

$$m_1 := 4$$

$$m_2 := 5$$

$$k_1 := 2500$$

$$k_2 := 4500$$

$$k_3 := 3200$$



$$\mathbf{M} := \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$$

$$\mathbf{M} = \begin{pmatrix} 4 & 0 \\ 0 & 5 \end{pmatrix}$$

$$\mathbf{K} := \begin{bmatrix} (k_1 + k_2) & -k_2 \\ -k_2 & (k_2 + k_3) \end{bmatrix}$$

$$\mathbf{K} = \begin{pmatrix} 7000 & -4500 \\ -4500 & 7700 \end{pmatrix}$$

$$\Delta(\omega) := \mathbf{K} - \omega^2 \cdot \mathbf{M}$$

$$\Delta(\omega) \rightarrow \begin{pmatrix} k_1 - m_1 \cdot \omega^2 + k_2 & -k_2 \\ -k_2 & k_2 - m_2 \cdot \omega^2 + k_3 \end{pmatrix}$$

$$\det(\omega) := |\Delta(\omega)|$$

$$\det(\omega) \text{ collect, } \omega \rightarrow m_1 \cdot m_2 \cdot \omega^4 + (-k_1 \cdot m_2 - k_2 \cdot m_1 - k_2 \cdot m_2 - k_3 \cdot m_1) \cdot \omega^2 + k_1 \cdot k_2 + k_1 \cdot k_3 + k_2 \cdot k_3$$

$$\det(\omega) \text{ coeffs}, \omega \rightarrow \begin{pmatrix} k_1 \cdot k_2 + k_1 \cdot k_3 + k_2 \cdot k_3 \\ 0 \\ -k_1 \cdot m_2 - k_2 \cdot m_1 - k_2 \cdot m_2 - k_3 \cdot m_1 \\ 0 \\ m_1 \cdot m_2 \end{pmatrix}$$

**Coefficienti del polinomio caratteristico**

$$\underline{A} := m_1 \cdot m_2 = 20$$

$$\underline{B} := -k_1 \cdot m_2 - k_2 \cdot m_1 - k_2 \cdot m_2 - k_3 \cdot m_1 = -65800$$

$$\underline{C} := k_1 \cdot k_2 + k_1 \cdot k_3 + k_2 \cdot k_3 = 33650000$$

$$\gamma_1 := \frac{-B - \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} = 633.306$$

$$\omega_1 := \sqrt{\gamma_1} = 25.166$$

$$\gamma_2 := \frac{-B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} = 2656.694$$

$$\omega_2 := \sqrt{\gamma_2} = 51.543$$

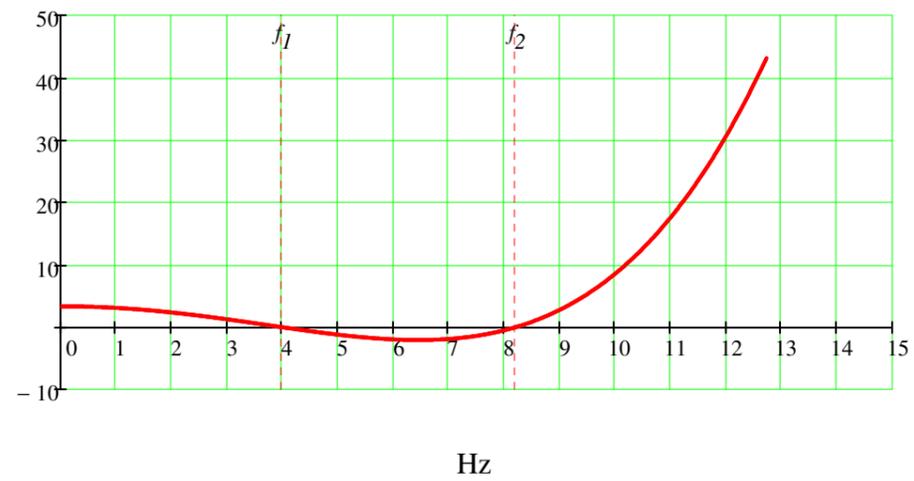
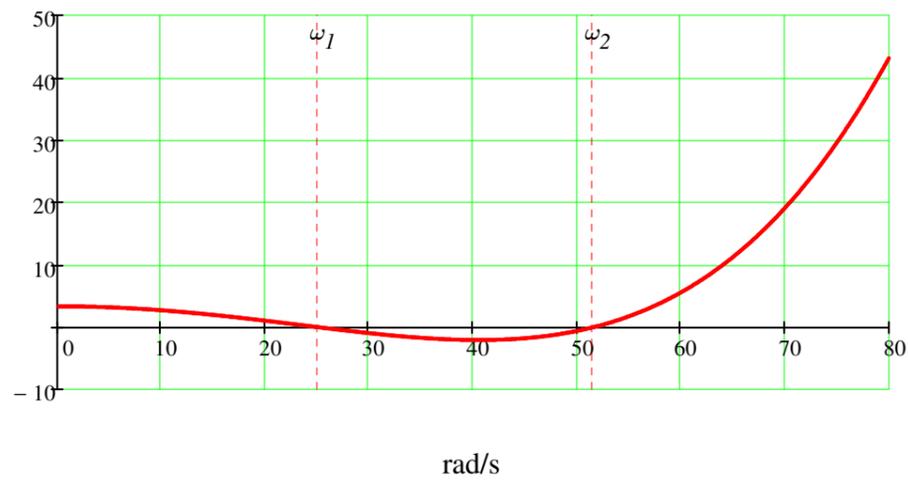
$$\text{coeff} := \begin{pmatrix} C \\ B \\ A \end{pmatrix} = \begin{pmatrix} 3.365 \times 10^7 \\ -65800 \\ 20 \end{pmatrix}$$

