$ORIGIN := 1$

$m_1 := 5$

$m_2 := 12$

$k_2 := 10000$

$k_1 := 70000$

$c_1 := 20$

$c_2 := 40$

$d := 30 \cdot 10^{-3}$ $\qquad A_{sez} := \frac{\pi \cdot d^2}{4} = 7.069 \times 10^{-4}$

$p_0 := 100000$

$F_0 := p_0 \cdot A_{sez} = 70.686$

$\Omega := 30$ $\qquad \tau := \frac{2 \cdot \pi}{\Omega} = 0.209$ $\qquad \frac{1}{\tau} = 4.775$

Condizioni iniziali

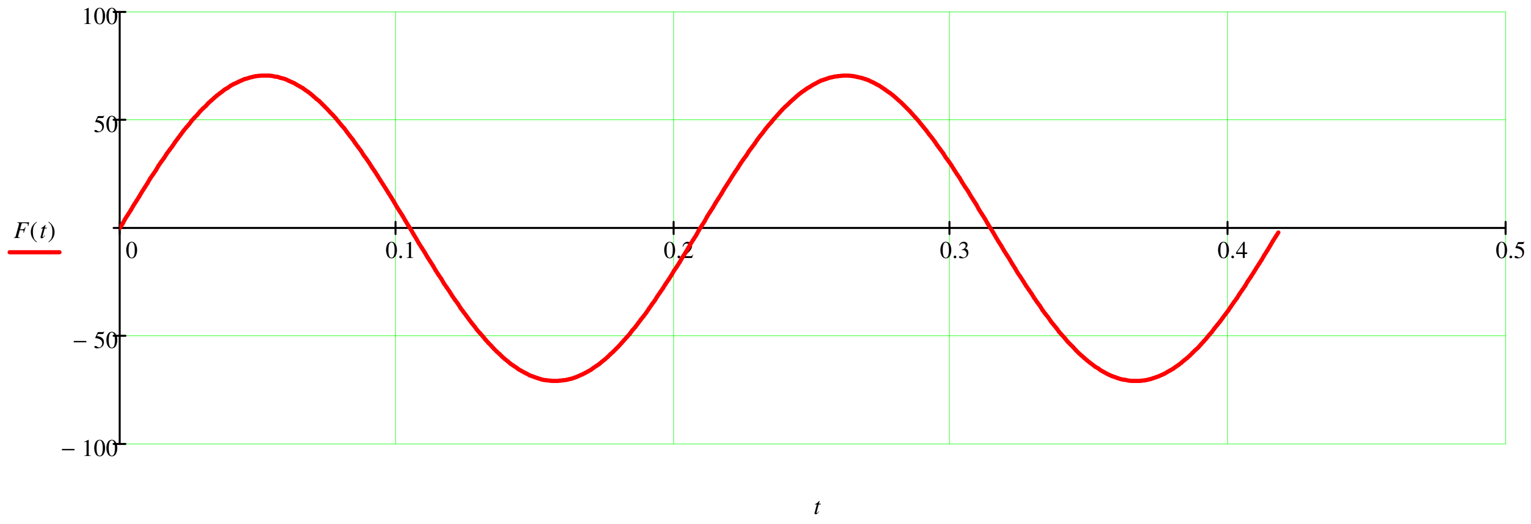
$x_{1\_ini} := 0.02$ $\qquad x_{2\_ini} := 0.03$ $\qquad v_{1\_ini} := 0.3$ $\qquad v_{2\_ini} := 0.2$

Durata simulazione e intervallo di campionamento

$T_{max} := 5$ $\qquad \Delta t := 0.001$

$F(t) := F_0 \cdot sin(\Omega \cdot t)$ $\qquad N_{per} := 2$

$t := 0, 0.001 .. N_{per} \cdot \tau$



$$\mathbf{M} := \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \qquad \mathbf{M} = \begin{pmatrix} 5 & 0 \\ 0 & 12 \end{pmatrix}$$

$$\mathbf{C} := \begin{pmatrix} c_1 & -c_1 \\ -c_1 & c_1 + c_2 \end{pmatrix} \qquad \mathbf{C} = \begin{pmatrix} 20 & -20 \\ -20 & 60 \end{pmatrix}$$

$$\mathbf{K} := \begin{pmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{pmatrix} \qquad \mathbf{K} = \begin{pmatrix} 70000 & -70000 \\ -70000 & 80000 \end{pmatrix}$$

$$omega := \sqrt{eigenvals\left(\mathbf{M}^{-1}\cdot\mathbf{K}\right)} = \begin{pmatrix} 141.724 \\ 24.101 \end{pmatrix}$$

Moto a regime con forzante sinusoidale

$$\mathbf{Z} := \left(\mathbf{K} - \Omega^2\cdot\mathbf{M}\right) + i\cdot\Omega\cdot\mathbf{C} \qquad \mathbf{Z} = \begin{pmatrix} 65500 + 600i & -70000 - 600i \\ -70000 - 600i & 69200 + 1800i \end{pmatrix}$$

$$\mathbf{X_{reg}} := \mathbf{Z}^{-1}\cdot\begin{pmatrix} F_0 \\ 0 \end{pmatrix} = \begin{pmatrix} -0.013 - 2.944i \times 10^{-3} \\ -0.013 - 2.753i \times 10^{-3} \end{pmatrix}$$

$$X_1 := \left|\mathbf{X_{reg}}_1\right| = 0.01302 \qquad \varphi_1 := arg\left(\mathbf{X_{reg}}_1\right) = -166.932\cdot deg$$

$$X_2 := \left|\mathbf{X_{reg}}_2\right| = 0.01317 \qquad \varphi_2 := arg\left(\mathbf{X_{reg}}_2\right) = -167.93\cdot deg$$

