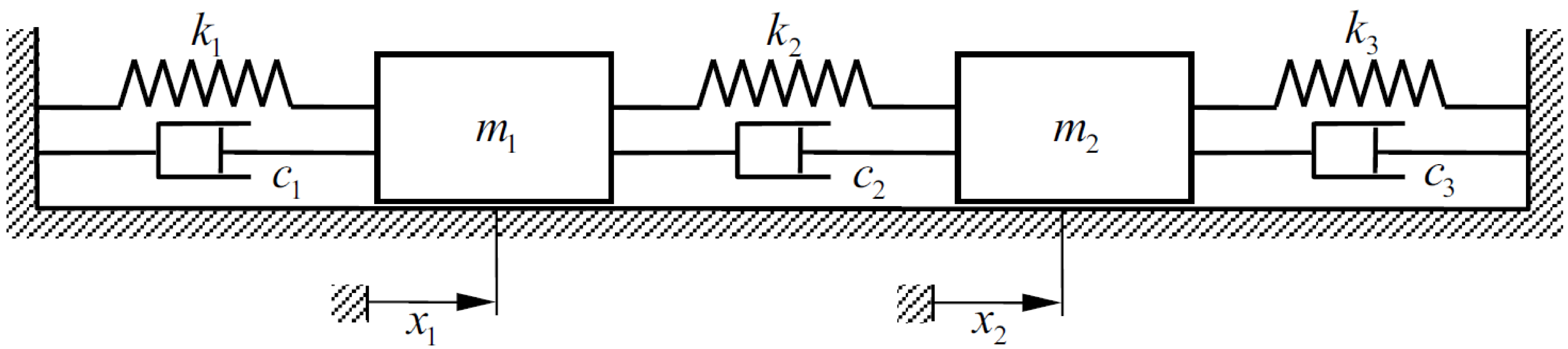
Vibrazioni libere smorzate a 2 GDL: soluzione mediante sviluppo simbolico del determinante e calcolo del polinomio caratteristico



Calcolo simbolico senza valori numerici

$$\mathbf{M} := \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \qquad \mathbf{C} := \begin{bmatrix} (c_1 + c_2) & -c_2 \\ -c_2 & (c_2 + c_3) \end{bmatrix} \qquad \mathbf{K} := \begin{bmatrix} (k_1 + k_2) & -k_2 \\ -k_2 & (k_2 + k_3) \end{bmatrix}$$

$$\Delta_{\mathbf{star}}(\lambda) := \mathbf{K} + \lambda \cdot \mathbf{C} + \lambda^2 \cdot \mathbf{M}$$

$$\Delta_{\mathbf{star}}(\lambda) \rightarrow \begin{bmatrix} m_1 \cdot \lambda^2 + (c_1 + c_2) \cdot \lambda + k_1 + k_2 & -k_2 - \lambda \cdot c_2 \\ -k_2 - \lambda \cdot c_2 & m_2 \cdot \lambda^2 + (c_2 + c_3) \cdot \lambda + k_2 + k_3 \end{bmatrix}$$

$$\left|\Delta_{\mathbf{star}}(\lambda)\right| \ \mathit{collect}, \lambda \ \rightarrow m_1 \cdot m_2 \cdot \lambda^4 + (c_1 \cdot m_2 + c_2 \cdot m_1 + c_2 \cdot m_2 + c_3 \cdot m_1) \cdot \lambda^3 + (c_1 \cdot c_2 + c_1 \cdot c_3 + c_2 \cdot c_3 + k_1 \cdot m_2 + k_2 \cdot m_1 + k_2 \cdot m_2 + k_3 \cdot m_1) \cdot \lambda^2 + (c_1 \cdot k_2 + c_2 \cdot k_1 + c_1 \cdot k_3 + c_3 \cdot k_1 + c_2 \cdot k_3 + c_3 \cdot k_2) \cdot \lambda + k_1 \cdot k_2 + k_1 \cdot k_3 + k_2 \cdot k_3$$

