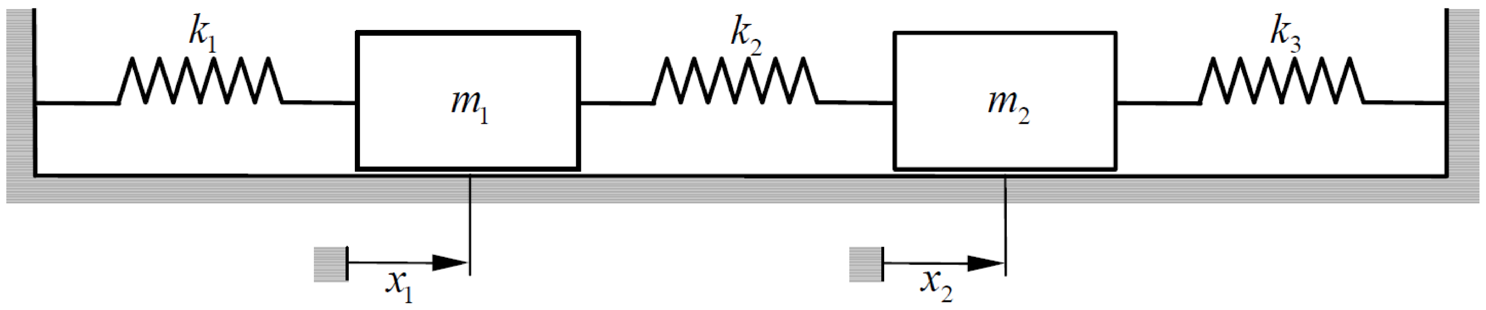


$m_1 := 2.5$

$m_2 := 3$

$k_1 := 2000$

$k_2 := 4000$

$k_3 := 3000$

$$\mathbf{M} := \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \qquad \mathbf{M} = \begin{pmatrix} 2.5 & 0 \\ 0 & 3 \end{pmatrix}$$

$$\mathbf{K} := \begin{bmatrix} (k_1 + k_2) & -k_2 \\ -k_2 & (k_2 + k_3) \end{bmatrix} \qquad \mathbf{K} = \begin{pmatrix} 6000 & -4000 \\ -4000 & 7000 \end{pmatrix}$$

$$\mathbf{\Delta}(\omega) := \mathbf{K} - \omega^2 \cdot \mathbf{M}$$

$$\mathbf{\Delta}(\omega) \rightarrow \begin{pmatrix} k_1 - m_1 \cdot \omega^2 + k_2 & -k_2 \\ -k_2 & k_2 - m_2 \cdot \omega^2 + k_3 \end{pmatrix}$$

$$det(\omega) := \left| \mathbf{\Delta}(\omega) \right|$$

$$det(\omega) \text{ collect}, \omega \rightarrow m_1 \cdot m_2 \cdot \omega^4 + \left(-k_1 \cdot m_2 - k_2 \cdot m_1 - k_2 \cdot m_2 - k_3 \cdot m_1 \right) \cdot \omega^2 + k_1 \cdot k_2 + k_1 \cdot k_3 + k_2 \cdot k_3$$

$$det(\omega)\ coeffs, \omega \ \rightarrow \begin{pmatrix} k_1 \cdot k_2 + k_1 \cdot k_3 + k_2 \cdot k_3 \\ 0 \\ -k_1 \cdot m_2 - k_2 \cdot m_1 - k_2 \cdot m_2 - k_3 \cdot m_1 \\ 0 \\ m_1 \cdot m_2 \end{pmatrix}$$

Coefficienti del polinomio caratteristico

$A := m_1 \cdot m_2 = 7.5$

$B := -k_1 \cdot m_2 - k_2 \cdot m_1 - k_2 \cdot m_2 - k_3 \cdot m_1 = -35500$

$C := k_1 \cdot k_2 + k_1 \cdot k_3 + k_2 \cdot k_3 = 26000000$

$\gamma_1 := \dfrac{-B - \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} = 905.693$ $\qquad \omega_1 := \sqrt{\gamma_1} = 30.095$