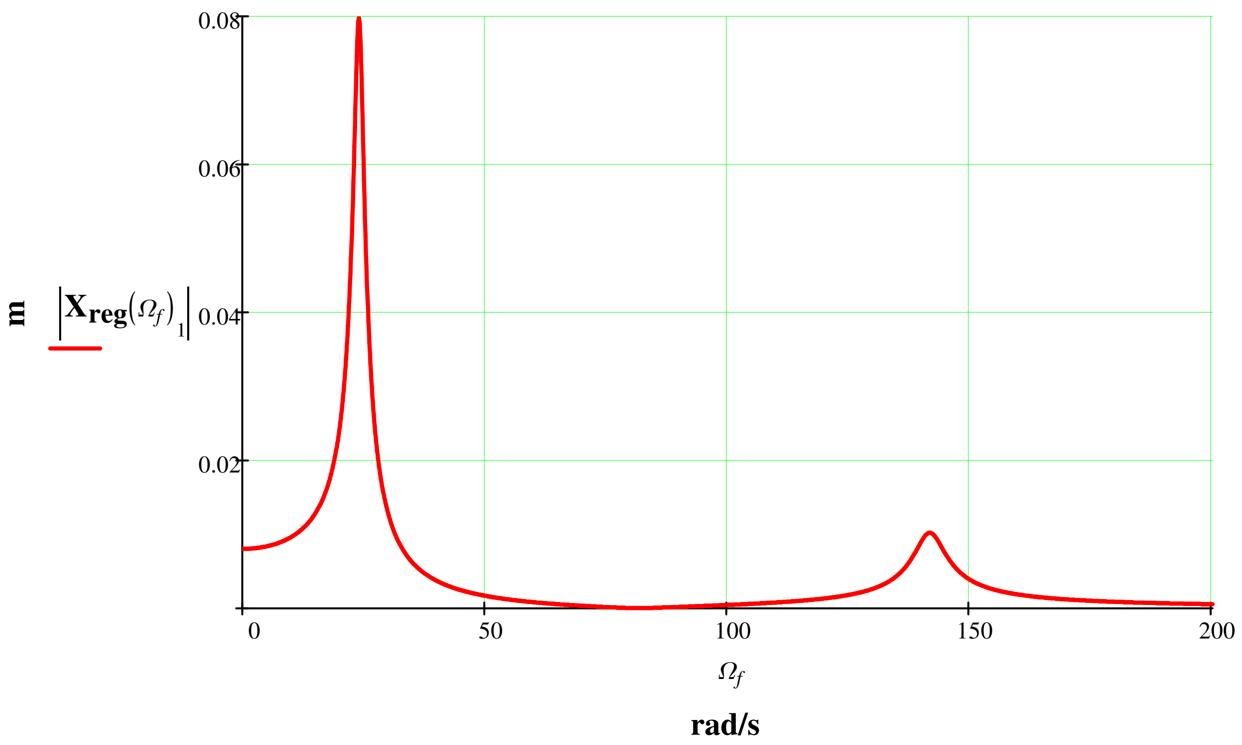
Diagrammi di risposta in frequenza

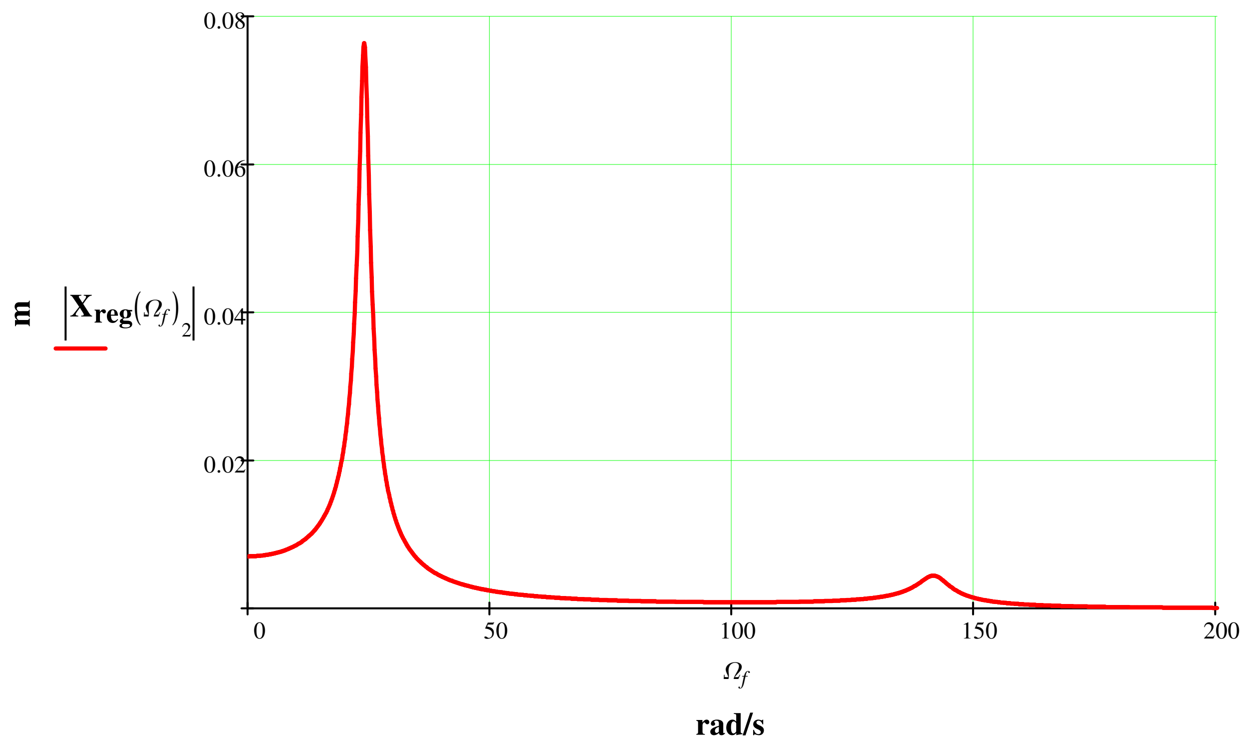
$\Omega_f := 0, 0.1 .. 200$

