Risposta al gradino (per cond. iniziali non nulle) - caso sottosmorzato

Spiegazione del metodo

Esempio numerico

$m := 10$ $x_0 := 0.05$

$k := 10000$ $v_0 := -2$

$c := 200$ $F_0 := 250$

$\omega := \sqrt{\frac{k}{m}} = 31.623$ $\xi := \frac{c}{2\cdot m\cdot\omega} = 31.623\cdot\%$ $\omega_s := \omega\cdot\sqrt{1-\xi^2} = 30$ $\delta_{st} := \frac{F_0}{k} = 0.025$

Metodo 1

$A := x_0 - \delta_{st} = 0.025$

$B := \frac{v_0 - \xi\cdot\omega\cdot\delta_{st} + \xi\cdot\omega\cdot x_0}{\omega_s} = -0.058$

$x_{omo}(t) := e^{-\xi\cdot\omega\cdot t}\cdot\left(A\cdot cos\left(\omega_s\cdot t\right) + B\cdot sin\left(\omega_s\cdot t\right)\right)$

$x_{part}(t) := \delta_{st}$

$x_{Metodo\_1}(t) := x_{omo}(t) + x_{part}(t)$

Metodo 1

Metodo 2

$x_{lib}(t) := e^{-\xi\cdot\omega\cdot t}\cdot\left(x_0\cdot cos\left(\omega_s\cdot t\right) + \frac{v_0 + \xi\cdot\omega\cdot x_0}{\omega_s}\cdot sin\left(\omega_s\cdot t\right)\right)$

$F(t) := F_0$

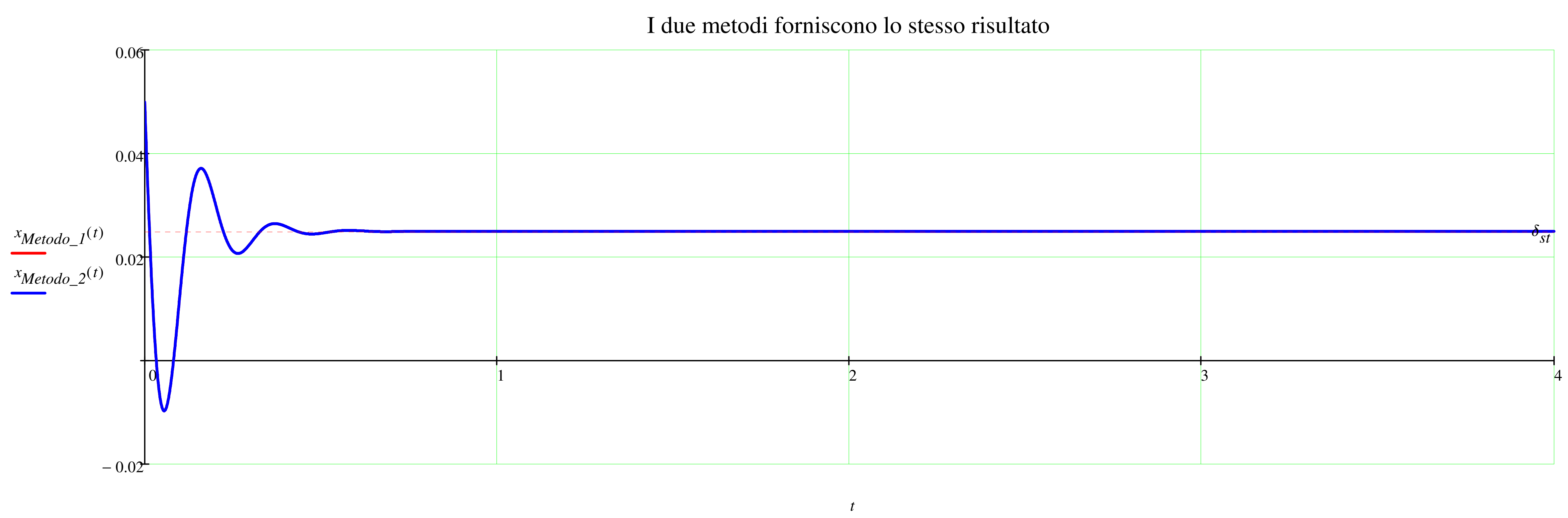
$h(t) := \frac{e^{-\xi\cdot\omega\cdot t}}{m\cdot\omega_s}\cdot sin\left(\omega_s\cdot t\right)$