$$\sqrt{\text{polyroots}(\text{coeff})} = \begin{pmatrix} 25.166 \\ 51.543 \end{pmatrix}$$

$$f_1 := \frac{\omega_1}{2 \cdot \pi} = 4.005$$

$$f_2 := \frac{\omega_2}{2 \cdot \pi} = 8.203$$

$$\omega := 0, 0.1..80$$



$$r(\omega) := \frac{k_2}{(k_1 + k_2) - \omega^2 \cdot m_1}$$

$$r_{bis}(\omega) := \frac{(k_2 + k_3) - \omega^2 \cdot m_2}{k_2}$$

$$\omega_1 = 25.166$$

$$r_1 := r(\omega_1) = 1.007 \quad r_{bis}(\omega_1) = 1.007$$

$$\omega_2 = 51.543$$

$$r_2 := r(\omega_2) = -1.241 \quad r_{bis}(\omega_2) = -1.241$$

$$\mathbf{x}_1 := \begin{pmatrix} r_1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1.007 \\ 1 \end{pmatrix} \quad \omega_1 = 25.166$$

$$\mathbf{x}_2 := \begin{pmatrix} r_2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1.241 \\ 1 \end{pmatrix} \quad \omega_2 = 51.543$$

### Condizioni iniziali

$$x_{10} := 0.03 \quad x'_{10} := 0.2$$

$$x_{20} := 0.02 \quad x'_{20} := 0.3$$

$$\begin{pmatrix} \underline{A} \\ \underline{B} \end{pmatrix} := \begin{pmatrix} r_1 & r_2 \\ 1 & 1 \end{pmatrix}^{-1} \cdot \begin{pmatrix} x_{10} \\ x_{20} \end{pmatrix} \quad \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0.024 \\ -0.004 \end{pmatrix}$$

$$\begin{pmatrix} \underline{C} \\ \underline{D} \end{pmatrix} := \begin{pmatrix} \omega_1 \cdot r_1 & \omega_2 \cdot r_2 \\ \omega_1 & \omega_2 \end{pmatrix}^{-1} \cdot \begin{pmatrix} x'_{10} \\ x'_{20} \end{pmatrix} \quad \begin{pmatrix} C \\ D \end{pmatrix} = \begin{pmatrix} 0.01 \\ 0.001 \end{pmatrix}$$

$$A = A_1 \cdot \sin(\alpha_1) \quad B = A_2 \cdot \sin(\alpha_2)$$

$$C = A_1 \cdot \cos(\alpha_1) \quad D = A_2 \cdot \cos(\alpha_2)$$

$$A_1 := \sqrt{A^2 + C^2} = 0.026$$

$$A_2 := \sqrt{B^2 + D^2} = 0.004$$

$$\frac{A}{C} = 2.411 \quad \frac{B}{D} = -4.967$$

$$\alpha_1 := \text{atan2}(C, A) = 1.178 \cdot \text{rad} \quad \alpha_1 = 67.47 \cdot \text{deg}$$

$$\alpha_2 := \text{atan2}(D, B) = -1.372 \cdot \text{rad} \quad \alpha_2 = -78.616 \cdot \text{deg}$$