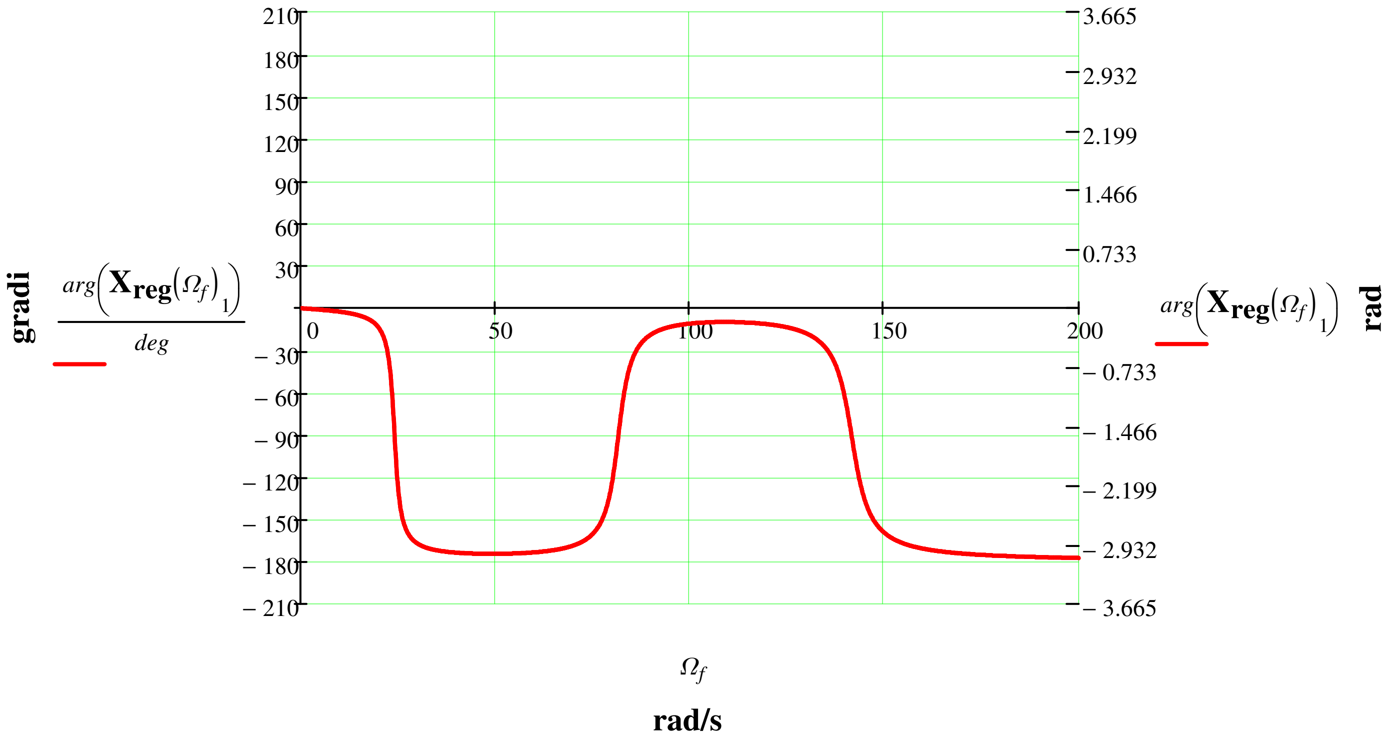
$\mathbf{Z}(\Omega_f) := \left(\mathbf{K} - \Omega_f{}^2 \cdot \mathbf{M}\right) + i \cdot \Omega_f \cdot \mathbf{C}$