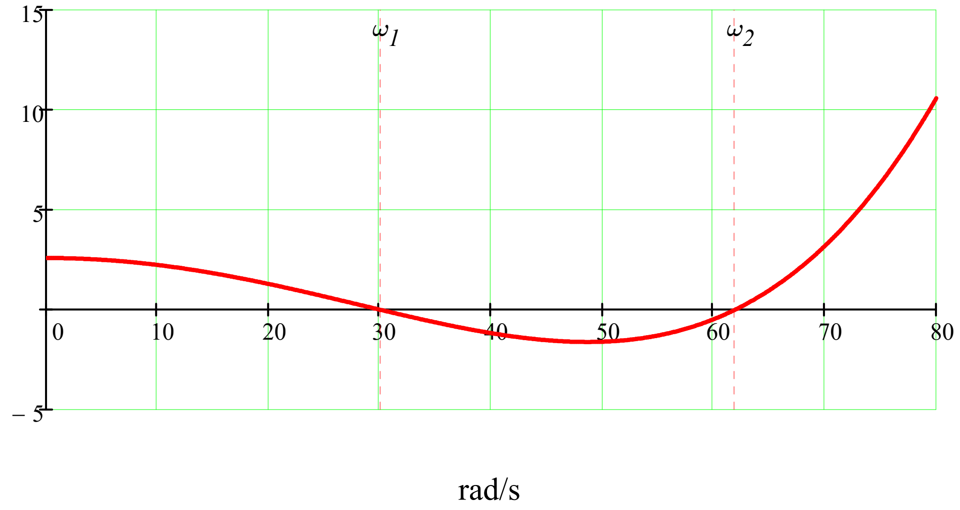
$\gamma_2 := \dfrac{-B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} = 3827.64$ $\qquad \omega_2 := \sqrt{\gamma_2} = 61.868$

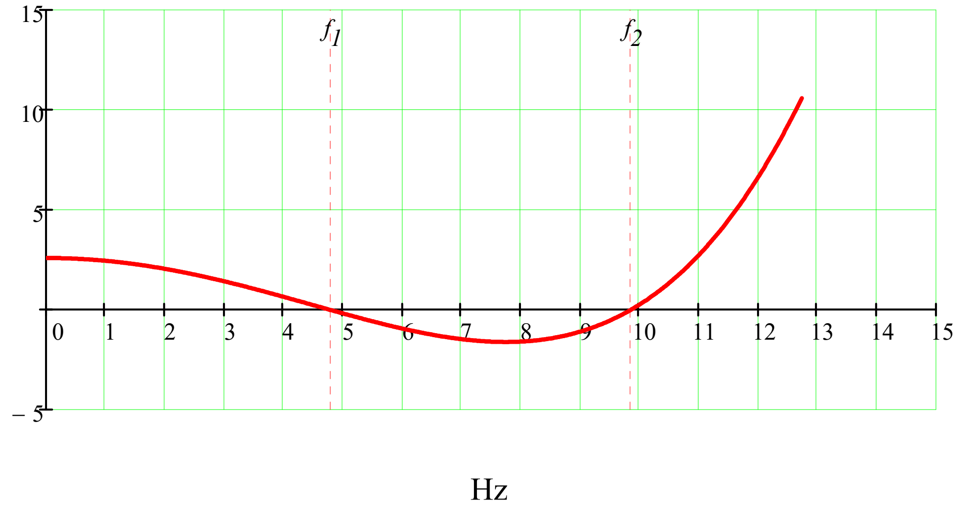
$$coeff := \begin{pmatrix} C \\ B \\ A \end{pmatrix} = \begin{pmatrix} 2.6 \times 10^7 \\ -35500 \\ 7.5 \end{pmatrix}$$

$$\sqrt{polyroots(coeff)} = \begin{pmatrix} 30.095 \\ 61.868 \end{pmatrix}$$

$f_1 := \dfrac{\omega_1}{2 \cdot \pi} = 4.79$ $\qquad f_2 := \dfrac{\omega_2}{2 \cdot \pi} = 9.847$