$$\left|\Delta_{\mathbf{star}}(\lambda)\right| \ \mathit{coeffs}, \lambda \ \rightarrow \begin{pmatrix} k_1 \cdot k_2 + k_1 \cdot k_3 + k_2 \cdot k_3 \\ c_1 \cdot k_2 + c_2 \cdot k_1 + c_1 \cdot k_3 + c_3 \cdot k_1 + c_2 \cdot k_3 + c_3 \cdot k_2 \\ c_1 \cdot c_2 + c_1 \cdot c_3 + c_2 \cdot c_3 + k_1 \cdot m_2 + k_2 \cdot m_1 + k_2 \cdot m_2 + k_3 \cdot m_1 \\ c_1 \cdot m_2 + c_2 \cdot m_1 + c_2 \cdot m_2 + c_3 \cdot m_1 \\ m_1 \cdot m_2 \end{pmatrix}$$

Parametri del sistema vibrante

$m_1 := 6$ $\quad m_2 := 7$

$k_1 := 600$ $\quad k_2 := 500$ $\quad k_3 := 100$

$c_1 := 5$ $\quad c_2 := 10$ $\quad c_3 := 8$

Condizioni iniziali

$x_{10} := 0.05$ $\quad x_{20} := 0.04$

$x'_{10} := 0.2$ $\quad x'_{20} := 0.3$

Matrici di massa, smorzamento e rigidezza

$$\mathbf{M} := \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 0 & 7 \end{pmatrix}$$

$$\mathbf{C} := \begin{bmatrix} (c_1 + c_2) & -c_2 \\ -c_2 & (c_2 + c_3) \end{bmatrix} = \begin{pmatrix} 15 & -10 \\ -10 & 18 \end{pmatrix}$$

$$\mathbf{K} := \begin{bmatrix} (k_1 + k_2) & -k_2 \\ -k_2 & (k_2 + k_3) \end{bmatrix} = \begin{pmatrix} 1100 & -500 \\ -500 & 600 \end{pmatrix}$$

$$\Delta_{\mathbf{star}}(\lambda) := \mathbf{K} + \lambda \cdot \mathbf{C} + \lambda^2 \cdot \mathbf{M}$$

$$\Delta_{\mathbf{star}}(\lambda) \rightarrow \begin{pmatrix} 6 \cdot \lambda^2 + 15 \cdot \lambda + 1100 & -10 \cdot \lambda - 500 \\ -10 \cdot \lambda - 500 & 7 \cdot \lambda^2 + 18 \cdot \lambda + 600 \end{pmatrix}$$

Coefficienti del polinomio caratteristico

$$\begin{pmatrix} E \\ D \\ C \\ B \\ A \end{pmatrix} := \begin{pmatrix} k_1 \cdot k_2 + k_1 \cdot k_3 + k_2 \cdot k_3 \\ c_1 \cdot k_2 + c_2 \cdot k_1 + c_1 \cdot k_3 + c_3 \cdot k_1 + c_2 \cdot k_3 + c_3 \cdot k_2 \\ c_1 \cdot c_2 + c_1 \cdot c_3 + c_2 \cdot c_3 + k_1 \cdot m_2 + k_2 \cdot m_1 + k_2 \cdot m_2 + k_3 \cdot m_1 \\ c_1 \cdot m_2 + c_2 \cdot m_1 + c_2 \cdot m_2 + c_3 \cdot m_1 \\ m_1 \cdot m_2 \end{pmatrix} = \begin{pmatrix} 410000 \\ 18800 \\ 11470 \\ 213 \\ 42 \end{pmatrix}$$