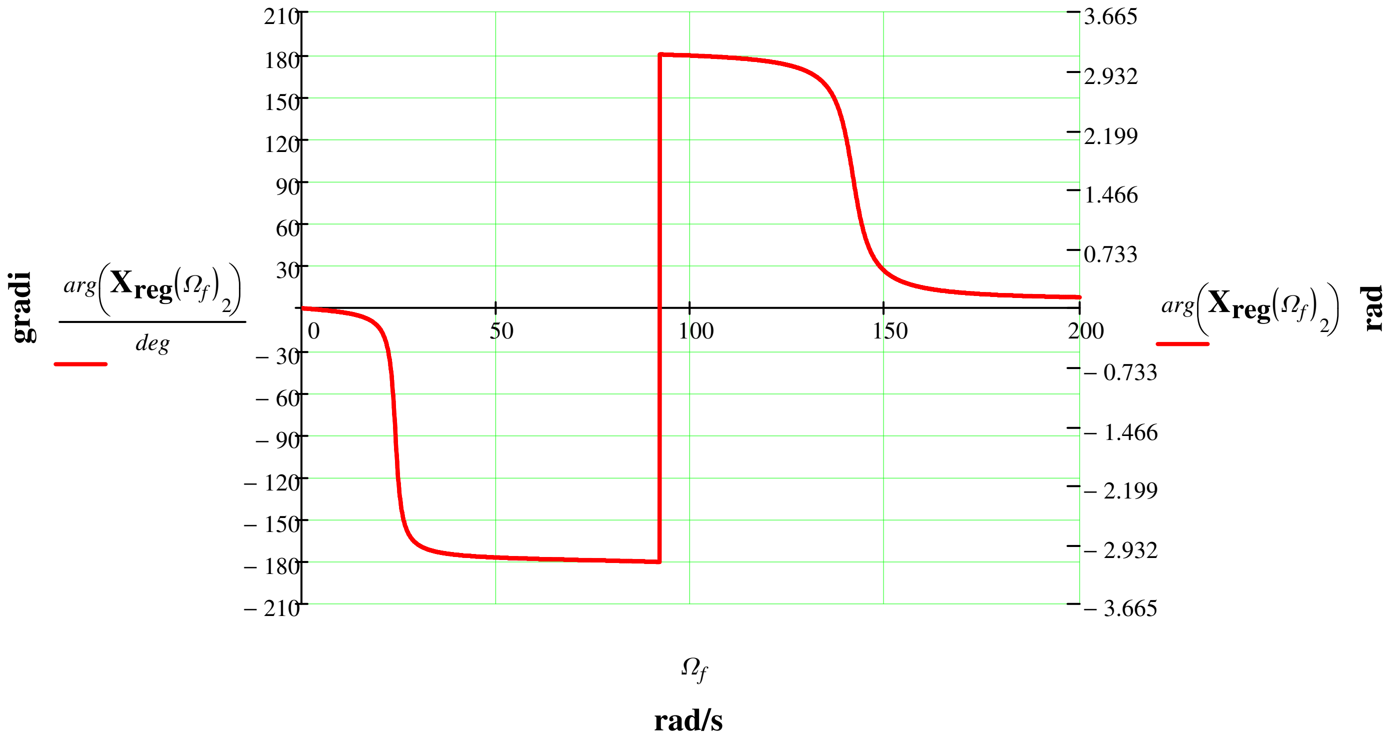
$max(omega) \cdot 1.2 = 170.069$

$\mathbf{X_{reg}}(\Omega_f) := \mathbf{Z}(\Omega_f)^{-1} \cdot \begin{pmatrix} F_0 \\ 0 \end{pmatrix}$









$$\mathbf{I} := identity(2) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathbf{O} := \mathbf{I} \cdot 0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

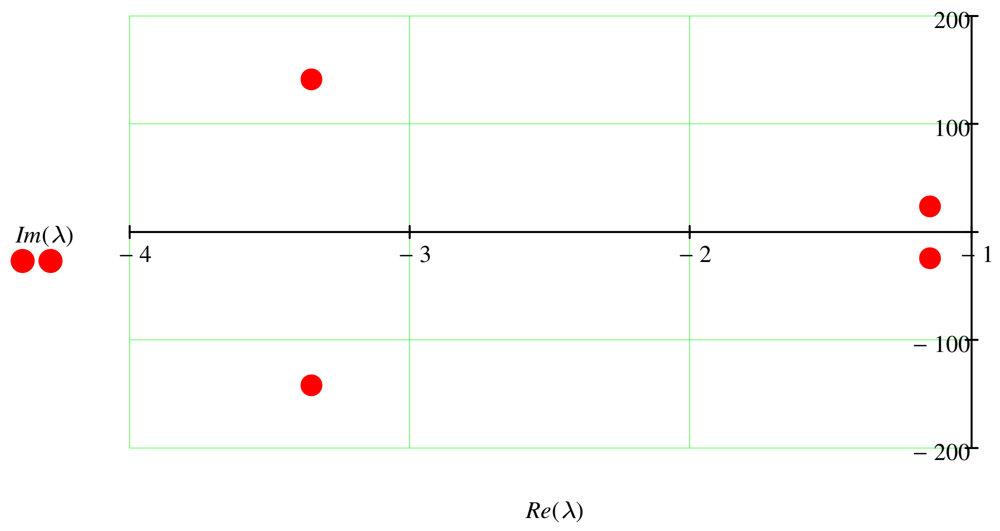
$$\mathbf{A_1} := augment(\mathbf{O}, \mathbf{I}) = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{A_2} := augment\left[-\left(\mathbf{M}^{-1} \cdot \mathbf{K}\right), -\left(\mathbf{M}^{-1} \cdot \mathbf{C}\right)\right] = \begin{pmatrix} -14000 & 14000 & -4 & 4 \\ 5833.333 & -6666.667 & 1.667 & -5 \end{pmatrix}$$