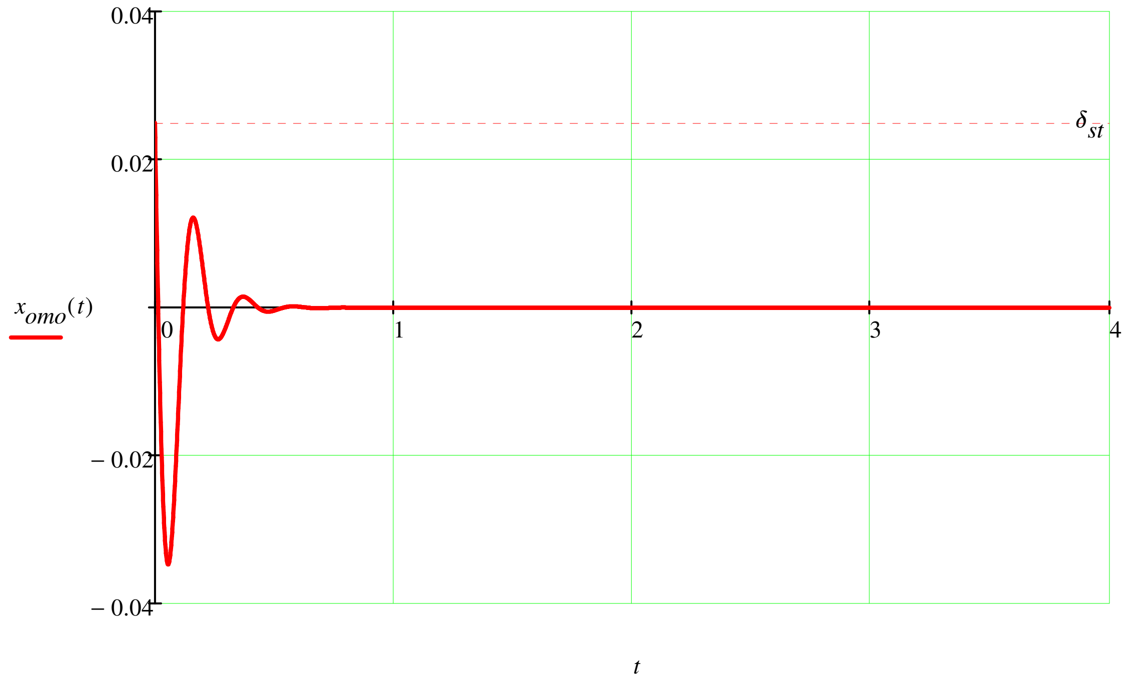
$x_{conv}(t) := \int_0^t F(\tau)\cdot h(t-\tau)\, d\tau$