$\omega := 0, 0.1 .. 80$





$$r(\omega) := \frac{k_2}{(k_1 + k_2) - \omega^2 \cdot m_1}$$

$$r_{bis}(\omega) := \frac{(k_2 + k_3) - \omega^2 \cdot m_2}{k_2}$$

$\omega_1 = 30.095$

$r_1 := r(\omega_1) = 1.071$ $\quad$ $r_{bis}(\omega_1) = 1.071$

$\omega_2 = 61.868$

$r_2 := r(\omega_2) = -1.121$ $\quad$ $r_{bis}(\omega_2) = -1.121$

$\mathbf{X_1} := \begin{pmatrix} r_1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1.071 \\ 1 \end{pmatrix}$ $\quad$ $\omega_1 = 30.095$

$\mathbf{X_2} := \begin{pmatrix} r_2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1.121 \\ 1 \end{pmatrix}$ $\quad$ $\omega_2 = 61.868$

Condizioni iniziali

$x_{10} := 0.03$ $x'_{10} := 0.2$

$x_{20} := 0.02$ $x'_{20} := 0.3$

$$\begin{pmatrix} A \\ B \end{pmatrix} := \begin{pmatrix} r_1 & r_2 \\ 1 & 1 \end{pmatrix}^{-1} \cdot \begin{pmatrix} x_{10} \\ x_{20} \end{pmatrix} \qquad \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0.024 \\ -0.004 \end{pmatrix}$$

$$\begin{pmatrix} C \\ D \end{pmatrix} := \begin{pmatrix} \omega_1 \cdot r_1 & \omega_2 \cdot r_2 \\ \omega_1 & \omega_2 \end{pmatrix}^{-1} \cdot \begin{pmatrix} x'_{10} \\ x'_{20} \end{pmatrix} \qquad \begin{pmatrix} C \\ D \end{pmatrix} = \begin{pmatrix} 0.008 \\ 0.001 \end{pmatrix}$$

$A = A_1 \cdot sin(\alpha_1)$ $B = A_2 \cdot sin(\alpha_2)$

$C = A_1 \cdot cos(\alpha_1)$ $D = A_2 \cdot cos(\alpha_2)$

$A_1 := \sqrt{A^2 + C^2} = 0.025$

$A_2 := \sqrt{B^2 + D^2} = 0.004$

$\frac{A}{C} = 2.942$ $\frac{B}{D} = -4.382$

$\alpha_1 := atan2(C, A) = 1.243 \cdot rad$ $\alpha_1 = 71.225 \cdot deg$

$\alpha_2 := atan2(D, B) = -1.346 \cdot rad$ $\alpha_2 = -77.144 \cdot deg$

$$\mathbf{X}(t) := \left[A_1\cdot\begin{pmatrix} r_1 \\ 1 \end{pmatrix}\cdot sin(\omega_1\cdot t + \alpha_1)\right] + \left[A_2\cdot\begin{pmatrix} r_2 \\ 1 \end{pmatrix}\cdot sin(\omega_2\cdot t + \alpha_2)\right] \qquad x_1(t) := \mathbf{X}(t)_1 \qquad x_2(t) := \mathbf{X}(t)_2$$

spostamenti

$$\mathbf{X'}(t) := \left[\omega_1\cdot A_1\cdot\begin{pmatrix} r_1 \\ 1 \end{pmatrix}\cdot cos(\omega_1\cdot t + \alpha_1)\right] + \left[\omega_2\cdot A_2\cdot\begin{pmatrix} r_2 \\ 1 \end{pmatrix}\cdot cos(\omega_2\cdot t + \alpha_2)\right] \qquad x'_1(t) := \mathbf{X'}(t)_1 \qquad x'_2(t) := \mathbf{X'}(t)_2$$

velocità

$$\mathbf{X''}(t) := \left[-\omega_1^{\,2}\cdot A_1\cdot\begin{pmatrix} r_1 \\ 1 \end{pmatrix}\cdot sin(\omega_1\cdot t + \alpha_1)\right] + \left[-\omega_2^{\,2}\cdot A_2\cdot\begin{pmatrix} r_2 \\ 1 \end{pmatrix}\cdot sin(\omega_2\cdot t + \alpha_2)\right] \qquad x''_1(t) := \mathbf{X''}(t)_1 \qquad x''_2(t) := \mathbf{X''}(t)_2$$

accelerazioni

$\Delta t := 1\cdot 10^{-3}$ $\qquad T_{max} := 4$ $\qquad$ **Passo di calcolo e tempo max. di simulazione**