$A = 42$ $\quad B = 213$ $\quad C = 11470$ $\quad D = 18800$ $\quad E = 410000$

Polinomio caratteristico

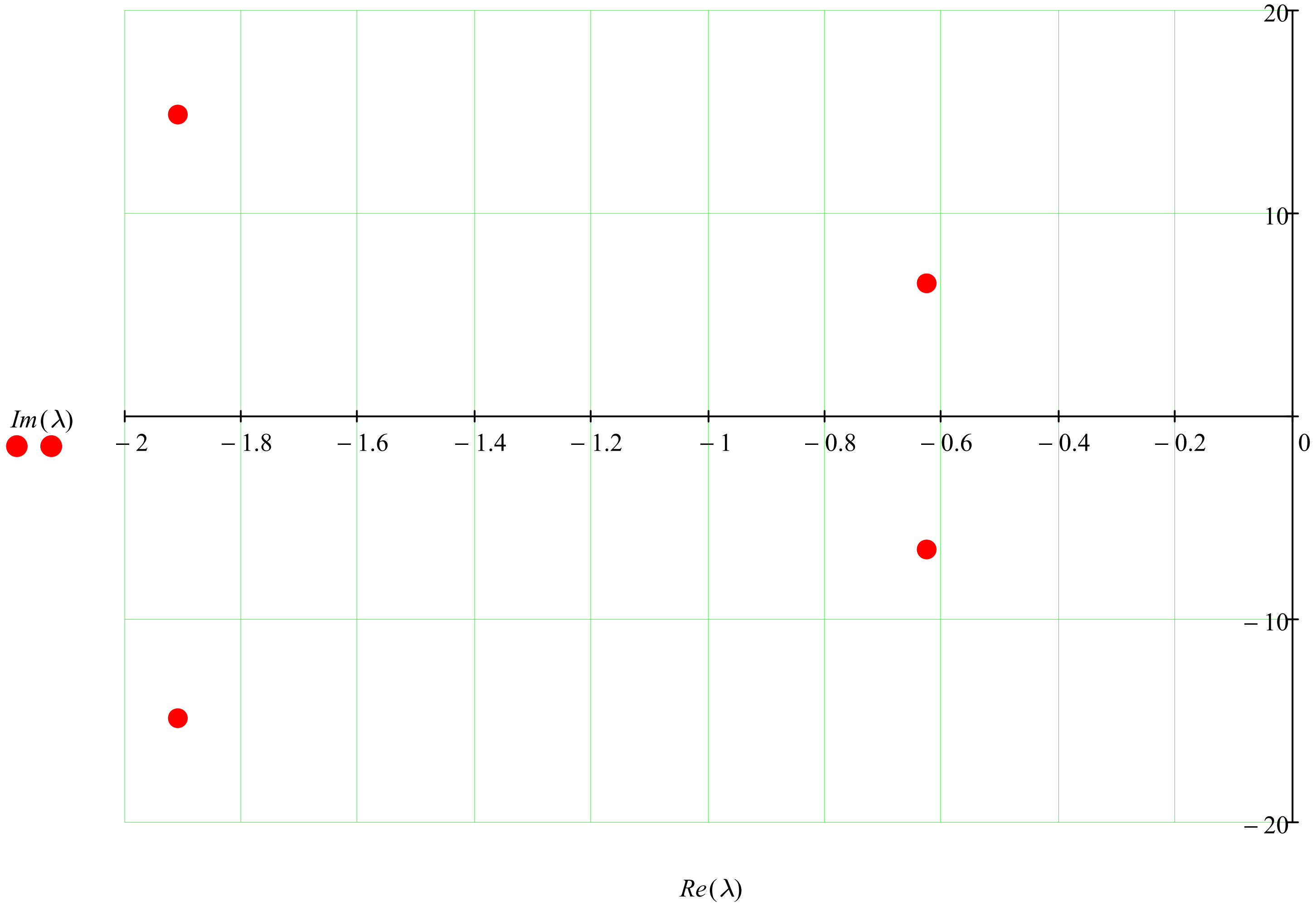
$$P(\lambda) := A\cdot\lambda^4 + B\cdot\lambda^3 + C\cdot\lambda^2 + D\cdot\lambda + E$$

$$P(\lambda) \rightarrow 42\cdot\lambda^4 + 213\cdot\lambda^3 + 11470\cdot\lambda^2 + 18800\cdot\lambda + 410000$$