$$\mathbf{X}(t) := \left[ A_1 \cdot \begin{pmatrix} r_1 \\ 1 \end{pmatrix} \cdot \sin(\omega_1 \cdot t + \alpha_1) \right] + \left[ A_2 \cdot \begin{pmatrix} r_2 \\ 1 \end{pmatrix} \cdot \sin(\omega_2 \cdot t + \alpha_2) \right]$$

$$\mathbf{X}'(t) := \left[ \omega_1 \cdot A_1 \cdot \begin{pmatrix} r_1 \\ 1 \end{pmatrix} \cdot \cos(\omega_1 \cdot t + \alpha_1) \right] + \left[ \omega_2 \cdot A_2 \cdot \begin{pmatrix} r_2 \\ 1 \end{pmatrix} \cdot \cos(\omega_2 \cdot t + \alpha_2) \right]$$

$$\mathbf{X}''(t) := \left[ -\omega_1^2 \cdot A_1 \cdot \begin{pmatrix} r_1 \\ 1 \end{pmatrix} \cdot \sin(\omega_1 \cdot t + \alpha_1) \right] + \left[ -\omega_2^2 \cdot A_2 \cdot \begin{pmatrix} r_2 \\ 1 \end{pmatrix} \cdot \sin(\omega_2 \cdot t + \alpha_2) \right]$$

$$x_1(t) := \mathbf{X}(t)_1$$

$$x_2(t) := \mathbf{X}(t)_2$$

**spostamenti**

$$x'_1(t) := \mathbf{X}'(t)_1$$

$$x'_2(t) := \mathbf{X}'(t)_2$$

**velocità**

$$x''_1(t) := \mathbf{X}''(t)_1$$

$$x''_2(t) := \mathbf{X}''(t)_2$$

**accelerazioni**

$$\Delta t := 1 \cdot 10^{-3}$$

$$T_{max} := 4$$

**Passo di calcolo e tempo max. di simulazione**

## Integrazione numerica (Runge Kutta)

$$ACCEL(x_1, x_2) := -\mathbf{M}^{-1} \cdot \mathbf{K} \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

**Accelerazioni esplicitate (in forma matriciale)**

$$f(x_1, x_2) := ACCEL(x_1, x_2)_1$$

$$g(x_1, x_2) := ACCEL(x_1, x_2)_2$$

$$EQMOTO(t, u) := \begin{pmatrix} u_3 \\ u_4 \\ f(u_1, u_2) \\ g(u_1, u_2) \end{pmatrix}$$

**Equazioni di moto da risolvere**

$$u := \begin{pmatrix} x_{10} \\ x_{20} \\ x'_{10} \\ x'_{20} \end{pmatrix} = \begin{pmatrix} 0.03 \\ 0.02 \\ 0.2 \\ 0.3 \end{pmatrix}$$

**Condizioni iniziali**

$$N := \frac{T_{max}}{\Delta t} = 4000$$

$TAB := rkfixed(u, 0, T_{max}, N, EQMOTO)$

Calcolo della soluzione per via numerica

$tempo := TAB^{(1)}$

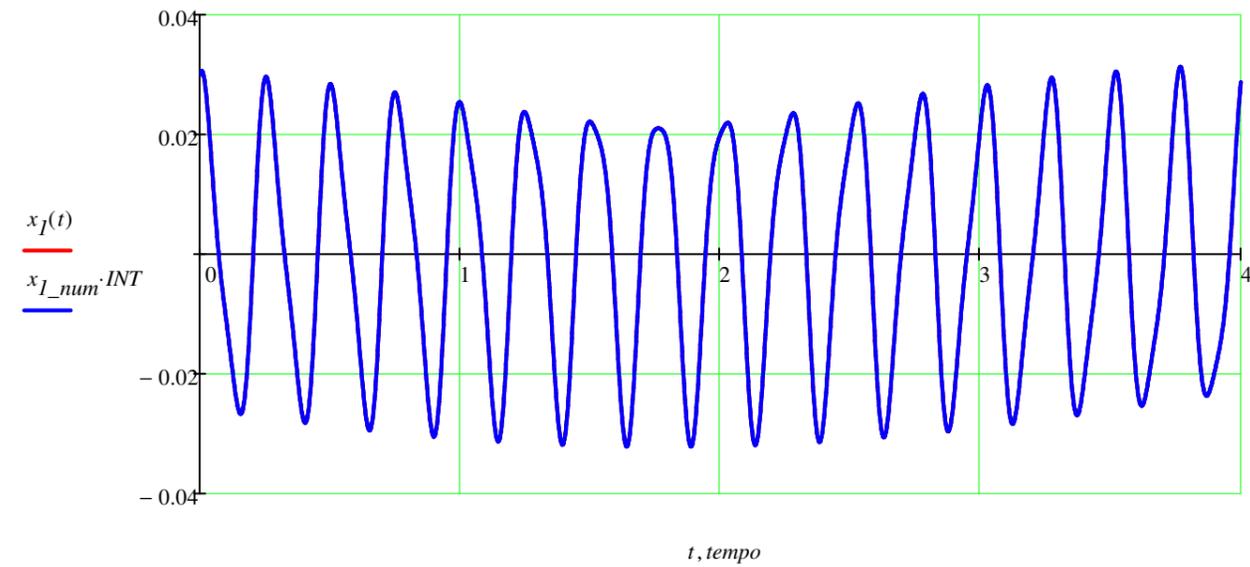
$x_{1\_num} := TAB^{(2)}$        $x_{2\_num} := TAB^{(3)}$

