Vibrazioni con forzante esponenziale

Calcoli simbolici

$$x_{omo}(t) := e^{-\xi\cdot\omega\cdot t}\cdot\left(A\cdot cos\left(\omega_s\cdot t\right) + B\cdot sin\left(\omega_s\cdot t\right)\right)$$

$$x_{part}(t) := Q - P\cdot e^{\frac{-t}{\gamma}}$$

$$x(t) := x_{omo}(t) + x_{part}(t)$$

$$x(t) \rightarrow Q - P\cdot e^{-\frac{t}{\gamma}} + e^{-\xi\cdot\omega\cdot t}\cdot\left(A\cdot cos\left(t\cdot\omega_s\right) + B\cdot sin\left(t\cdot\omega_s\right)\right)$$

$$x'(t) := \frac{d}{dt}x(t)$$

$$x'(t) \rightarrow e^{-\xi\cdot\omega\cdot t}\cdot\left(B\cdot\omega_s\cdot cos\left(t\cdot\omega_s\right) - A\cdot\omega_s\cdot sin\left(t\cdot\omega_s\right)\right) + \frac{P\cdot e^{-\frac{t}{\gamma}}}{\gamma} - \xi\cdot\omega\cdot e^{-\xi\cdot\omega\cdot t}\cdot\left(A\cdot cos\left(t\cdot\omega_s\right) + B\cdot sin\left(t\cdot\omega_s\right)\right)$$

Condizioni iniziali

$$x(0) \rightarrow A - P + Q$$

$$x'(0) \rightarrow \frac{P}{\gamma} + B\cdot\omega_s - A\cdot\xi\cdot\omega$$

Calcolo delle costanti A e B per condizioni iniziali nulle

Given

$$x(0) = 0$$

$$x'(0) = 0$$

$$\begin{pmatrix} A \\ B \end{pmatrix} := Find(A, B)$$

$$\begin{pmatrix} A \\ B \end{pmatrix} \rightarrow \begin{pmatrix} P - Q \\ -\frac{P - P\cdot\xi\cdot\omega\cdot\gamma + Q\cdot\xi\cdot\omega\cdot\gamma}{\omega_s\cdot\gamma} \end{pmatrix}$$

$$A \rightarrow P - Q$$

$$B \rightarrow -\frac{P - P\cdot\xi\cdot\omega\cdot\gamma + Q\cdot\xi\cdot\omega\cdot\gamma}{\omega_s\cdot\gamma}$$