Integrazione numerica (Runge Kutta)

$$ACCEL(x_1, x_2) := -\mathbf{M}^{-1}\cdot\mathbf{K}\cdot\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Accelerazioni esplicitate (in forma matriciale)

$$f(x_1, x_2) := ACCEL(x_1, x_2)_1$$

$$g(x_1, x_2) := ACCEL(x_1, x_2)_2$$

$$EQMOTO(t, u) := \begin{pmatrix} u_3 \\ u_4 \\ f(u_1, u_2) \\ g(u_1, u_2) \end{pmatrix}$$

Equazioni di moto da risolvere

$$u := \begin{pmatrix} x_{10} \\ x_{20} \\ x'_{10} \\ x'_{20} \end{pmatrix} = \begin{pmatrix} 0.03 \\ 0.02 \\ 0.2 \\ 0.3 \end{pmatrix}$$

Condizioni iniziali

$$N := \frac{T_{max}}{\Delta t} = 4000$$

$$TAB := rkfixed(u, 0, T_{max}, N, EQMOTO)$$

Calcolo della soluzione per via numerica

$tempo := TAB^{\langle 1 \rangle}$

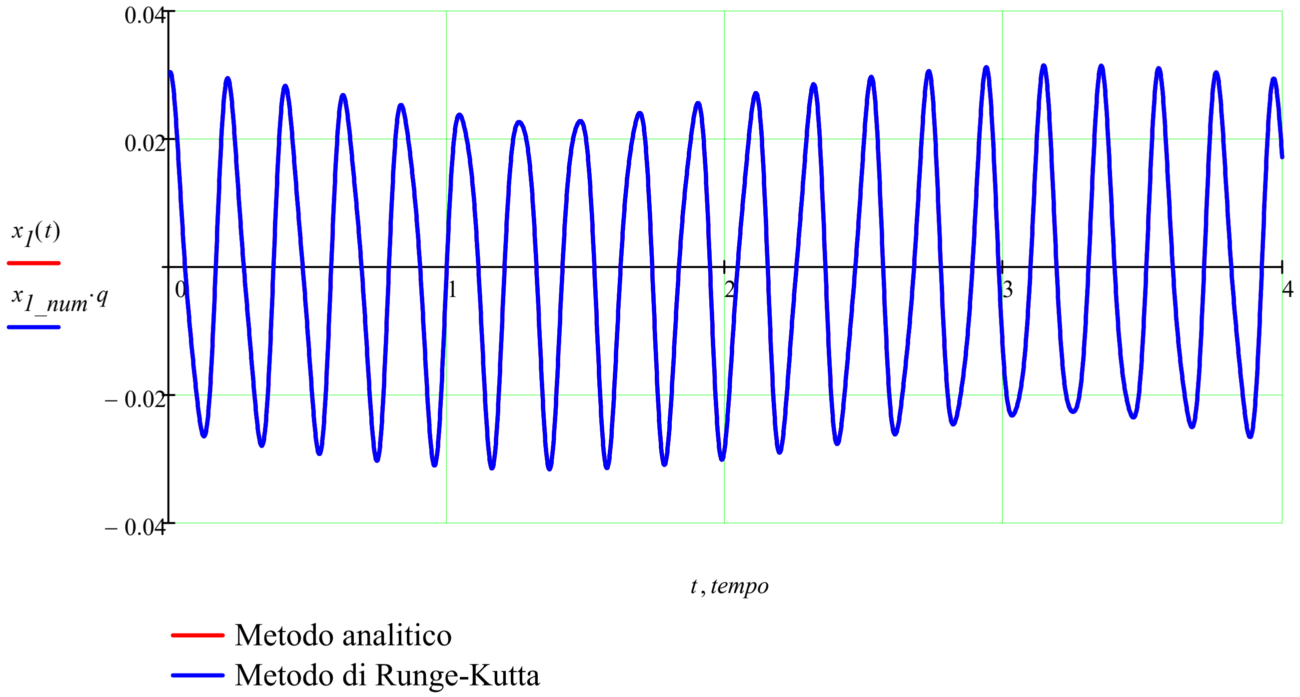
$x_{1\_num} := TAB^{\langle 2 \rangle}$ $\qquad$ $x_{2\_num} := TAB^{\langle 3 \rangle}$