$x'_{1\_num} := TAB^{(4)}$        $x'_{2\_num} := TAB^{(5)}$

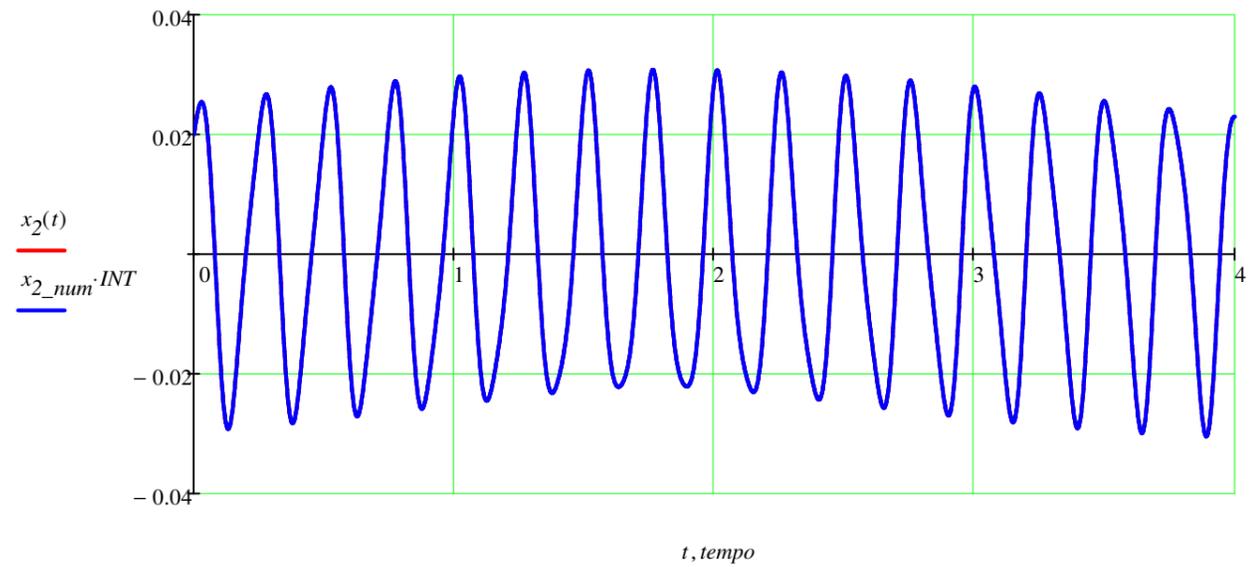
$x''_{1\_num} := \overrightarrow{f(x_{1\_num}, x_{2\_num})}$

