2 := TAB^{\langle 3 \rangle}$ $theta_4 := TAB^{\langle 4 \rangle}$ $theta_3 := \tau \cdot theta_2$

$theta\_vel_1 := TAB^{\langle 5 \rangle}$ $theta\_vel_2 := TAB^{\langle 6 \rangle}$ $theta\_vel_4 := TAB^{\langle 7 \rangle}$ $theta\_vel_3 := \tau \cdot theta\_vel_2$









Caso rigido

Momento d'inerzia equivalente ridotto all'asse del motore

$J_{eq} := (J_1 + J_2) + \tau^2 \cdot (J_3 + J_4) = 0.135$

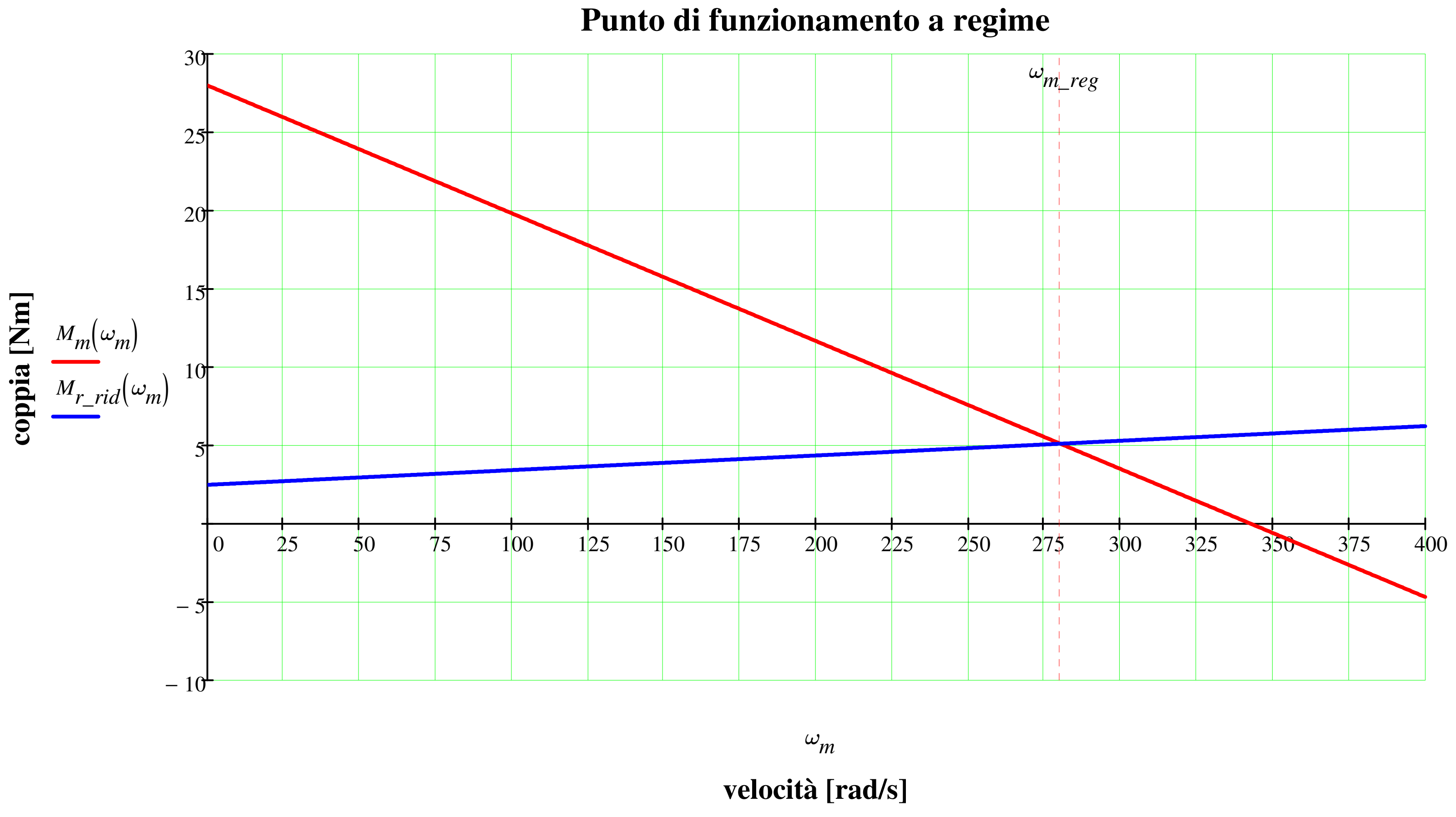
Velocità del motore a regime

$\omega_{m\_reg} := \dfrac{A - \tau \cdot C}{B + \tau^2 \cdot D} = 280.092$

Momento resistente ridotto all'asse del motore

$M_{r\_rid}(\omega_m) := \tau \cdot (C + D \cdot \tau \cdot \omega_m)$

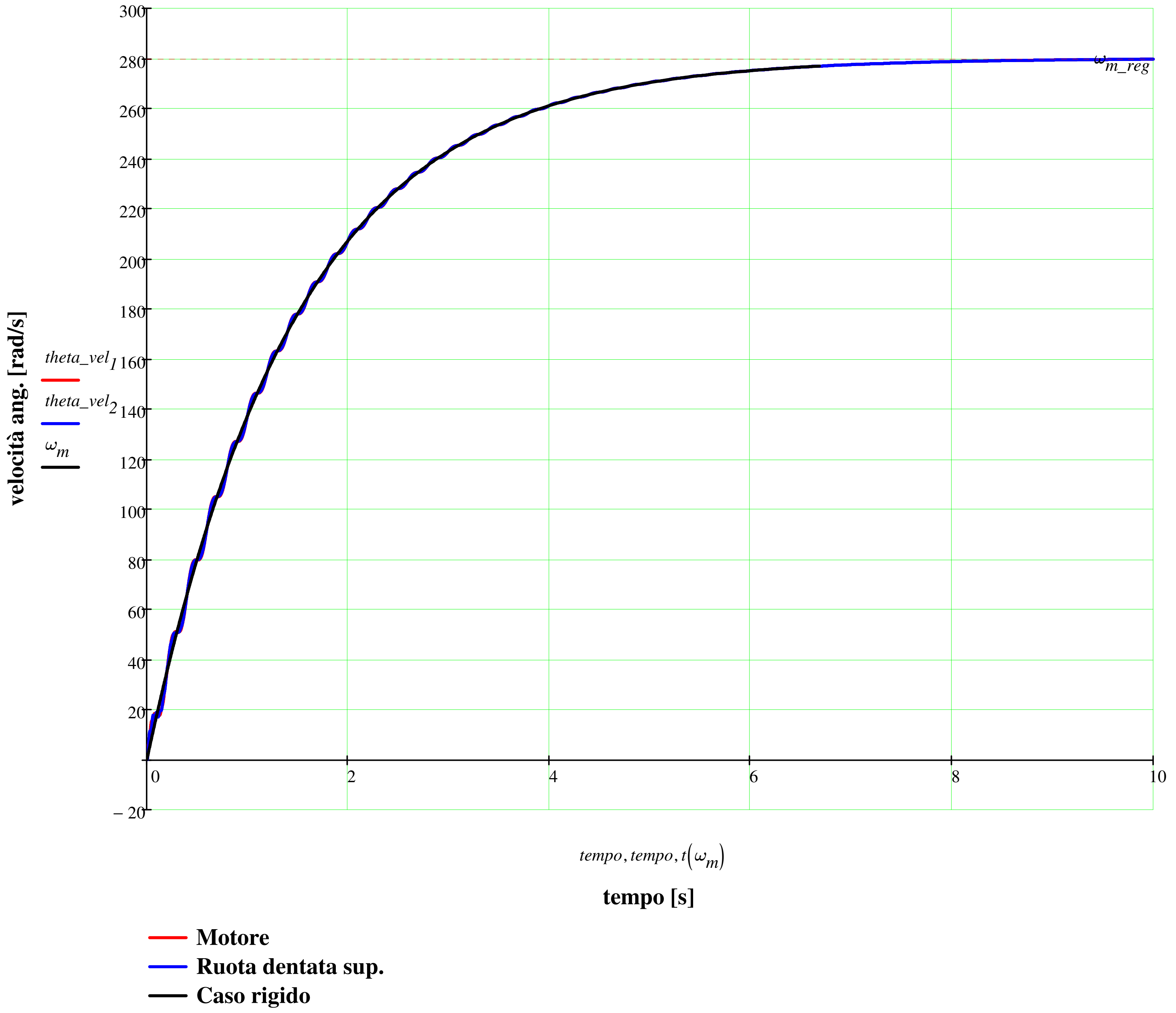
$M_m(\omega_{m\_reg}) = 5.126$ $M_{r\_rid}(\omega_{m\_reg}) = 5.126$