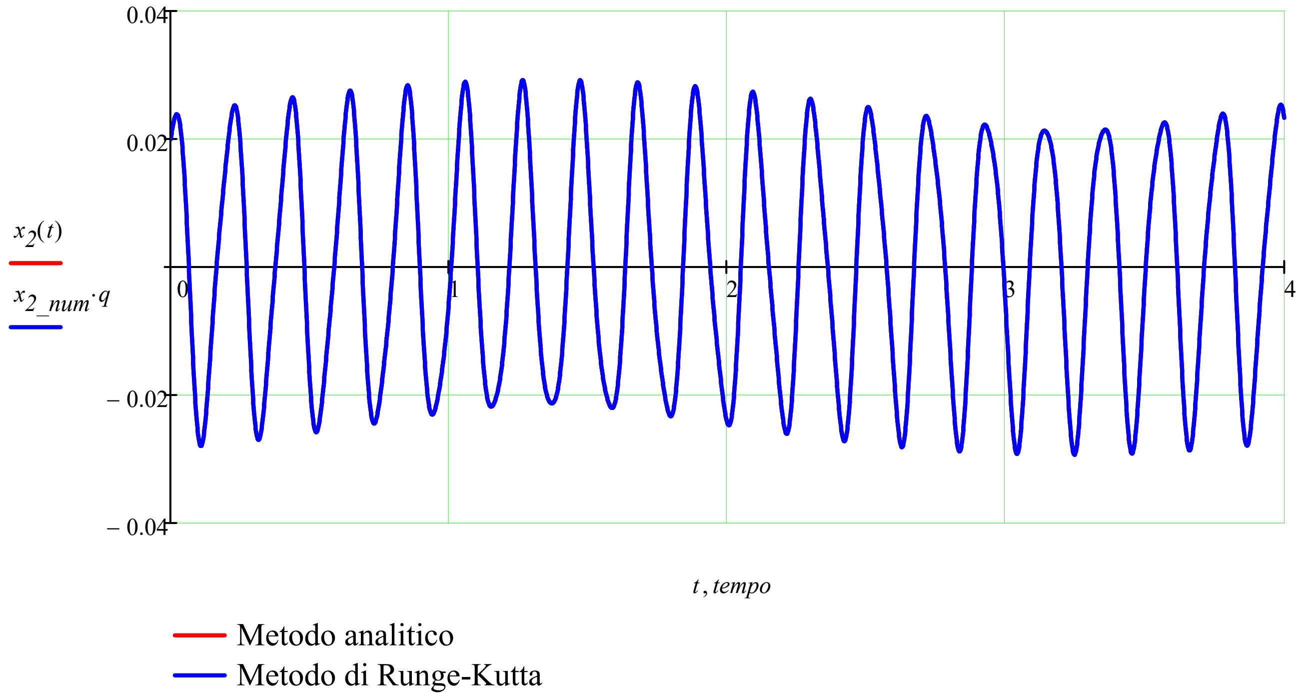
$x'_{1\_num} := TAB^{\langle 4 \rangle}$ $\qquad$ $x'_{2\_num} := TAB^{\langle 5 \rangle}$