Matrice di stato

$$\mathbf{A} := stack(\mathbf{A_1}, \mathbf{A_2}) = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -14000 & 14000 & -4 & 4 \\ 5833.333 & -6666.667 & 1.667 & -5 \end{pmatrix}$$

$$\lambda := eigenvals(\mathbf{A}) \qquad \lambda = \begin{pmatrix} -3.3522 + 141.6773i \\ -3.3522 - 141.6773i \\ -1.1478 + 24.0746i \\ -1.1478 - 24.0746i \end{pmatrix} \qquad omega := \sqrt{genvals(\mathbf{K}, \mathbf{M})} = \begin{pmatrix} 141.7245 \\ 24.1006 \end{pmatrix}$$



$$\mathbf{Y} := eigenvecs(\mathbf{A})$$

Y =

	1	2	3	4
1	-0.0002-0.0065i	-0.0002+0.0065i	-0.0014-0.0299i	-0.0014+0.0299i
2	0+0.0028i	0-0.0028i	-0.0015-0.0286i	-0.0015+0.0286i
3	0.917	0.917	0.7212	0.7212
4	-0.3986-0.009i	-0.3986+0.009i	0.6914-0.0026i	0.6914+0.0026i

$X_1 = 0.013022 \qquad \varphi_1 = -166.932 \cdot deg$

$X_2 = 0.013168 \qquad \varphi_2 = -167.93 \cdot deg$

$$\mathbf{y_{part}}(t) := \begin{pmatrix} X_1 \cdot sin(\Omega \cdot t + \varphi_1) \\ X_2 \cdot sin(\Omega \cdot t + \varphi_2) \\ \Omega \cdot X_1 \cdot cos(\Omega \cdot t + \varphi_1) \\ \Omega \cdot X_2 \cdot cos(\Omega \cdot t + \varphi_2) \end{pmatrix} \qquad \mathbf{d} := \begin{pmatrix} x_{1\_ini} - X_1 \cdot sin(\varphi_1) \\ x_{2\_ini} - X_2 \cdot sin(\varphi_2) \\ v_{1\_ini} - \Omega \cdot X_1 \cdot cos(\varphi_1) \\ v_{2\_ini} - \Omega \cdot X_2 \cdot cos(\varphi_2) \end{pmatrix} = \begin{pmatrix} 0.023 \\ 0.033 \\ 0.681 \\ 0.586 \end{pmatrix}$$

$$C := \mathbf{Y}^{-1} \cdot \mathbf{d} = \begin{pmatrix} 0.023 - 0.602i \\ 0.023 + 0.602i \\ 0.443 + 0.535i \\ 0.443 - 0.535i \end{pmatrix}$$

$$\mathbf{y}(t) := \sum_{k=1}^{4} \left(C_k \cdot \mathbf{Y}^{\langle k \rangle} \cdot e^{\lambda_k \cdot t} \right) + \mathbf{y_{part}}(t)$$

Soluzione mediante integrazione numerica

$$u := \begin{pmatrix} x_{1\_ini} \\ x_{2\_ini} \\ v_{1\_ini} \\ v_{2\_ini} \end{pmatrix} \qquad u = \begin{pmatrix} 0.02 \\ 0.03 \\ 0.3 \\ 0.2 \end{pmatrix}$$

$$ACC(x_1, x_2, v_1, v_2, t) := \mathbf{M}^{-1} \cdot \left[\begin{pmatrix} F_0 \cdot sin(\Omega \cdot t) \\ 0 \end{pmatrix} - \mathbf{C} \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} - \mathbf{K} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right]$$