$x''_{2\_num} := \overrightarrow{g(x_{1\_num}, x_{2\_num})}$

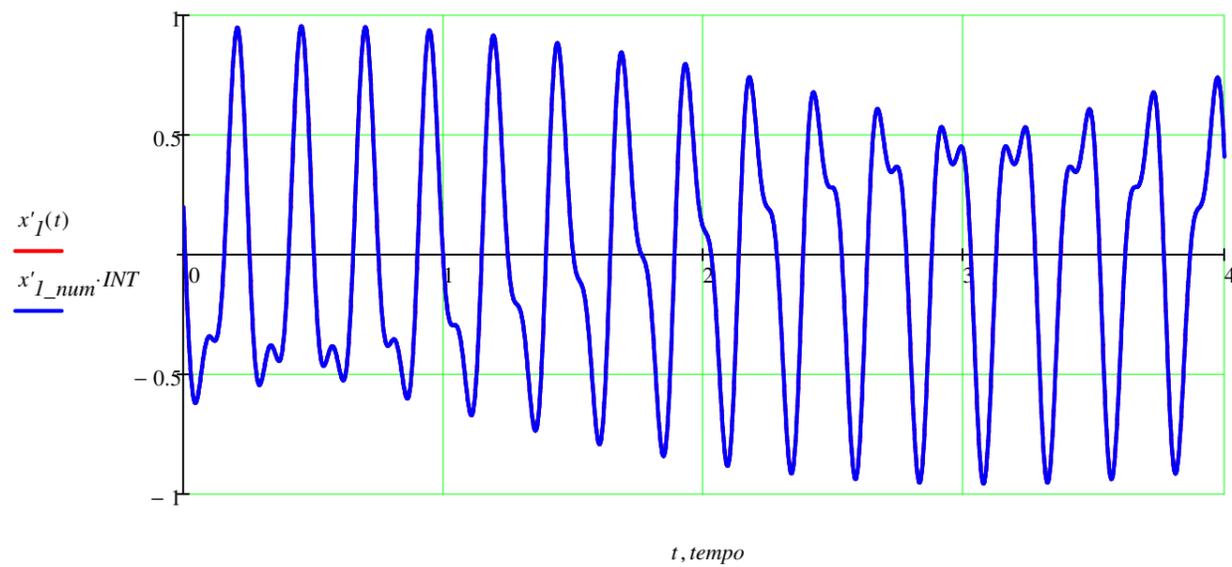
$t := 0, \Delta t .. T_{max}$        $INT := 1$



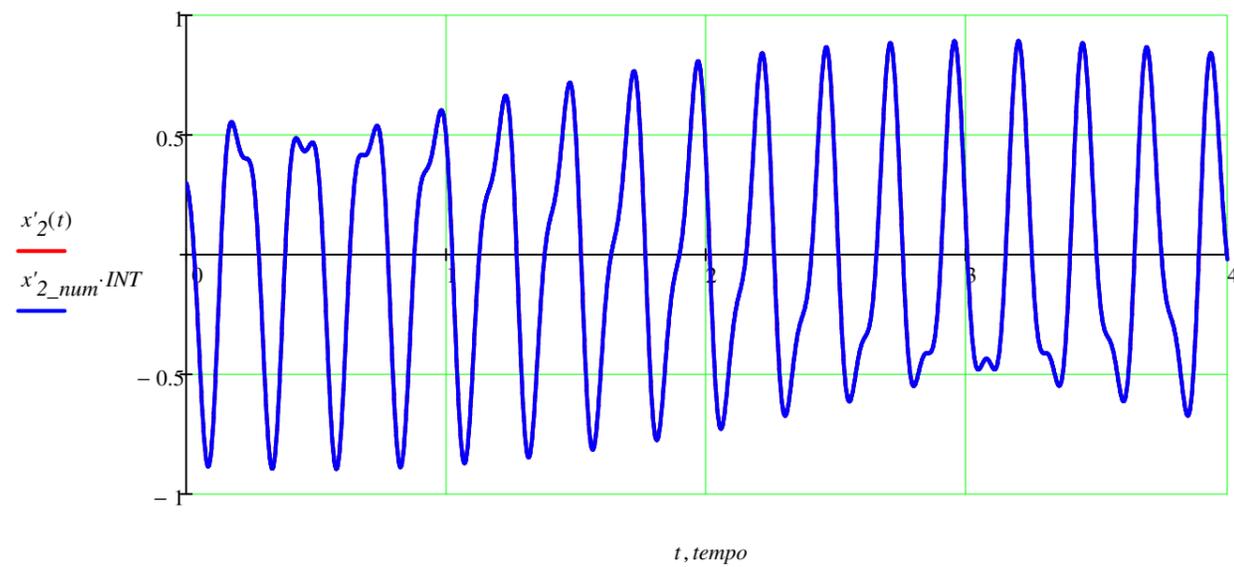
— Metodo analitico  
— Metodo di Runge-Kutta



