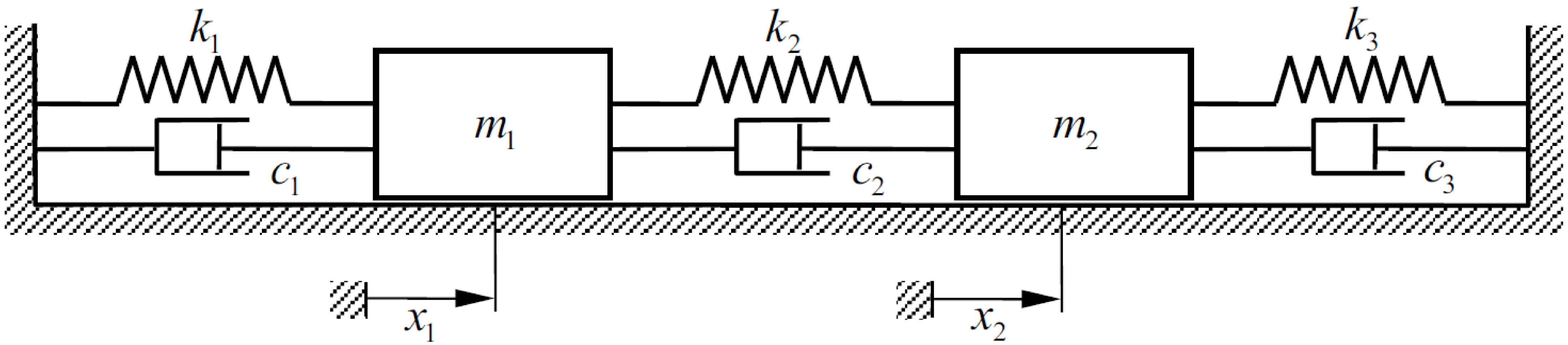
Vibrazioni libere smorzate a 2 GDL: soluzione mediante equazioni di stato



$ORIGIN := 1$

$m_1 := 6$

$m_2 := 7$

$c_1 := 5$ $\qquad k_1 := 600$

$c_2 := 10$ $\qquad k_2 := 500$

$c_3 := 8$ $\qquad k_3 := 100$

$$\mathbf{M} := diag\left(\begin{pmatrix} m_1 \\ m_2 \end{pmatrix}\right) = \begin{pmatrix} 6 & 0 \\ 0 & 7 \end{pmatrix}$$

$$\mathbf{C} := \begin{bmatrix} (c_1 + c_2) & -c_2 \\ -c_2 & (c_2 + c_3) \end{bmatrix} = \begin{pmatrix} 15 & -10 \\ -10 & 18 \end{pmatrix}$$

$$\mathbf{K} := \begin{bmatrix} (k_1 + k_2) & -k_2 \\ -k_2 & (k_2 + k_3) \end{bmatrix} = \begin{pmatrix} 1100 & -500 \\ -500 & 600 \end{pmatrix}$$

$$\mathbf{I} := identity(2) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \mathbf{O} := 0 \cdot \mathbf{I} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\mathbf{A_{sup}} := augment(\mathbf{O}, \mathbf{I}) = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{A_{inf}} := augment\left(-\mathbf{M}^{-1} \cdot \mathbf{K}, -\mathbf{M}^{-1} \cdot \mathbf{C}\right) = \begin{pmatrix} -183.333 & 83.333 & -2.5 & 1.667 \\ 71.429 & -85.714 & 1.429 & -2.571 \end{pmatrix}$$

$$\mathbf{A} := stack(\mathbf{A_{sup}}, \mathbf{A_{inf}}) = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -183.333 & 83.333 & -2.5 & 1.667 \\ 71.429 & -85.714 & 1.429 & -2.571 \end{pmatrix}$$