Parameteri del sistema vibrante

$m := 4$

$c := 15$

$k := 2500$

Parametri della forzante

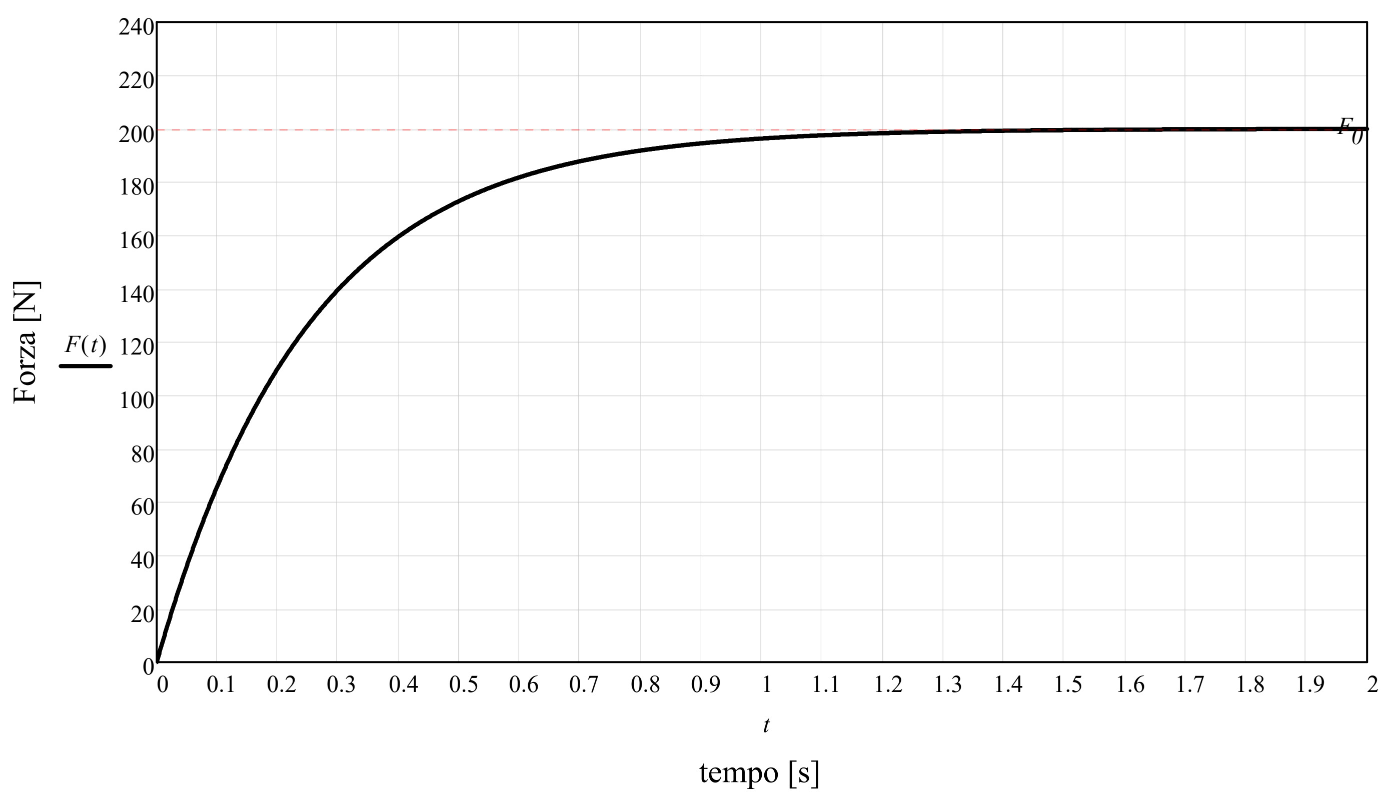
$F_0 := 200$ **Valore di regime**

$\gamma := 0.25$ **Costante di tempo**

$\Delta t := 10^{-3}$ $T_{max} := 2$

$t := 0, \Delta t .. T_{max}$

$F(t) := F_0 \cdot \left(1 - e^{\frac{-t}{\gamma}}\right)$ $\gamma = 0.25$



$$P := \frac{F_0}{\frac{m}{\gamma^2} - \frac{c}{\gamma} + k} = 0.08 \qquad Q := \frac{F_0}{k} = 0.08$$

$$x_{part}(t) := Q - P \cdot e^{\frac{-t}{\gamma}}$$

$$\omega := \sqrt{\frac{k}{m}} = 25 \qquad \xi := \frac{c}{2 \cdot m \cdot \omega} = 0.075$$

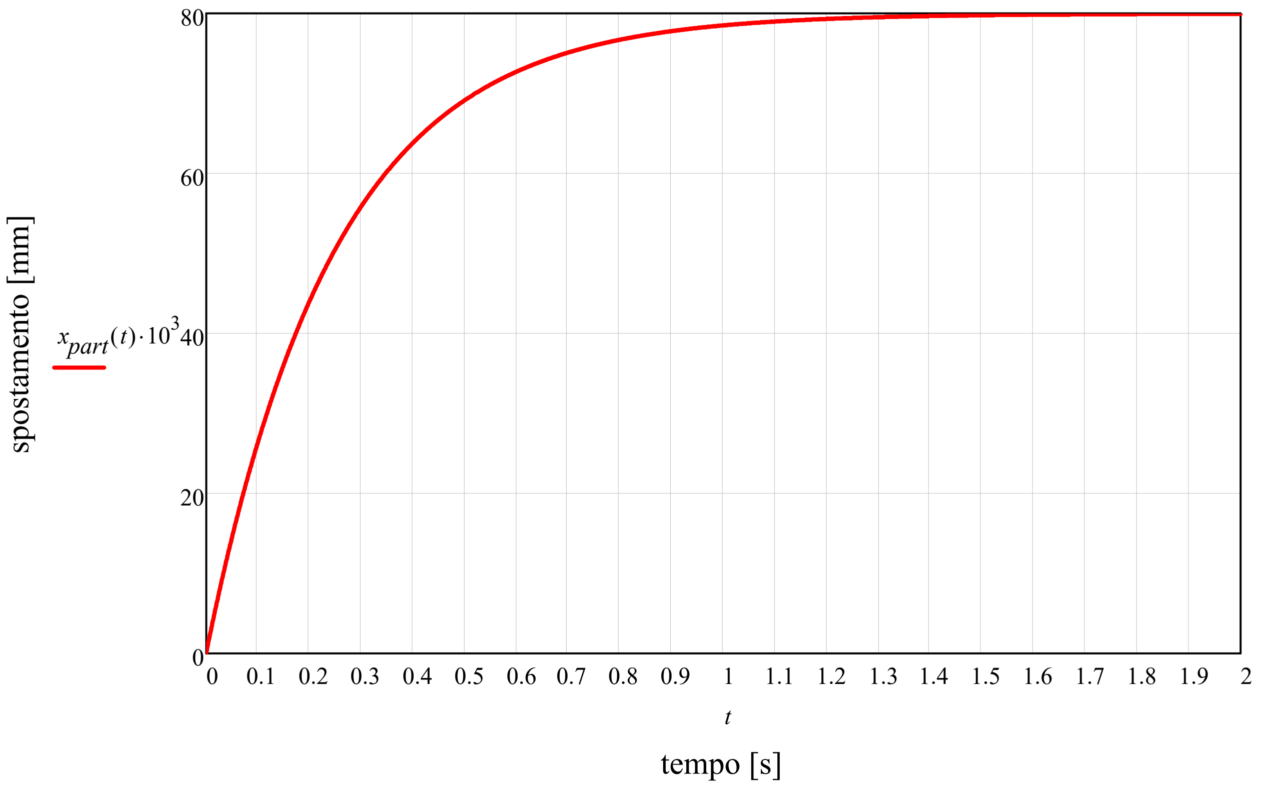
$$\omega_s := \omega \cdot \sqrt{1 - \xi^2} = 24.93$$