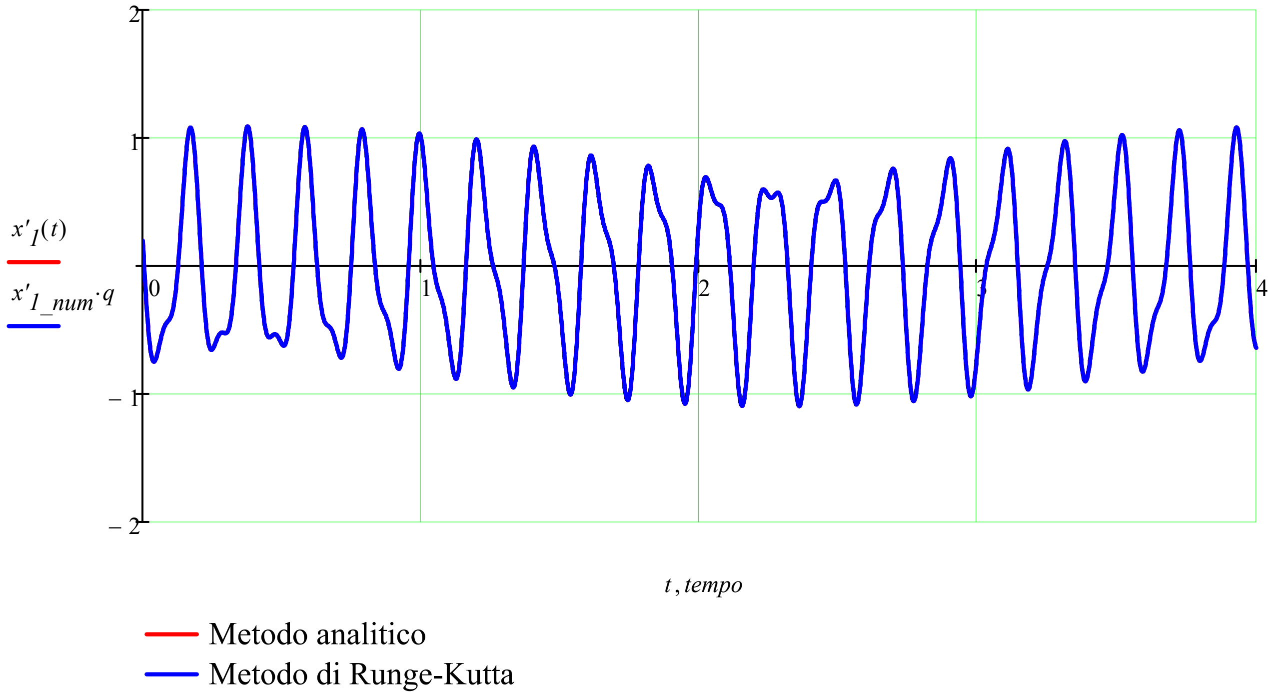
$x''_{1\_num} := \overrightarrow{f\left(x_{1\_num}, x_{2\_num}\right)}$