Matrice di stato

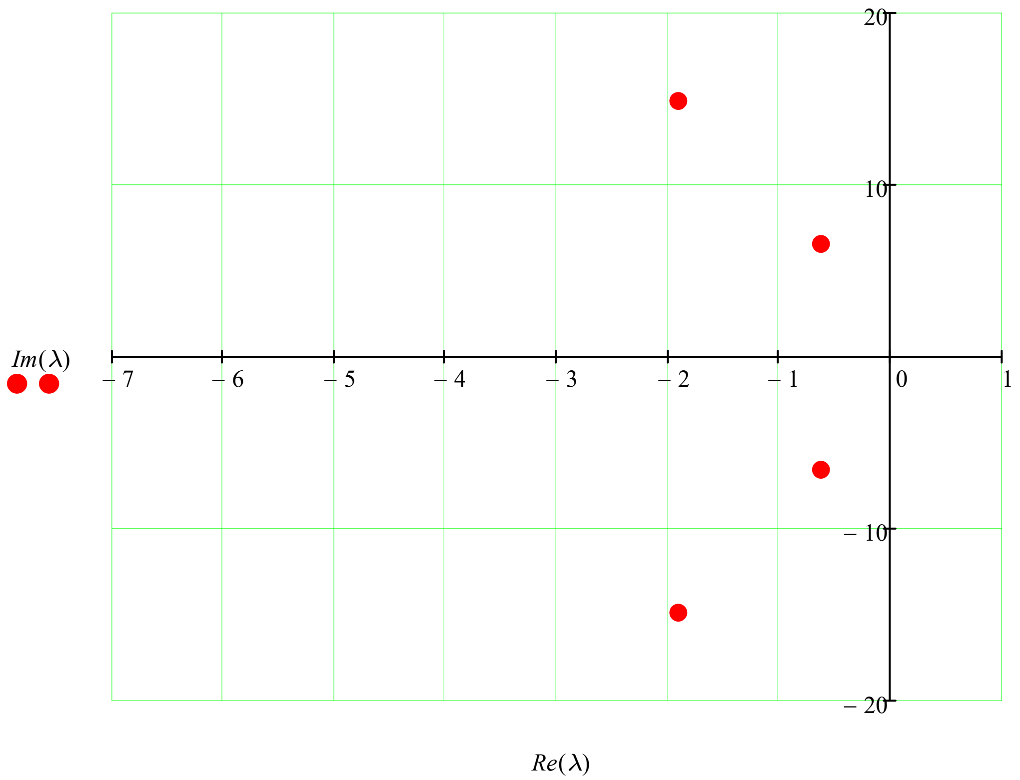
$$\mathbf{A_{bis}} := \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1 + k_2}{m_1} & \frac{k_2}{m_1} & -\frac{c_1 + c_2}{m_1} & \frac{c_2}{m_1} \\ \frac{k_2}{m_2} & -\frac{k_2 + k_3}{m_2} & \frac{c_2}{m_2} & -\frac{c_2 + c_3}{m_2} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -183.333 & 83.333 & -2.5 & 1.667 \\ 71.429 & -85.714 & 1.429 & -2.571 \end{pmatrix}$$

$$\lambda := eigenvals(\mathbf{A}) = \begin{pmatrix} -1.909 + 14.875i \\ -1.909 - 14.875i \\ -0.627 + 6.558i \\ -0.627 - 6.558i \end{pmatrix}$$

Autovalori della matrice di stato

$$\mathbf{Y} := eigenvecs(\mathbf{A}) = \begin{pmatrix} 0.00751 + 0.05853i & 0.00751 - 0.05853i & -0.00168 - 0.07677i & -0.00168 + 0.07677i \\ 0.00116 - 0.03071i & 0.00116 + 0.03071i & -0.01226 - 0.12835i & -0.01226 + 0.12835i \\ -0.88493 & -0.88493 & 0.50452 + 0.03708i & 0.50452 - 0.03708i \\ 0.45466 + 0.07583i & 0.45466 - 0.07583i & 0.84945 & 0.84945 \end{pmatrix}$$

Autovettori della matrice di stato (sono le colonne della matrice Y)



Condizioni iniziali

$x_{10} := 50 \cdot 10^{-3} = 0.05$

$x_{20} := 40 \cdot 10^{-3} = 0.04$

$x'_{10} := 0.2$

$x'_{20} := 0.3$

$$\mathbf{y_0} := \begin{pmatrix} x_{10} \\ x_{20} \\ x'_{10} \\ x'_{20} \end{pmatrix} = \begin{pmatrix} 0.05 \\ 0.04 \\ 0.2 \\ 0.3 \end{pmatrix}$$