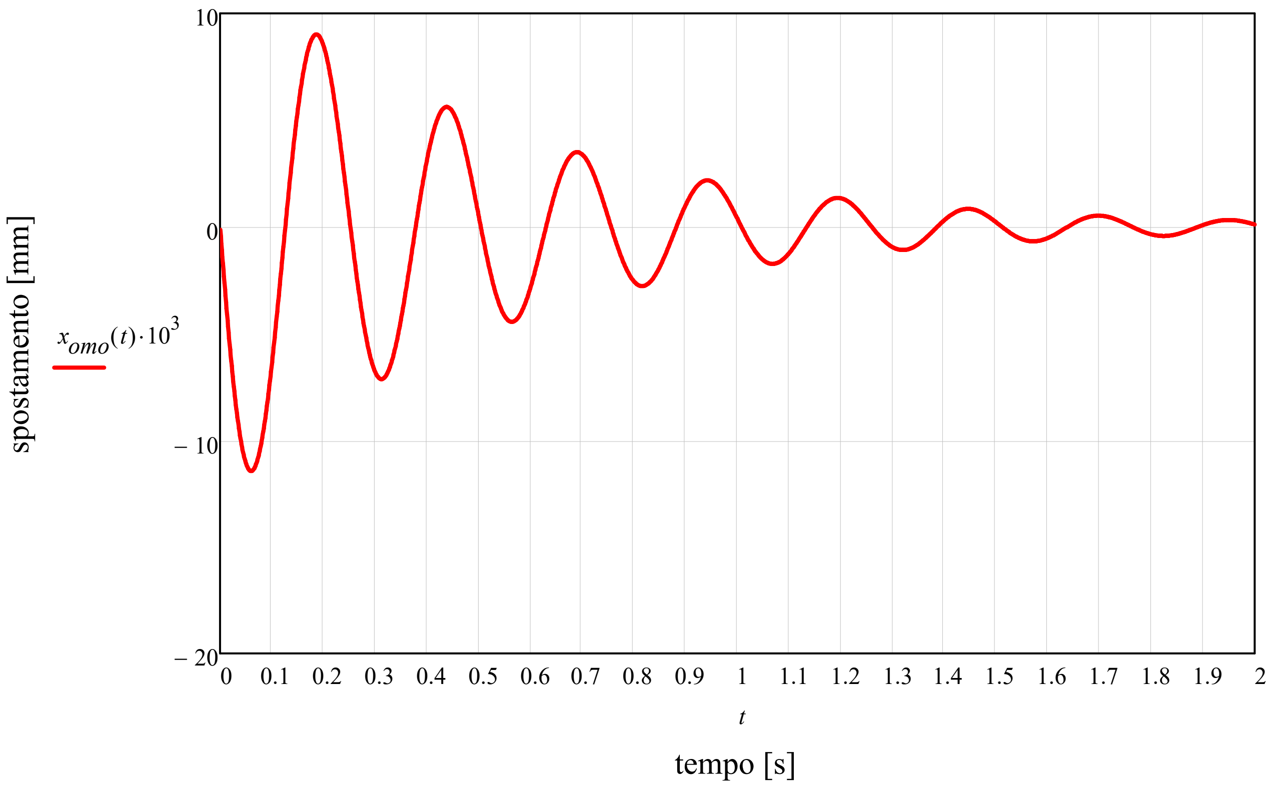
$$\begin{pmatrix} A \\ B \end{pmatrix} := \begin{pmatrix} P - Q \\ -\frac{P - P \cdot \xi \cdot \omega \cdot \gamma + Q \cdot \xi \cdot \omega \cdot \gamma}{\omega_s \cdot \gamma} \end{pmatrix} = \begin{pmatrix} -1.278 \times 10^{-4} \\ -0.013 \end{pmatrix}$$