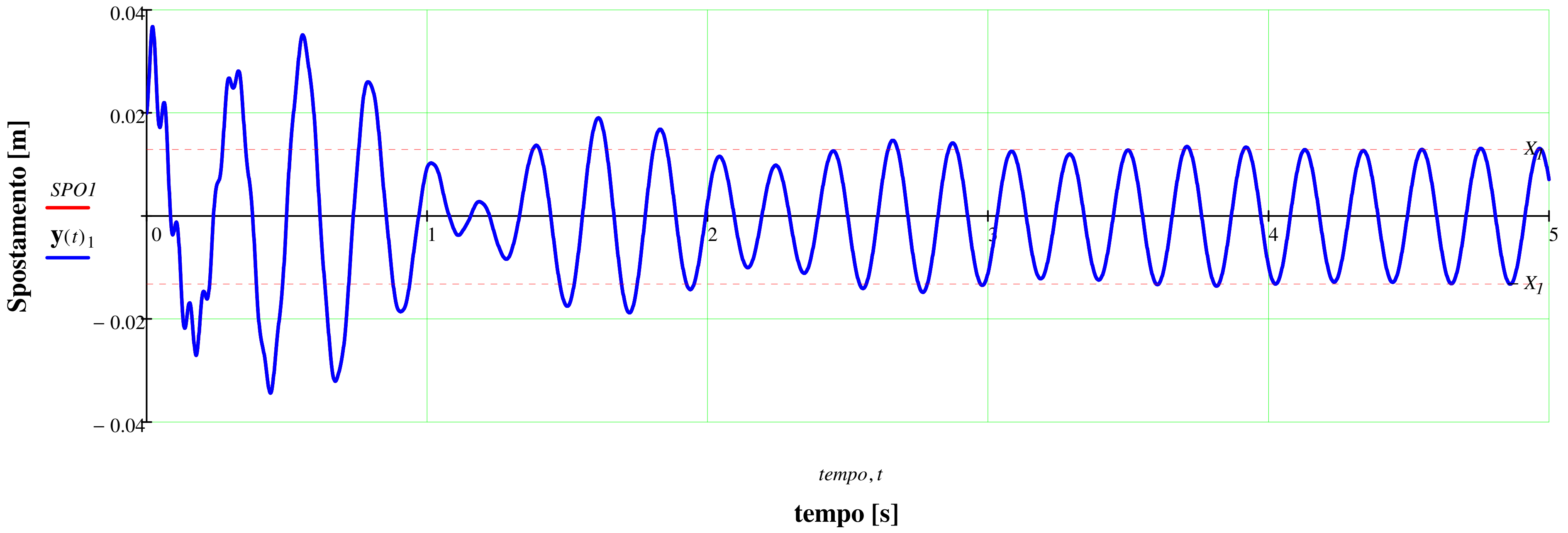
$$EQMOTO(t, u) := \begin{pmatrix} u_3 \\ u_4 \\ ACC(u_1, u_2, u_3, u_4, t)_1 \\ ACC(u_1, u_2, u_3, u_4, t)_2 \end{pmatrix}$$

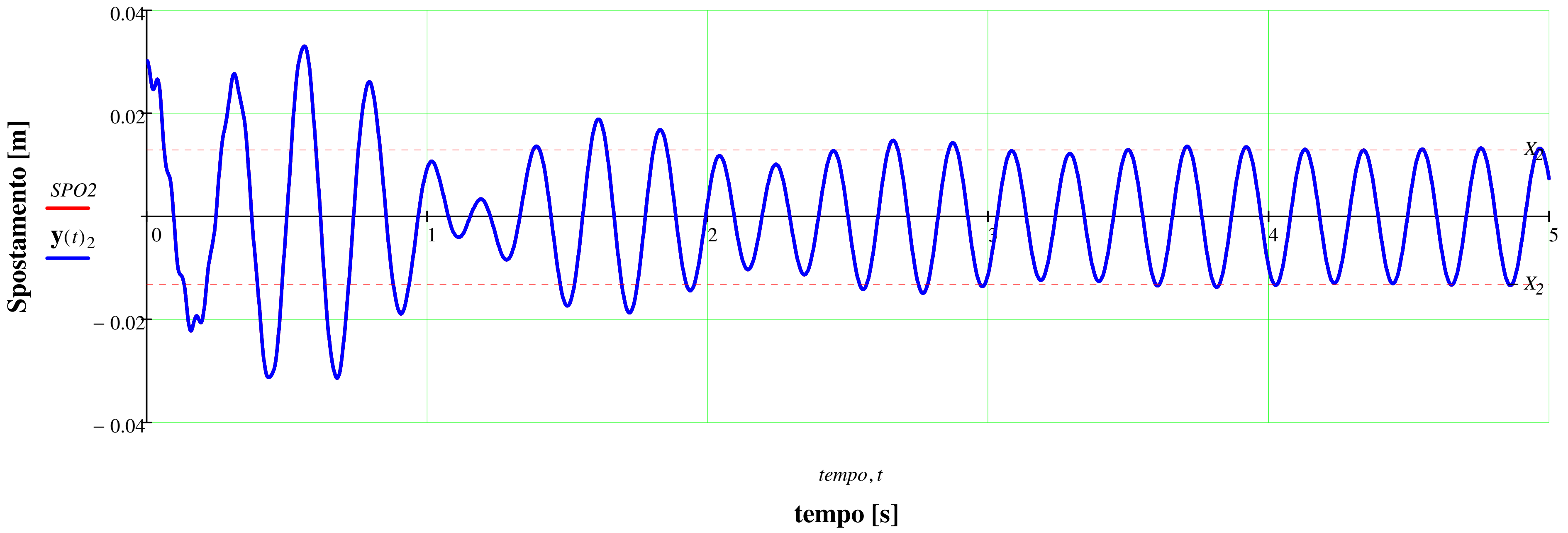
$$N_{pti} := ceil\left(\frac{T_{max}}{\Delta t} \right) = 5000$$

$TAB := rkfixed(u, 0, T_{max}, N_{pti}, EQMOTO)$

$tempo := TAB^{\langle 1 \rangle} \qquad SPO1 := TAB^{\langle 2 \rangle} \qquad SPO2 := TAB^{\langle 3 \rangle}$