Condizioni iniziali

$$\mathbf{c} := \mathbf{Y}^{-1} \cdot \mathbf{y_0} = \begin{pmatrix} -0.0224 - 0.1587i \\ -0.0224 + 0.1587i \\ 0.1744 + 0.2107i \\ 0.1744 - 0.2107i \end{pmatrix}$$

Calcolo delle costanti di integrazione in base alle condizioni iniziali

$$\mathbf{y}(t) := \sum_{k=1}^{4} \left(\mathbf{c}_k \cdot \mathbf{Y}^{\langle k \rangle} \cdot e^{\lambda_k \cdot t} \right)$$

$$\mathbf{y'}(t) := \sum_{k=1}^{4} \left(\lambda_k \cdot \mathbf{c}_k \cdot \mathbf{Y}^{\langle k \rangle} \cdot e^{\lambda_k \cdot t} \right)$$

$\Delta t := 0.001$ $\quad T_{max} := 8$

$t := 0, \Delta t .. T_{max}$

$\omega := \sqrt{genvals(\mathbf{K}, \mathbf{M})} = \begin{pmatrix} 15.027 \\ 6.575 \end{pmatrix}$ $\quad \omega_{max} := max(\omega) = 15.027$ $\quad \tau_{min} := \frac{2 \cdot \pi}{\omega_{max}} = 0.418$

$\Delta t_{cons} := \frac{1}{20} \cdot \tau_{min} = 0.021$

$x_1(t) := \mathbf{y}(t)_1$ $\quad x_2(t) := \mathbf{y}(t)_2$

$v_1(t) := \mathbf{y}(t)_3$ $\quad v_2(t) := \mathbf{y}(t)_4$

$a_1(t) := \mathbf{y'}(t)_3$ $\quad a_2(t) := \mathbf{y'}(t)_4$

Verifica numerica (Runge Kutta)

$$y := \begin{pmatrix} x_{10} \\ x_{20} \\ x'_{10} \\ x'_{20} \end{pmatrix} = \begin{pmatrix} 0.05 \\ 0.04 \\ 0.2 \\ 0.3 \end{pmatrix}$$