Radici del polinomio caratteristico

$$\lambda := polyroots\left(\begin{pmatrix} E \\ D \\ C \\ B \\ A \end{pmatrix}\right) \qquad \lambda = \begin{pmatrix} -1.90901 + 14.87484i \\ -1.90901 - 14.87484i \\ -0.62671 - 6.55834i \\ -0.62671 + 6.55834i \end{pmatrix}$$

Posizione delle radici del polinomio caratteristico nel piano complesso



$$rapp(\lambda) := -\left(\frac{\Delta_{\mathbf{star}}(\lambda)_{1,2}}{\Delta_{\mathbf{star}}(\lambda)_{1,1}}\right)$$

Calcolo dei rapporti r_k:

$i := 1..4$

$r_i := rapp(\lambda_i)$

$$\lambda = \begin{pmatrix} -1.90901 + 14.87484i \\ -1.90901 - 14.87484i \\ -0.62671 - 6.55834i \\ -0.62671 + 6.55834i \end{pmatrix} \qquad r = \begin{pmatrix} -1.8937 + 0.31583i \\ -1.8937 - 0.31583i \\ 0.59394 - 0.04366i \\ 0.59394 + 0.04366i \end{pmatrix}$$

$$\mathbf{S} := \begin{pmatrix} r_1 & r_2 & r_3 & r_4 \\ 1 & 1 & 1 & 1 \\ r_1 \cdot \lambda_1 & r_2 \cdot \lambda_2 & r_3 \cdot \lambda_3 & r_4 \cdot \lambda_4 \\ \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \end{pmatrix} = \begin{pmatrix} -1.8937 + 0.31583i & -1.8937 - 0.31583i & 0.59394 - 0.04366i & 0.59394 + 0.04366i \\ 1 & 1 & 1 & 1 \\ -1.08284 - 28.77138i & -1.08283 + 28.77139i & -0.65855 - 3.86792i & -0.65855 + 3.86792i \\ -1.90901 + 14.87484i & -1.90901 - 14.87484i & -0.62671 - 6.55834i & -0.62671 + 6.55834i \end{pmatrix}$$

Calcolo delle costanti X_{2k} con le condizioni iniziali

$$X_2 := \mathbf{S}^{-1} \cdot \begin{pmatrix} x_{10} \\ x_{20} \\ x'_{10} \\ x'_{20} \end{pmatrix} = \begin{pmatrix} -0.0049 + 0.0005i \\ -0.0049 - 0.0005i \\ 0.0249 + 0.02497i \\ 0.0249 - 0.02497i \end{pmatrix}$$

Calcolo delle costanti X_{1k} con le condizioni iniziali

$i := 1..4$

$X_{1_i} := r_i \cdot X_{2_i}$

Soluzione nel dominio del tempo (movimento delle due masse)

$$x_1(t) := \sum_{i=1}^{4} \left(X_{1_i} \cdot e^{\lambda_i \cdot t} \right)$$

$$x_2(t) := \sum_{i=1}^{4} \left(X_{2_i} \cdot e^{\lambda_i \cdot t} \right)$$

$$x'_1(t) := \sum_{i=1}^{4} \left(\lambda_i \cdot X_{1_i} \cdot e^{\lambda_i \cdot t} \right)$$

$$x'_2(t) := \sum_{i=1}^{4} \left(\lambda_i \cdot X_{2_i} \cdot e^{\lambda_i \cdot t} \right)$$

$$x''_1(t) := \sum_{i=1}^{4} \left[\left(\lambda_i\right)^2 \cdot X_{1_i} \cdot e^{\lambda_i \cdot t} \right]$$

$$x''_2(t) := \sum_{i=1}^{4} \left[\left(\lambda_i\right)^2 \cdot X_{2_i} \cdot e^{\lambda_i \cdot t} \right]$$