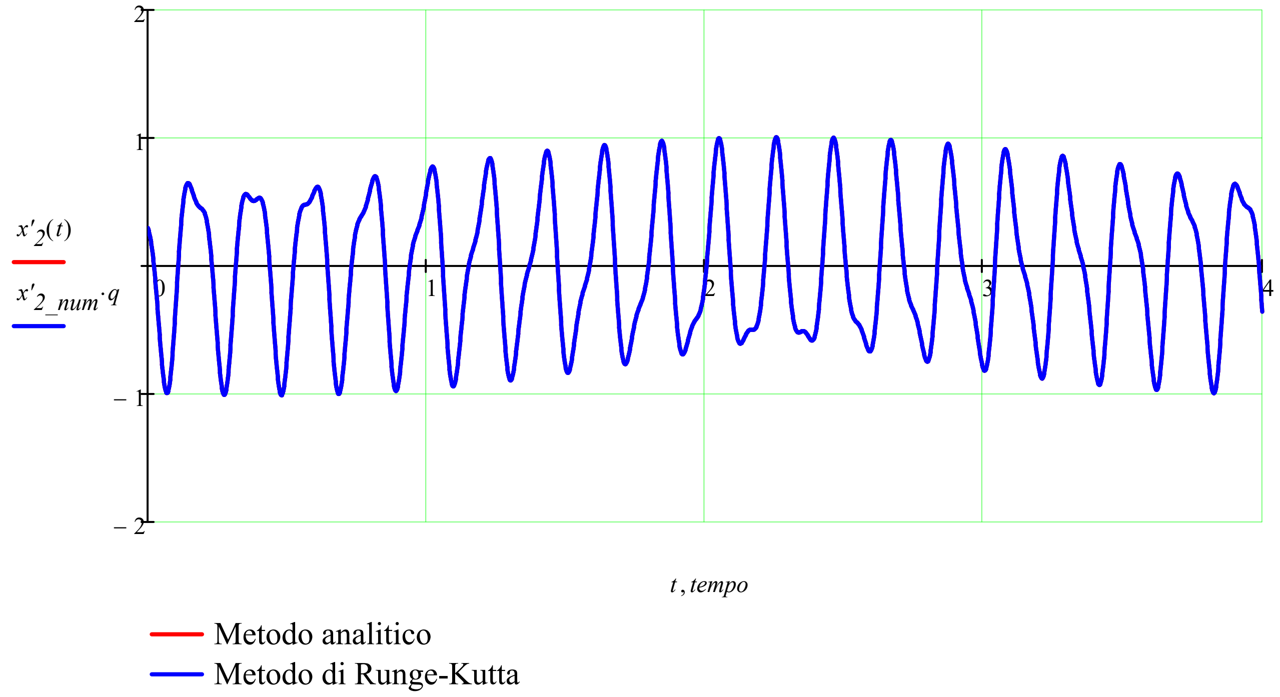
$x''_{2\_num} := \overrightarrow{g\left(x_{1\_num}, x_{2\_num}\right)}$

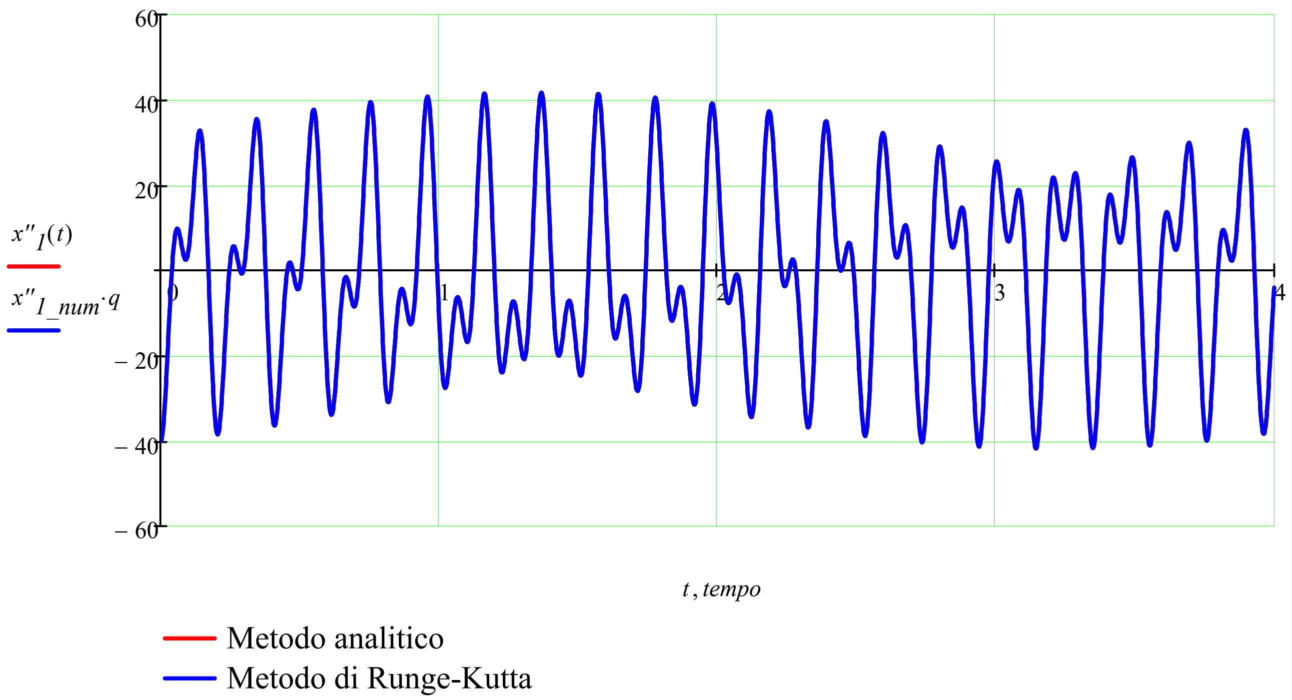
$t := 0, \Delta t .. T_{max}$ $\qquad$ $q := 1$