Assegnazione delle condiz. iniziali

$N_{interv} := ceil\left(\frac{T_{max}}{\Delta t}\right) = 8000$

$EQMOTO(t, y) := \mathbf{A} \cdot y$

$TAB := rkfixed(y, 0, T_{max}, N_{interv}, EQMOTO)$

Calcolo della soluzione in forma tabellare

$tempo := TAB^{\langle 1 \rangle}$

$SPO1 := TAB^{\langle 2 \rangle}$ $\qquad$ $SPO2 := TAB^{\langle 3 \rangle}$

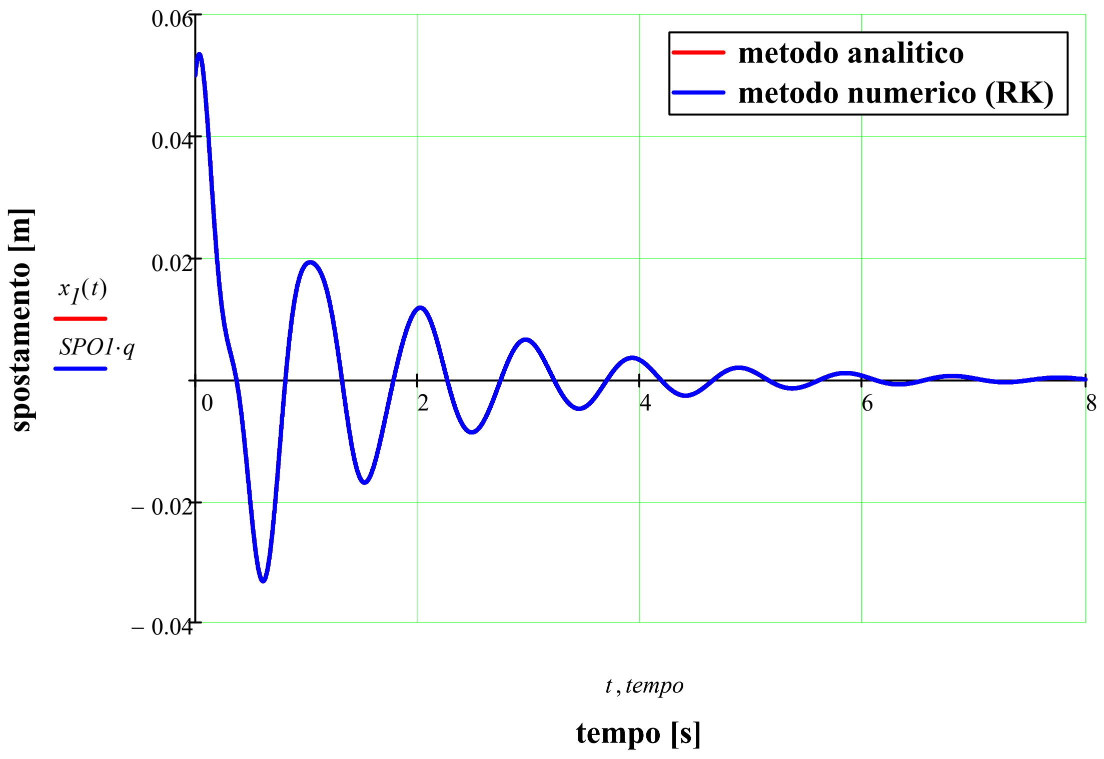
$VEL1 := TAB^{\langle 4 \rangle}$ $\qquad$ $VEL2 := TAB^{\langle 5 \rangle}$