$T_{max} := 8$ $\quad\quad$ $\Delta t := 0.001$

$t := 0, \Delta t .. T_{max}$

Verifica numerica (Runge Kutta)

$$u := \begin{pmatrix} x_{10} \\ x_{20} \\ x'_{10} \\ x'_{20} \end{pmatrix} = \begin{pmatrix} 0.05 \\ 0.04 \\ 0.2 \\ 0.3 \end{pmatrix}$$

Assegnazione delle condiz. iniziali

$$N_{interv} := ceil\left(\frac{T_{max}}{\Delta t}\right) = 8000$$

$$ACC\left(x_1, x_2, x'_1, x'_2\right) := -\mathbf{M}^{-1} \cdot \left[\mathbf{C} \cdot \begin{pmatrix} x'_1 \\ x'_2 \end{pmatrix} + \mathbf{K} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \right]$$

$$f\left(x_1, x_2, x'_1, x'_2\right) := ACC\left(x_1, x_2, x'_1, x'_2\right)_1$$

Accelerazione massa 1

$$g\left(x_1, x_2, x'_1, x'_2\right) := ACC\left(x_1, x_2, x'_1, x'_2\right)_2$$

Accelerazione massa 2

$$EQMOTO(t,u) := \begin{pmatrix} u_3 \\ u_4 \\ f(u_1,u_2,u_3,u_4) \\ g(u_1,u_2,u_3,u_4) \end{pmatrix}$$

Sistema delle equazioni di moto (4 eq. diff. del 1° ordine)