$t := 0, \Delta t .. T_{max}$

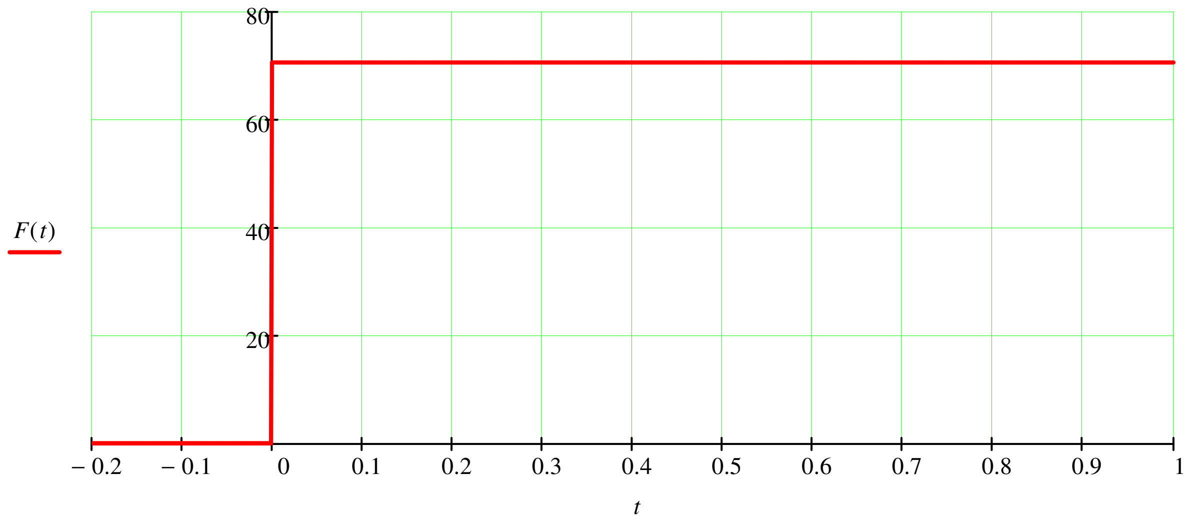




Caso con forzante a gradino

$$F(t) := \begin{cases} F_0 & \text{if } t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$t := -0.2, -0.199 .. 1$



$$\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} := \mathbf{K}^{-1} \cdot \begin{pmatrix} F_0 \\ 0 \end{pmatrix} = \begin{pmatrix} 8.078 \times 10^{-3} \\ 7.069 \times 10^{-3} \end{pmatrix}$$

$$\mathbf{y_{part}}(t) := \begin{pmatrix} A_1 \\ A_2 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{d} := \begin{pmatrix} x_{1\_ini} - A_1 \\ x_{2\_ini} - A_2 \\ v_{1\_ini} \\ v_{2\_ini} \end{pmatrix} = \begin{pmatrix} 0.012 \\ 0.023 \\ 0.3 \\ 0.2 \end{pmatrix}$$

$$C := \mathbf{Y}^{-1} \cdot \mathbf{d} = \begin{pmatrix} 0.031 - 0.64i \\ 0.031 + 0.64i \\ 0.169 + 0.346i \\ 0.169 - 0.346i \end{pmatrix}$$

$$\mathbf{y}(t) := \sum_{k=1}^{4} \left(C_k \cdot \mathbf{Y}^{\langle k \rangle} \cdot e^{\lambda_k \cdot t} \right) + \mathbf{y_{part}}(t)$$

Soluzione mediante integrazione numerica

$$u := \begin{pmatrix} x_{1\_ini} \\ x_{2\_ini} \\ v_{1\_ini} \\ v_{2\_ini} \end{pmatrix} \qquad u = \begin{pmatrix} 0.02 \\ 0.03 \\ 0.3 \\ 0.2 \end{pmatrix}$$

$$ACC(x_1, x_2, v_1, v_2, t) := \mathbf{M}^{-1} \cdot \left[\begin{pmatrix} F_0 \\ 0 \end{pmatrix} - \mathbf{C} \cdot \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} - \mathbf{K} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right]$$

$$EQMOTO(t,u) := \begin{pmatrix} u_3 \\ u_4 \\ ACC(u_1,u_2,u_3,u_4,t)_1 \\ ACC(u_1,u_2,u_3,u_4,t)_2 \end{pmatrix}$$