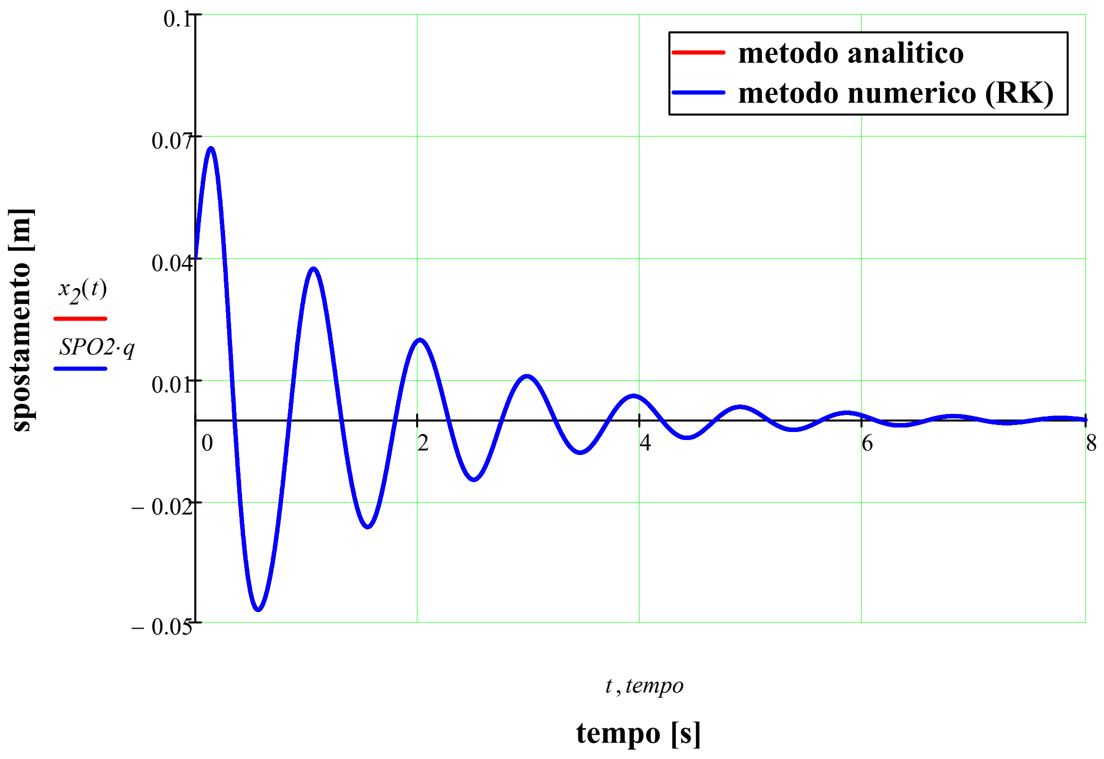
Calcolo delle accelerazioni

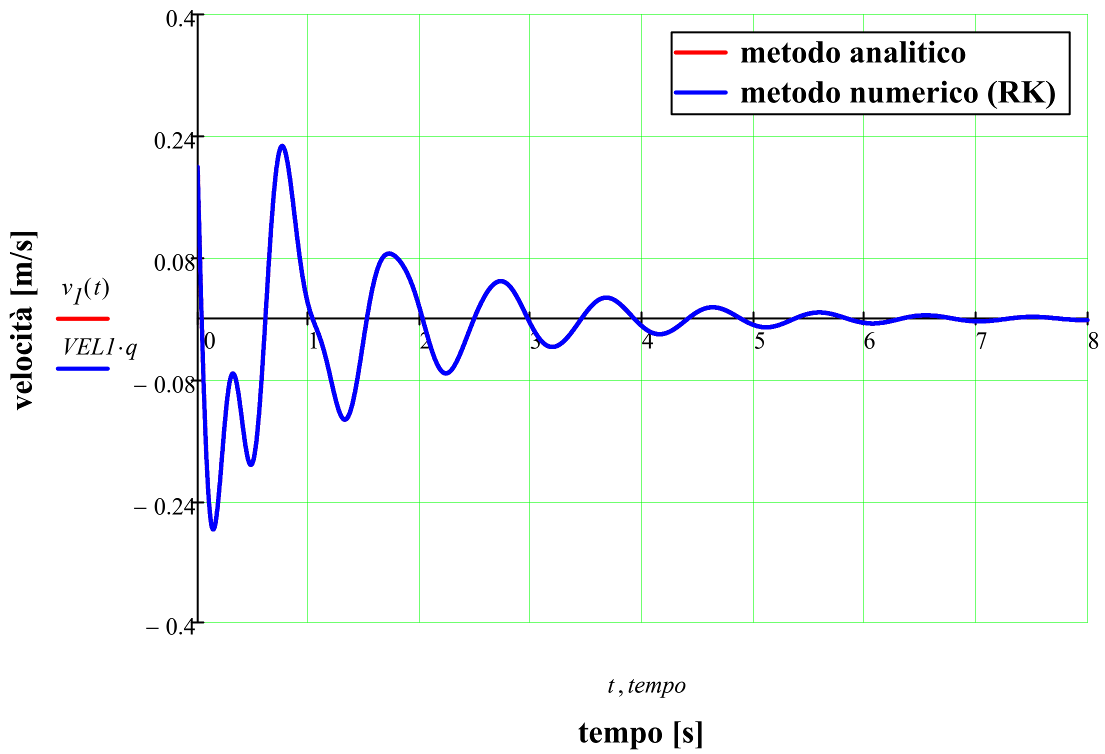
$i := 1 .. N_{interv} + 1$

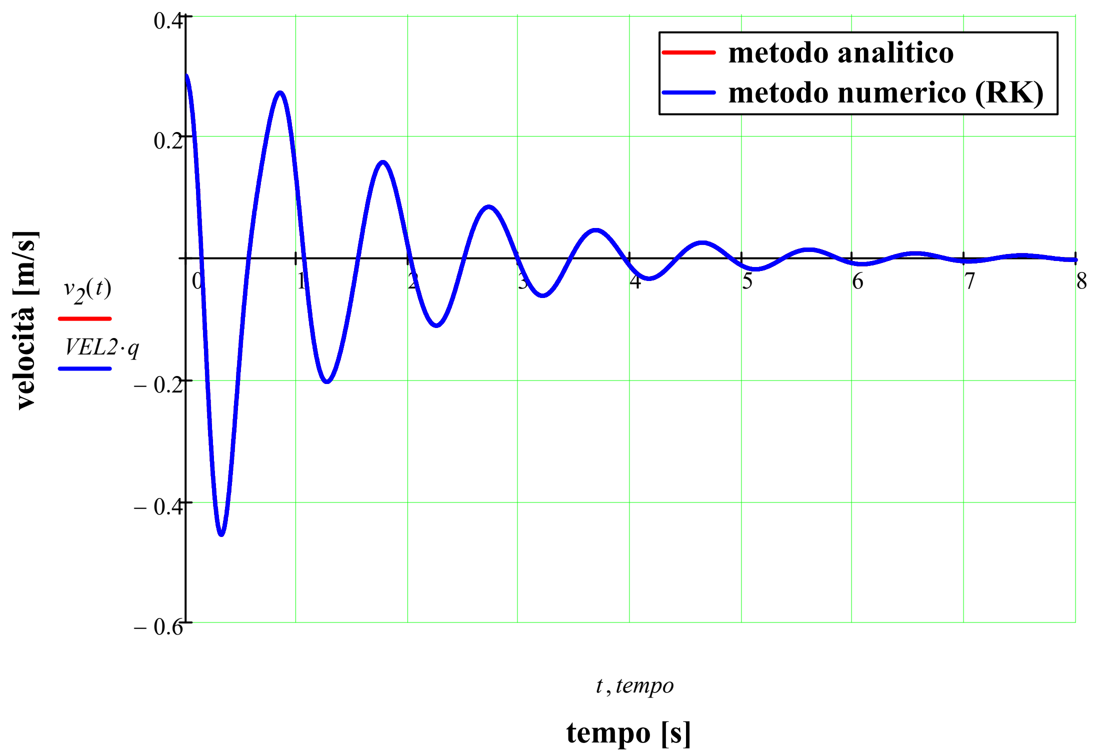
$$ACC1_i := \left[\mathbf{A} \cdot \begin{pmatrix} SPO1_i \\ SPO2_i \\ VEL1_i \\ VEL2_i \end{pmatrix} \right]_3 \qquad ACC2_i := \left[\mathbf{A} \cdot \begin{pmatrix} SPO1_i \\ SPO2_i \\ VEL1_i \\ VEL2_i \end{pmatrix} \right]_4$$

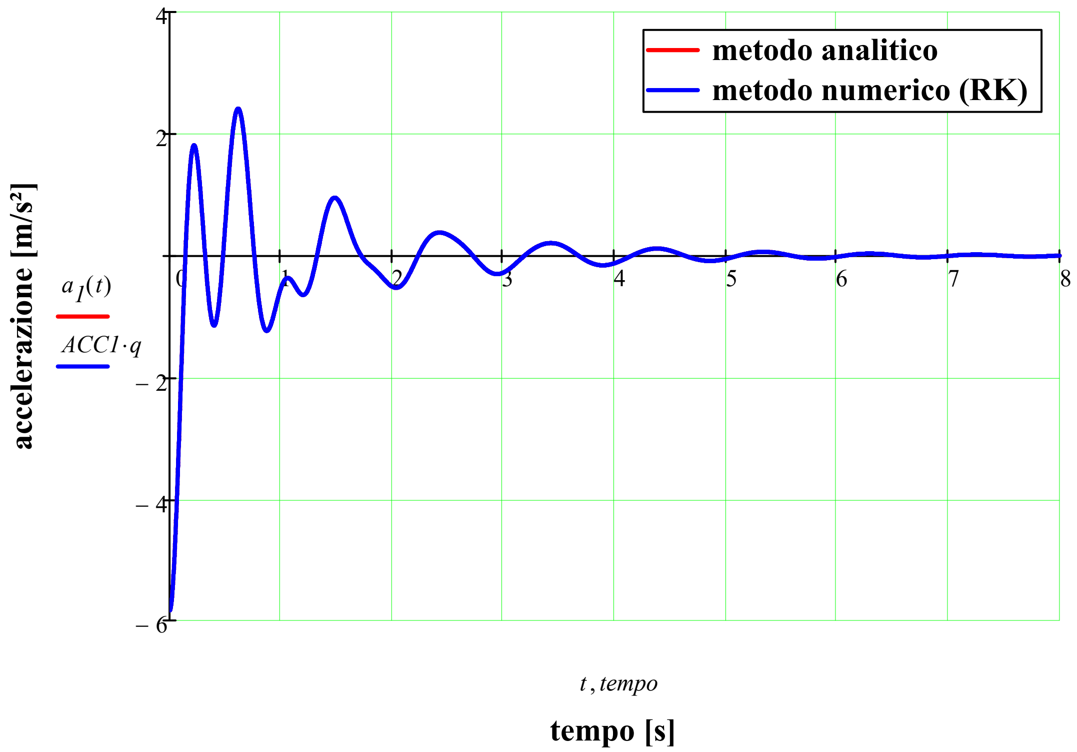
$q := 1$ $\qquad$ **Variabile on/off (attiva e disattiva i grafici della soluzione numerica**