$$TAB := rkfixed\left(u,0,T_{max},N_{interv},EQMOTO\right)$$

Calcolo della soluzione in forma tabellare

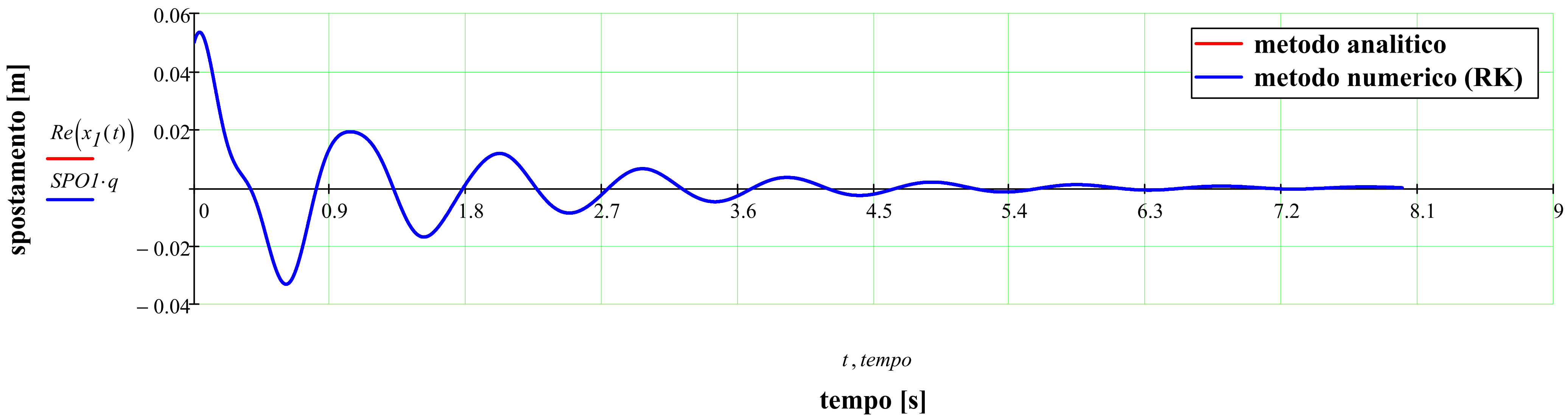
$tempo := TAB^{\langle 1\rangle}$

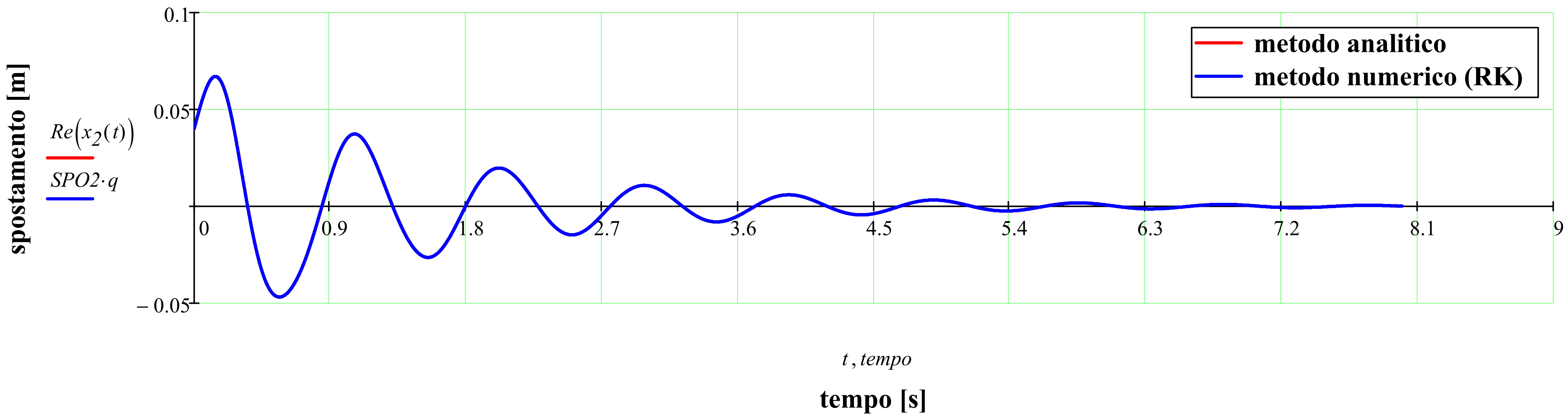
$SPO1 := TAB^{\langle 2\rangle}$ $\qquad$ $SPO2 := TAB^{\langle 3\rangle}$

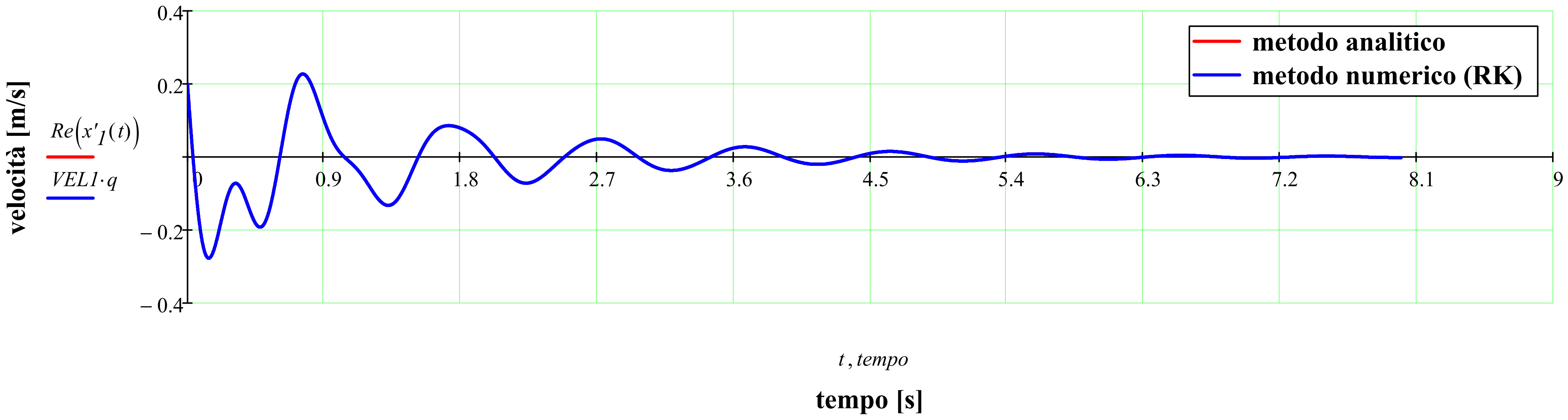
$VEL1 := TAB^{\langle 4\rangle}$ $\qquad$ $VEL2 := TAB^{\langle 5\rangle}$