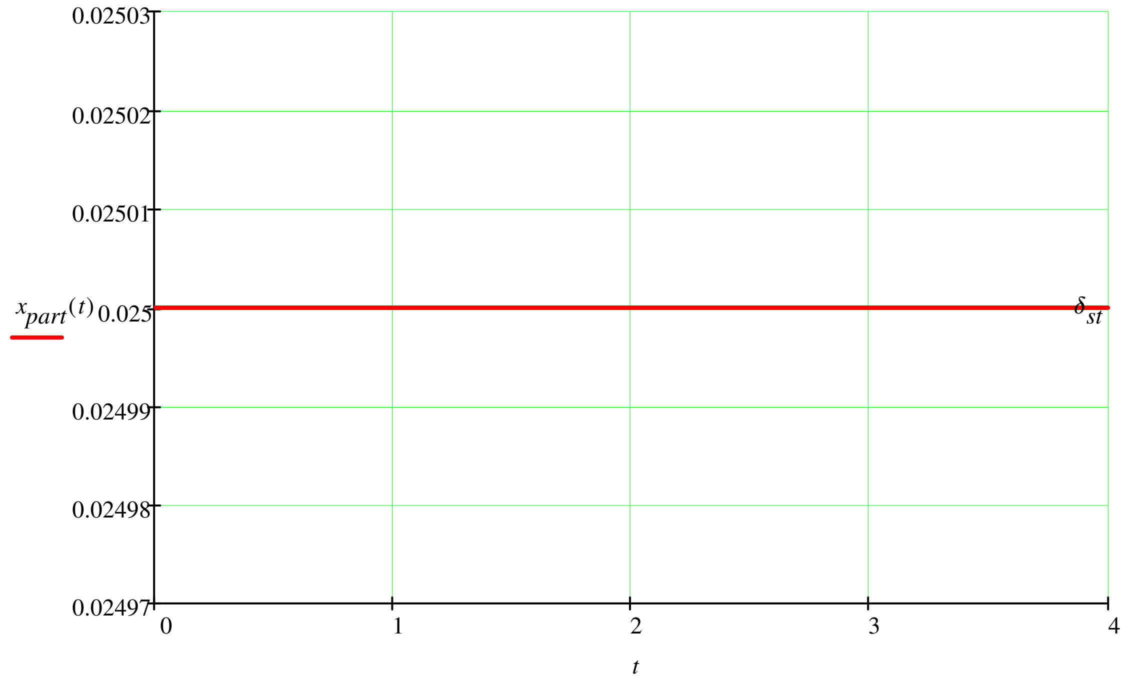
$x_{Metodo\_2}(t) := x_{lib}(t) + x_{conv}(t)$