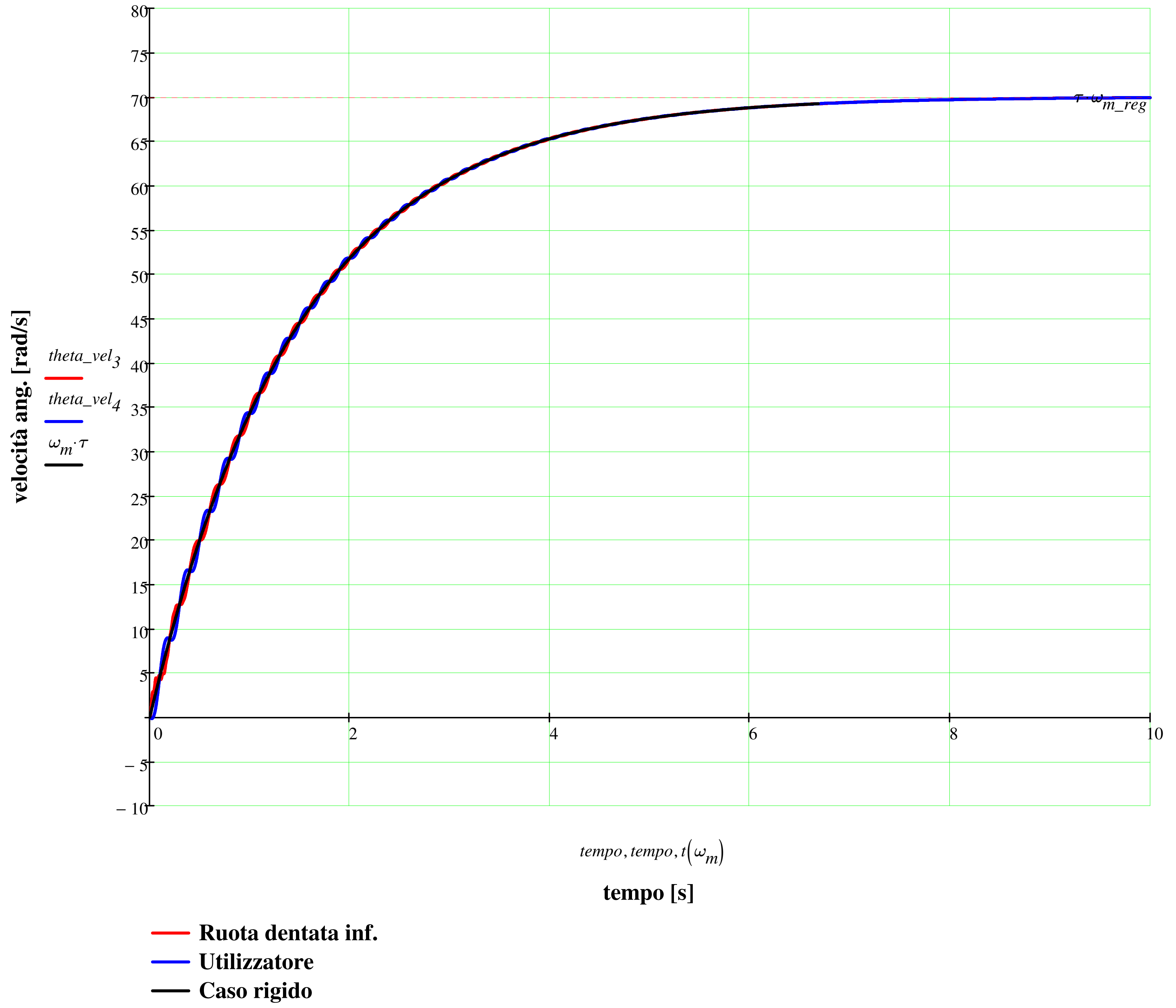


Studio del transitorio (caso rigido)

$$t(\omega_m) := J_{eq}\cdot\int_0^{\omega_m} \frac{1}{M_m(\omega_m) - M_{r\_rid}(\omega_m)}\, d\omega_m$$

$$\omega_m := 0, 0.5 .. \omega_{m\_reg}\cdot 0.99$$





$P := A - \tau \cdot C = 25.5$ $\quad$ $Q := B + \tau^2 \cdot D = 0.091$ $\quad$ $\frac{P}{Q} = 280.092$

$\omega_{m\_reg} = 280.092$

$\gamma := \frac{J_{eq}}{Q} = 1.483$ $\quad$ Costante di tempo [s]

$$omega_{mot}(t) := \frac{P}{Q} \cdot \left(1 - e^{\frac{-t}{\gamma}}\right)$$

$\mathbf{t} := 0, 0.01 .. 6.5$