$ACC1 := \overrightarrow{f(SPO1,SPO2,VEL1,VEL2)}$ $\qquad$ $ACC2 := \overrightarrow{g(SPO1,SPO2,VEL1,VEL2)}$

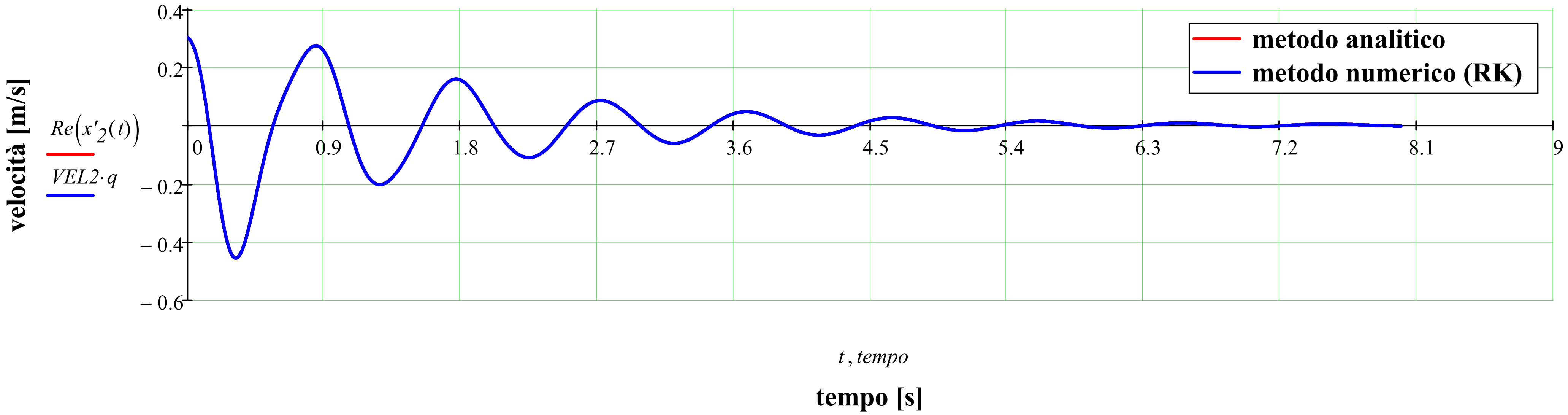
$q := 1$ Variabile on/off (attiva e disattiva i grafici della soluzione numerica