$$N_{pti} := ceil\left(\frac{T_{max}}{\Delta t}\right) = 5000$$

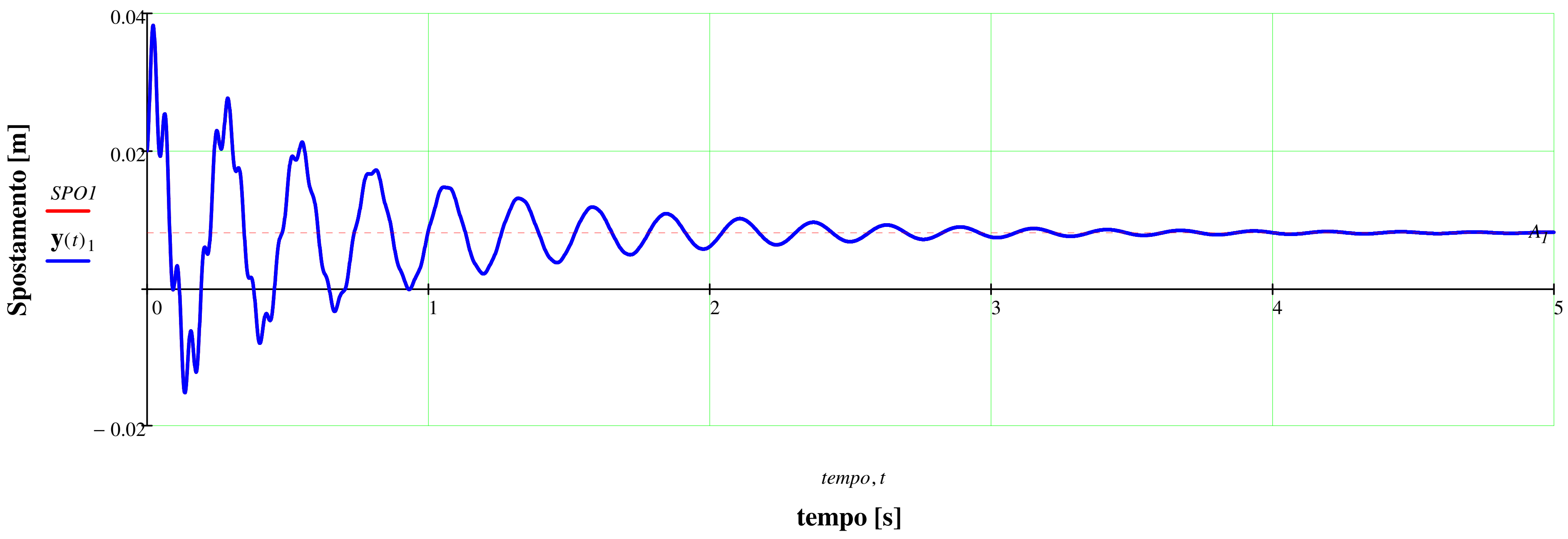
$$TAB := rkfixed(u, 0, T_{max}, N_{pti}, EQMOTO)$$

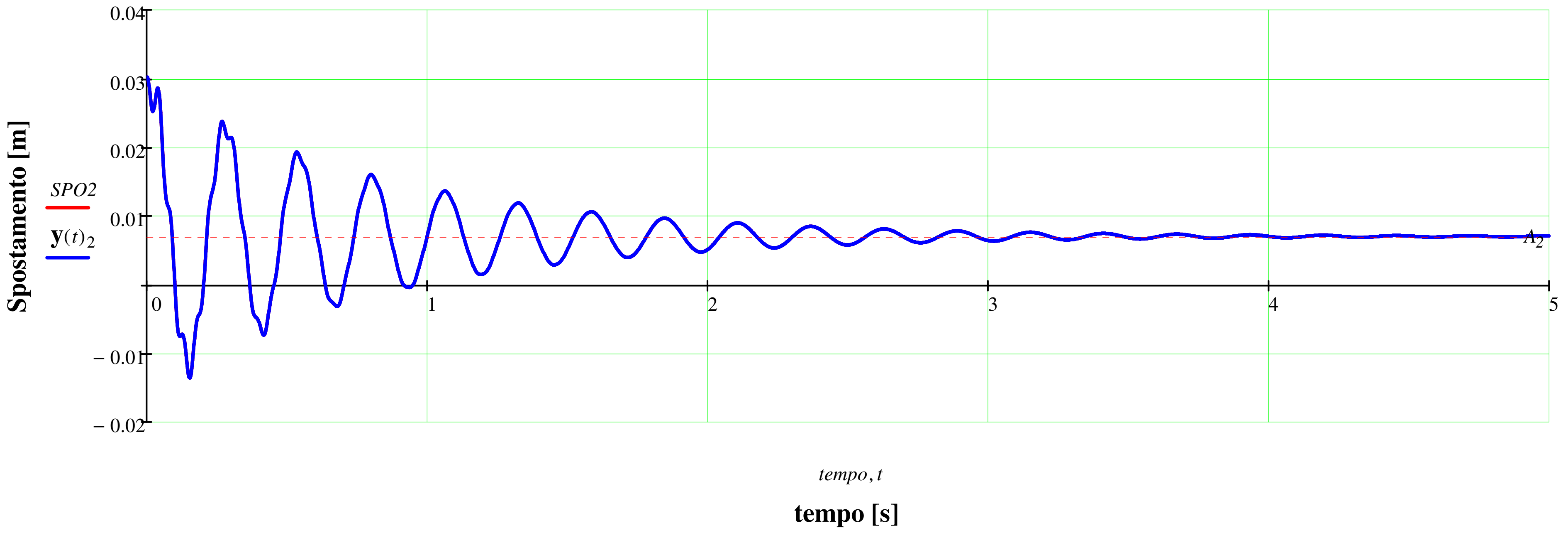
$TAB =$

	1	2	3	4	5	6
1	0	0.02	0.03	0.3	0.2	
2	$1\cdot10^{-3}$	0.02	0.03	0.452	0.117	
3	$2\cdot10^{-3}$	0.021	0.03	0.598	0.036	
4	$3\cdot10^{-3}$	0.022	0.03	0.736	-0.04	
5	$4\cdot10^{-3}$	0.022	0.03	0.862	-0.112	
6	$5\cdot10^{-3}$	0.023	0.03	0.974	-0.177	
7	$6\cdot10^{-3}$	0.024	0.03	1.069	...	

$tempo := TAB^{\langle 1 \rangle}$ $SPO1 := TAB^{\langle 2 \rangle}$ $SPO2 := TAB^{\langle 3 \rangle}$

$t := 0, \Delta t .. T_{max}$




0.04
0.03
0.02
0.01
− 0.01
− 0.02
Spostamento [m]
SPO2
y(t)2
A2
0
1
2
3
4
5
tempo, t
tempo [s]