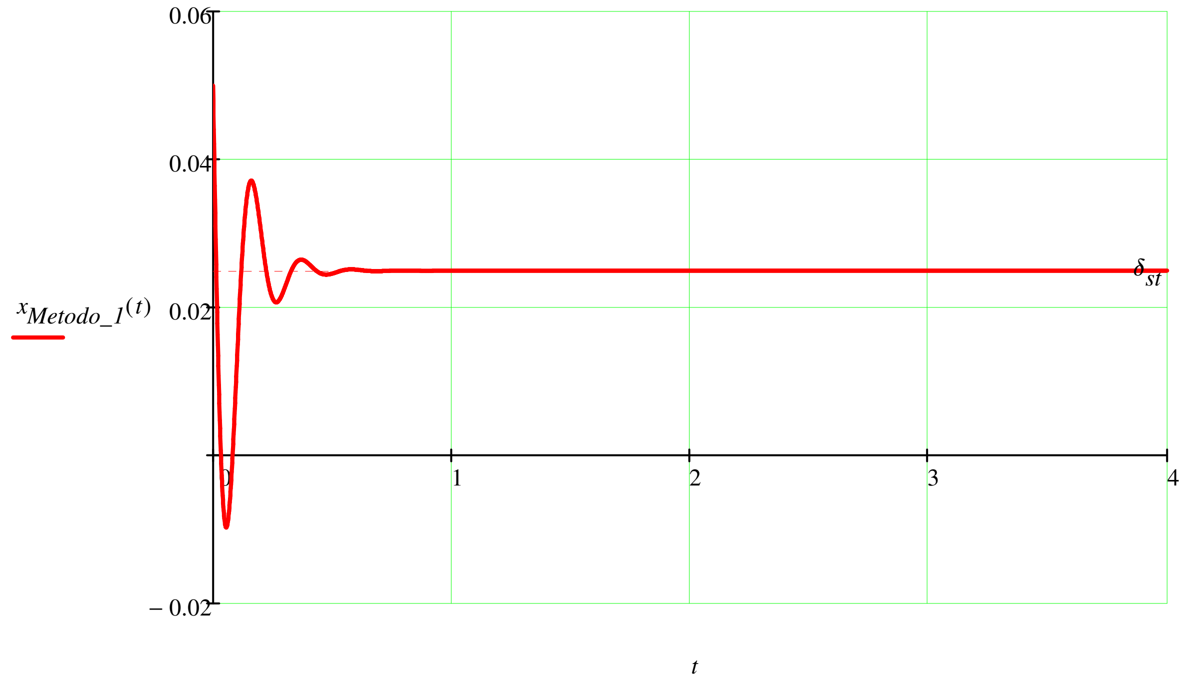
Metodo 2

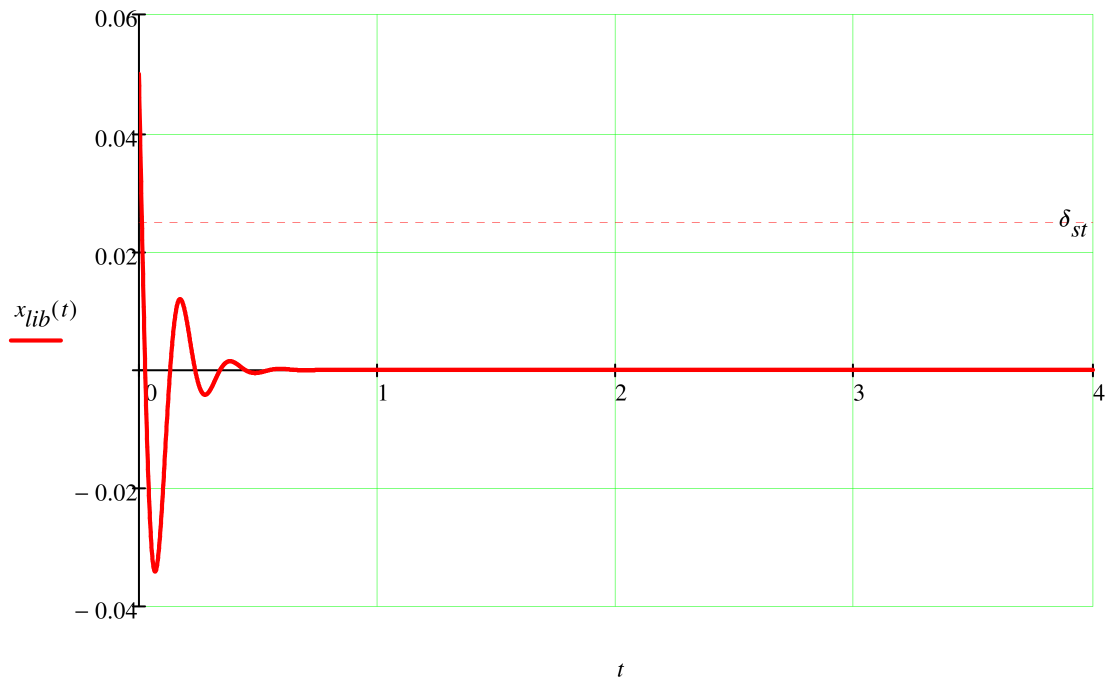
$t := 0, 0.001 .. 4$ $\omega = 31.623$ $T := \frac{2\cdot\pi}{\omega} = 0.199$ $\frac{T}{20} = 9.935\times 10^{-3}$