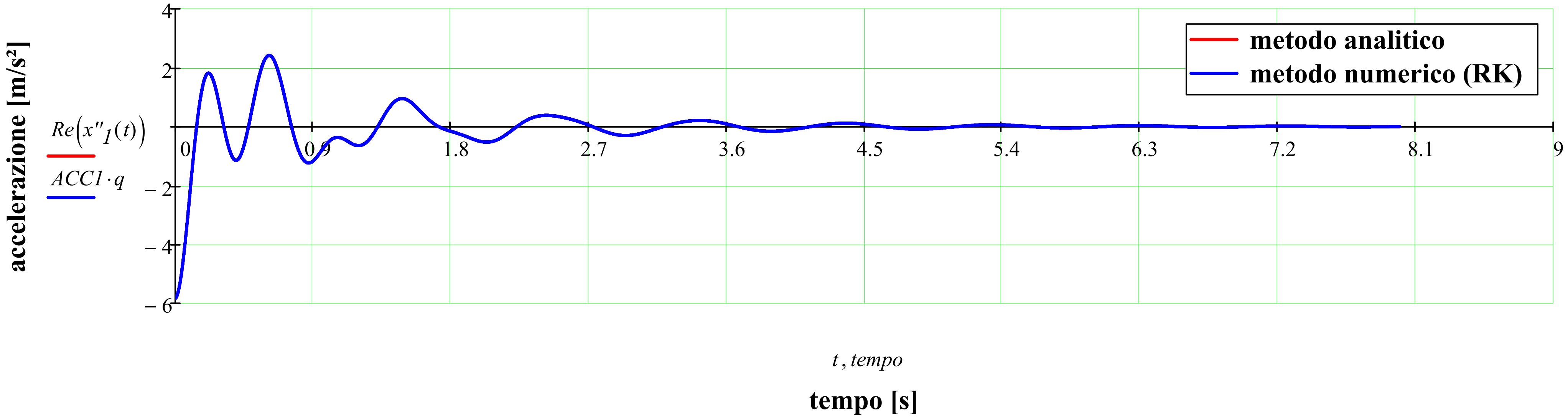


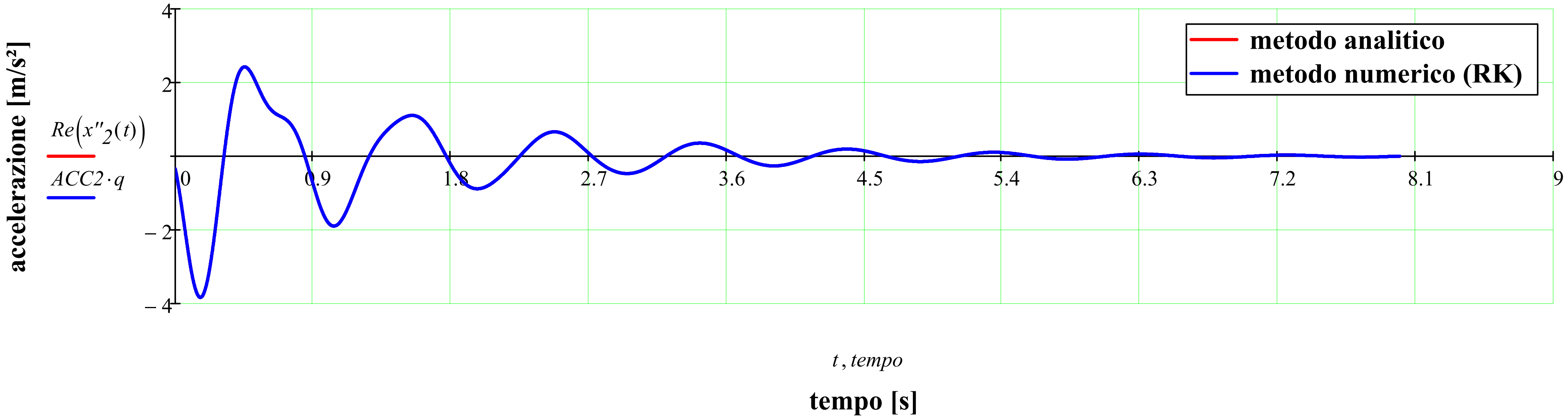


metodo analitico
metodo numerico (RK)
velocità [m/s]
Re(x'1(t))
VEL1·q
t, tempo
tempo [s]



metodo analitico
metodo numerico (RK)
velocità [m/s]
Re(x'2(t))
VEL2·q
t, tempo
tempo [s]


accelerazione [m/s²]
Re(x''1(t))
ACC1·q
metodo analitico
metodo numerico (RK)
4
2
−2
−4
−6
0
0.9
1.8
2.7
3.6
4.5
5.4
6.3
7.2
8.1
9
t, tempo
tempo [s]


accelerazione [m/s²]
Re(x''2(t))
ACC2·q
metodo analitico
metodo numerico (RK)
4
2
−2
−4
0
0.9
1.8
2.7
3.6
4.5
5.4
6.3
7.2
8.1
9
t, tempo
tempo [s]