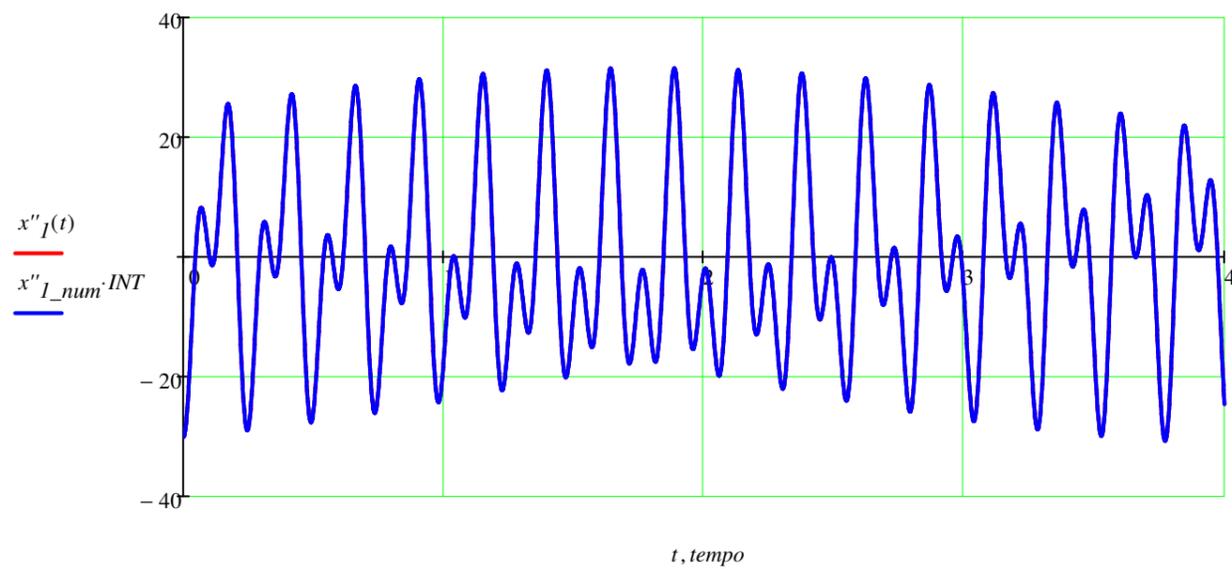
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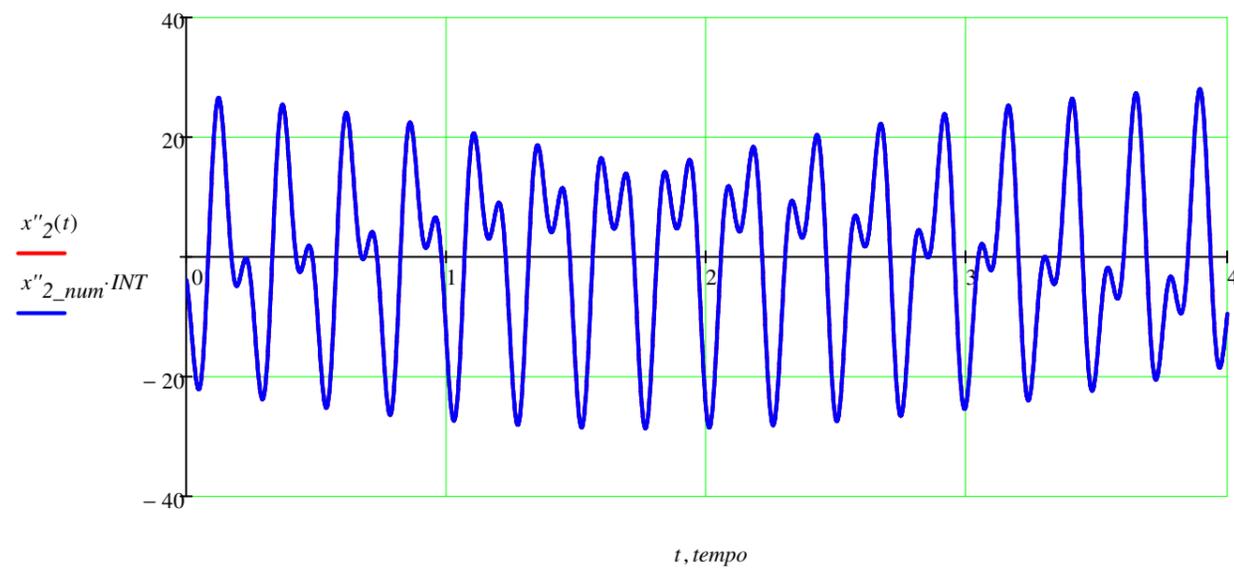
— Metodo analitico  
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— Metodo analitico  
 — Metodo di Runge-Kutta