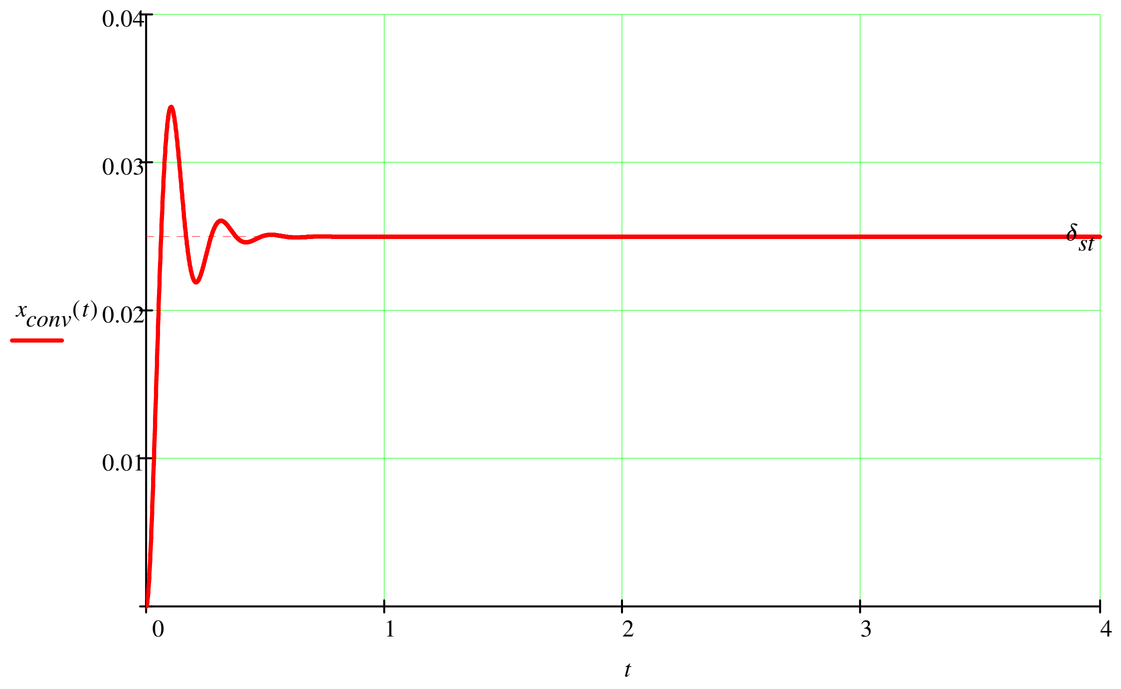


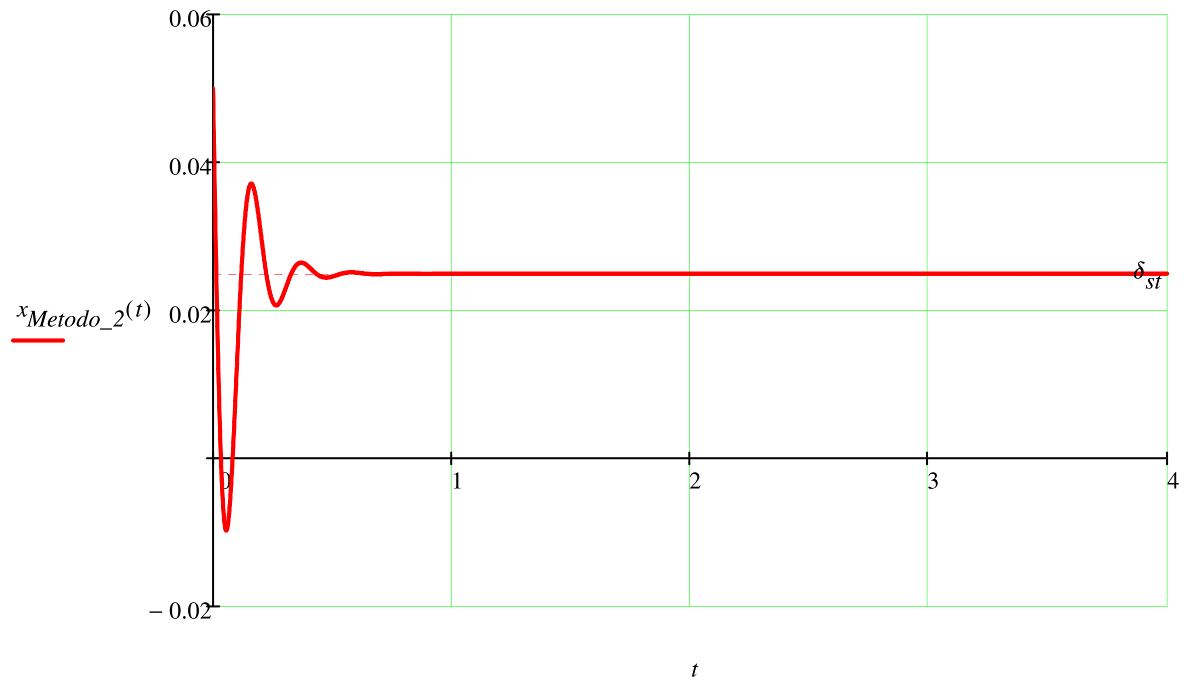




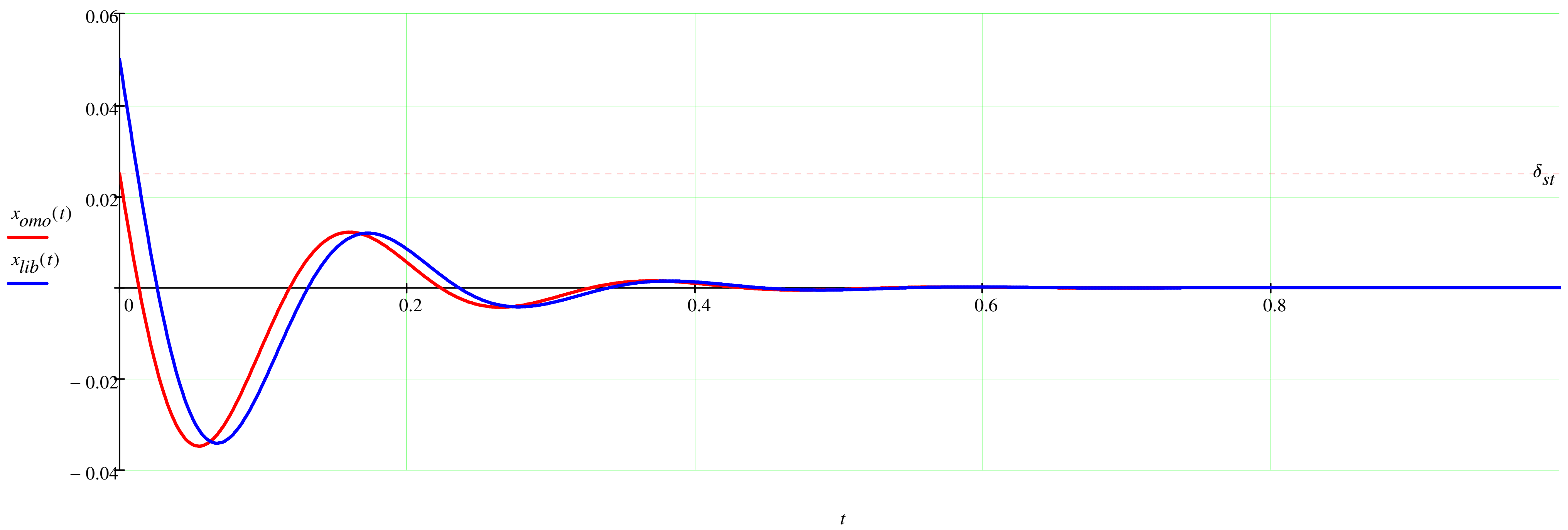








La soluzione $x_{omo}(t)$ è leggermente diversa dalla soluzione $x_{lib}(t)$; vedere grafico seguente:



$t_{prova} := 0.168$

$x_{omo}(t_{prova}) = 0.011793$	$x_{part}(t_{prova}) = 0.025$	$x_{Metodo\_1}(t_{prova}) = 0.036793$	Metodo 1
$x_{lib}(t_{prova}) = 0.011822$	$x_{conv}(t_{prova}) = 0.024971$	$x_{Metodo\_2}(t_{prova}) = 0.036793$	Metodo 2