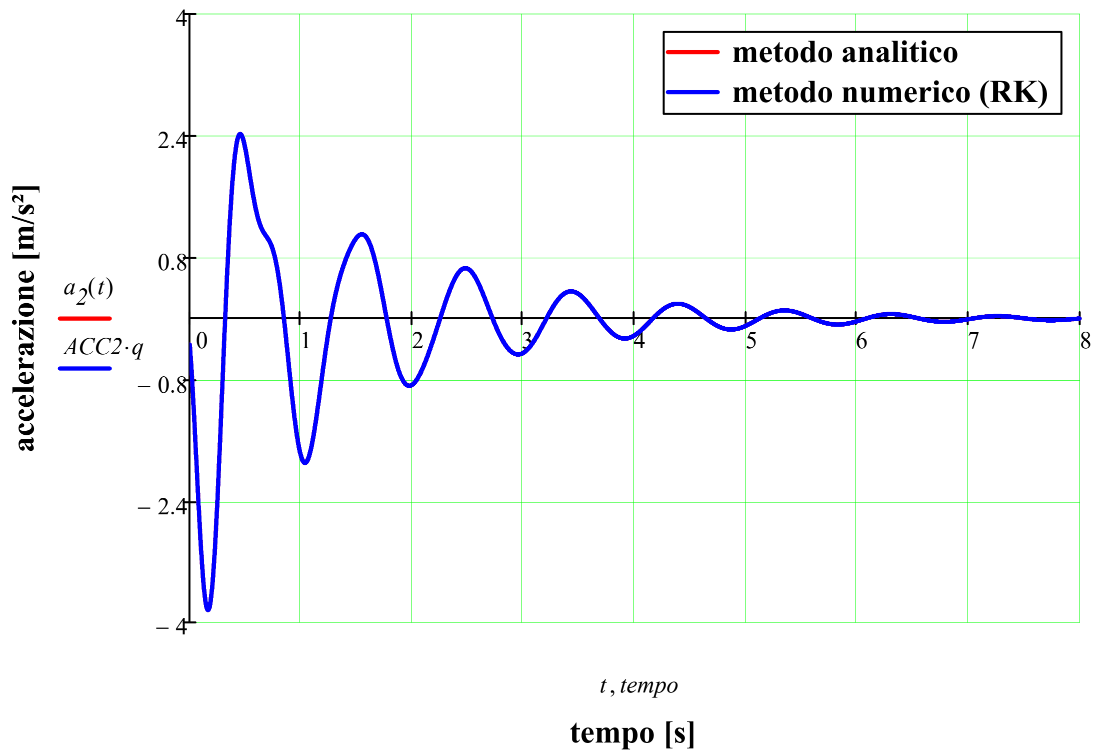





metodo analitico
metodo numerico (RK)
velocità [m/s]
v1(t)
VEL1·q
0.4
0.24
0.08
− 0.08
− 0.24
− 0.4
0
1
2
3
4
5
6
7
8
t , tempo
tempo [s]


metodo analitico
metodo numerico (RK)
velocità [m/s]
v2(t)
VEL2·q
0.4
0.2
− 0.2
− 0.4
− 0.6
0
1
2
3
4
5
6
7
8
t , tempo
tempo [s]


metodo analitico
metodo numerico (RK)
accelerazione [m/s²]
a1(t)
ACC1·q
4
2
− 2
− 4
− 6
0
1
2
3
4
5
6
7
8
t , tempo
tempo [s]


metodo analitico
metodo numerico (RK)
accelerazione [m/s²]
a2(t)
ACC2·q
4
2.4
0.8
− 0.8
− 2.4
− 4
0
1
2
3
4
5
6
7
8
t , tempo
tempo [s]