## Sistema con tre masse e quattro molle

$$V(x_1, x_2, x_3) := \frac{1}{2}\cdot\left[k_1\cdot x_1^2 + k_2\cdot(x_2 - x_1)^2 + k_3\cdot(x_3 - x_2)^2 + k_4\cdot x_3^2\right]$$

**Energia potenziale**

## Derivate dell'energia potenziale

$$\frac{d}{dx_1}V(x_1, x_2, x_3) \text{ collect}, x_1, x_2, x_3 \rightarrow (k_1 + k_2)\cdot x_1 + (-k_2)\cdot x_2$$

$$\frac{d}{dx_2}V(x_1, x_2, x_3) \text{ collect}, x_1, x_2, x_3 \rightarrow (-k_2)\cdot x_1 + (k_2 + k_3)\cdot x_2 + (-k_3)\cdot x_3$$

$$\frac{d}{dx_3}V(x_1, x_2, x_3) \text{ collect}, x_1, x_2, x_3 \rightarrow (-k_3)\cdot x_2 + (k_3 + k_4)\cdot x_3$$

$$\mathbf{M} := diag\left(\begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}\right) \qquad \mathbf{M} \rightarrow \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

$$\mathbf{K} := \begin{bmatrix} (k_1 + k_2) & -k_2 & 0 \\ -k_2 & (k_2 + k_3) & -k_3 \\ 0 & -k_3 & (k_3 + k_4) \end{bmatrix} \qquad \mathbf{K} \rightarrow \begin{pmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 + k_4 \end{pmatrix}$$

$$\mathbf{\Delta}(\omega) := \mathbf{K} - \omega^2\cdot\mathbf{M}$$

$$\mathbf{\Delta}(\omega) \rightarrow \begin{pmatrix} k_1 - m_1\cdot\omega^2 + k_2 & -k_2 & 0 \\ -k_2 & k_2 - m_2\cdot\omega^2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 - m_3\cdot\omega^2 + k_4 \end{pmatrix}$$

$$\left|\mathbf{\Delta}(\omega)\right| \text{ coeffs}, \omega \rightarrow \begin{pmatrix} k_1\cdot k_2\cdot k_3 + k_1\cdot k_2\cdot k_4 + k_1\cdot k_3\cdot k_4 + k_2\cdot k_3\cdot k_4 \\ 0 \\ -k_1\cdot k_2\cdot m_3 - k_1\cdot k_3\cdot m_2 - k_2\cdot k_3\cdot m_1 - k_1\cdot k_3\cdot m_3 - k_1\cdot k_4\cdot m_2 - k_2\cdot k_3\cdot m_2 - k_2\cdot k_4\cdot m_1 - k_2\cdot k_3\cdot m_3 - k_2\cdot k_4\cdot m_2 - k_3\cdot k_4\cdot m_1 \\ 0 \\ k_1\cdot m_2\cdot m_3 + k_2\cdot m_1\cdot m_3 + k_3\cdot m_1\cdot m_2 + k_2\cdot m_2\cdot m_3 + k_3\cdot m_1\cdot m_3 + k_4\cdot m_1\cdot m_2 \\ 0 \\ -m_1\cdot m_2\cdot m_3 \end{pmatrix}$$

$$\left|\Delta(\omega)\right| \text{ collect}, \omega \rightarrow (-m_1 \cdot m_2 \cdot m_3) \cdot \omega^6 + (k_1 \cdot m_2 \cdot m_3 + k_2 \cdot m_1 \cdot m_3 + k_3 \cdot m_1 \cdot m_2 + k_2 \cdot m_2 \cdot m_3 + k_3 \cdot m_1 \cdot m_3 + k_4 \cdot m_1 \cdot m_2) \cdot \omega^4 + (-k_1 \cdot k_2 \cdot m_3 - k_1 \cdot k_3 \cdot m_2 - k_2 \cdot k_3 \cdot m_1 - k_1 \cdot k_3 \cdot m_3 - k_1 \cdot k_4 \cdot m_2 - k_2 \cdot k_3 \cdot m_2 - k_2 \cdot k_4 \cdot m_1 - k_2 \cdot k_3 \cdot m_3 - k_2 \cdot k_4 \cdot m_2 - k_3 \cdot k_4 \cdot m_1) \cdot \omega^2 + k_1 \cdot k_2 \cdot k_3 + k_1 \cdot k_2 \cdot k_4 + k_1 \cdot k_3 \cdot k_4 + k_2 \cdot k_3 \cdot k_4$$

## Esempio numerico

$m_1 := 3$ $\quad m_2 := 5$ $\quad m_3 := 10$

$k_1 := 1500$ $\quad k_2 := 2000$ $\quad k_3 := 800$ $\quad k_4 := 1200$

$$\mathbf{M} := diag\left(\begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}\right) = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 10 \end{pmatrix}$$

$$\mathbf{K} := \begin{bmatrix} (k_1 + k_2) & -k_2 & 0 \\ -k_2 & (k_2 + k_3) & -k_3 \\ 0 & -k_3 & (k_3 + k_4) \end{bmatrix} = \begin{pmatrix} 3500 & -2000 & 0 \\ -2000 & 2800 & -800 \\ 0 & -800 & 2000 \end{pmatrix}$$

$$\Delta(\omega) := \mathbf{K} - \omega^2 \cdot \mathbf{M}$$

$$\Delta(\omega) \rightarrow \begin{pmatrix} 3500 - 3 \cdot \omega^2 & -2000 & 0 \\ -2000 & 2800 - 5 \cdot \omega^2 & -800 \\ 0 & -800 & 2000 - 10 \cdot \omega^2 \end{pmatrix}$$

## Equazione caratteristica e suoi coefficienti

$$COEFF := \left|\Delta(\omega)\right| \text{ coeffs}, \omega \rightarrow \begin{pmatrix} 9360000000 \\ 0 \\ -107880000 \\ 0 \\ 289000 \\ 0 \\ -150 \end{pmatrix} \qquad COEFF = \begin{pmatrix} 9360000000 \\ 0 \\ -107880000 \\ 0 \\ 289000 \\ 0 \\ -150 \end{pmatrix}$$

$$\left|\Delta(\omega)\right| \text{ collect}, \omega \rightarrow 289000 \cdot \omega^4 - 150 \cdot \omega^6 - 107880000 \cdot \omega^2 + 9360000000$$

## Pulsazioni proprie

$$\omega := polyroots(COEFF) = \begin{pmatrix} -38.272 \\ -18.292 \\ -11.284 \\ 11.284 \\ 18.292 \\ 38.272 \end{pmatrix}$$

Bisogna considerare solo le soluzioni positive

$$\omega := 0, 0.1 .. 40$$