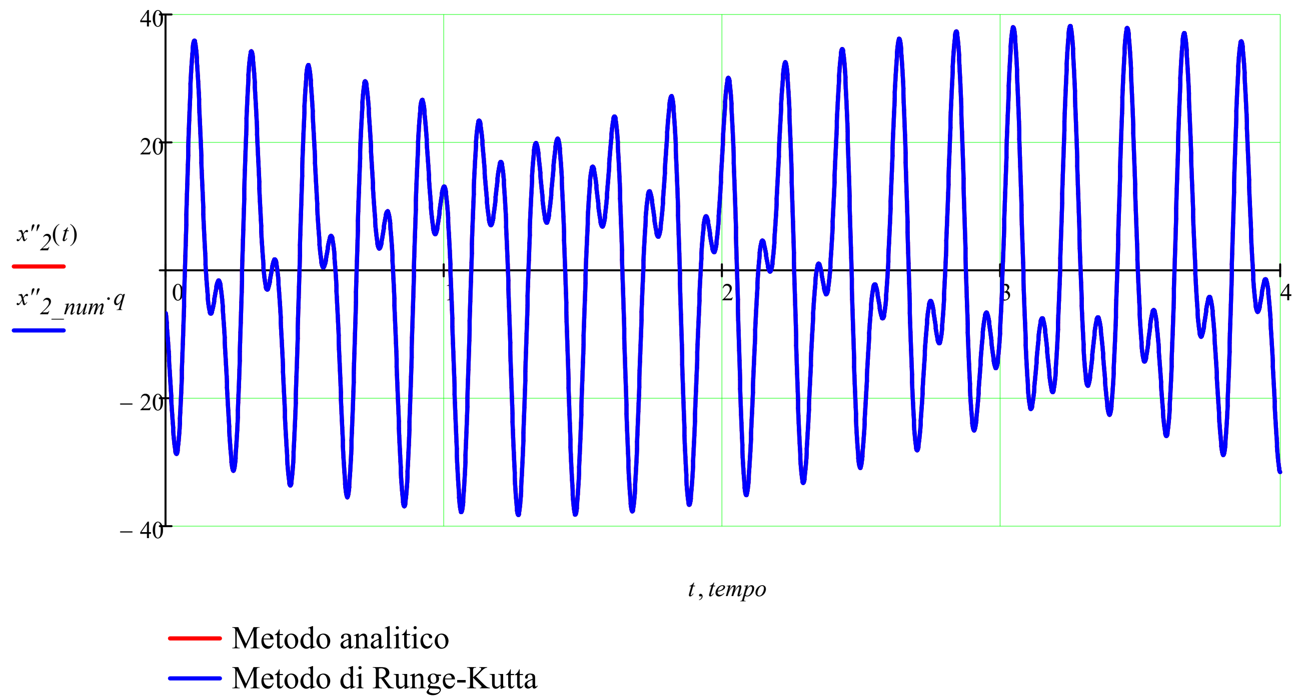





x'1(t)
x'1_num·q
t, tempo
Metodo analitico
Metodo di Runge-Kutta


x'2(t)
x'2_num·q
t, tempo
Metodo analitico
Metodo di Runge-Kutta


x''1(t)
x''1_num·q
t, tempo
Metodo analitico
Metodo di Runge-Kutta


x''2(t)
x''2_num·q
t, tempo
Metodo analitico
Metodo di Runge-Kutta