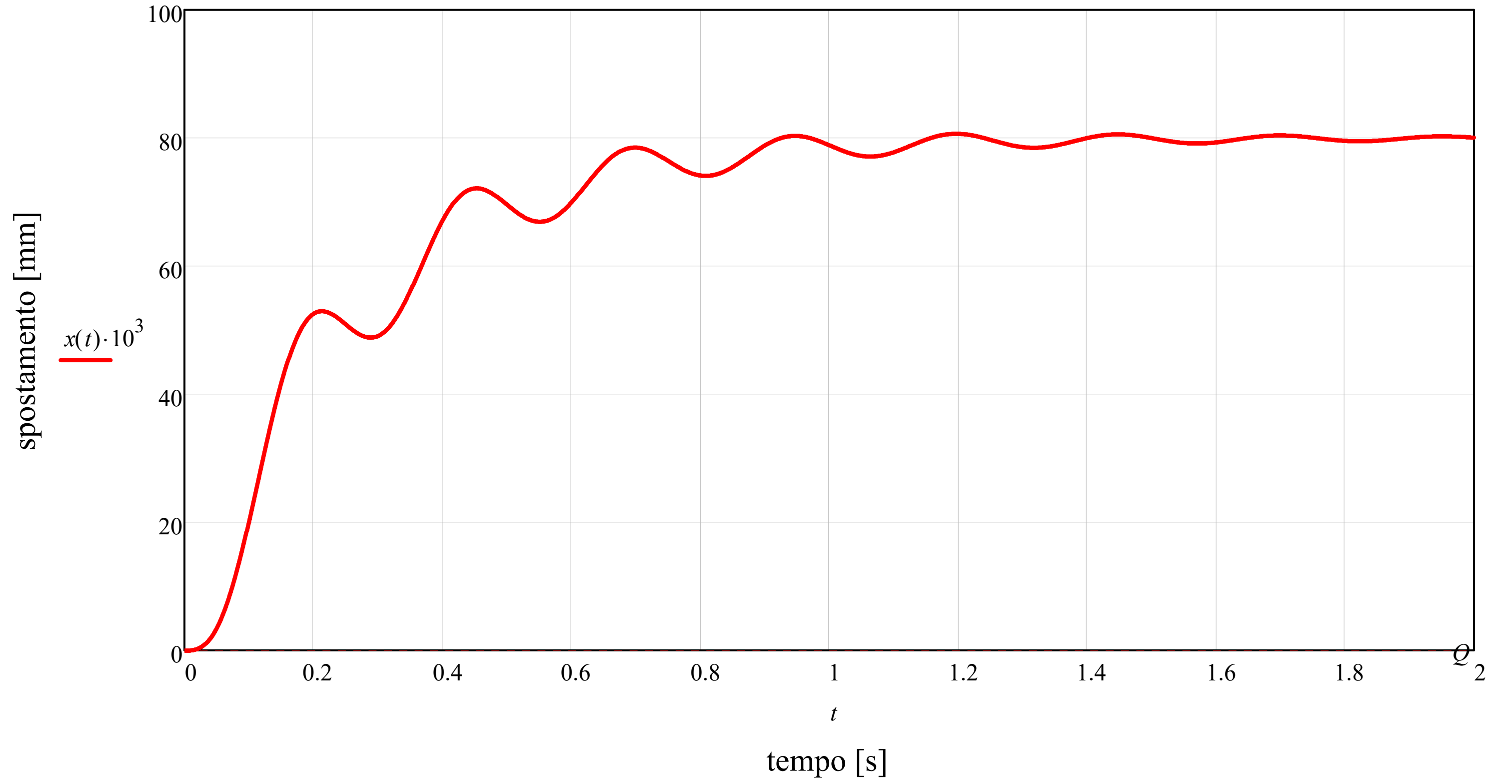
$$x_{omo}(t) := e^{-\xi \cdot \omega \cdot t} \cdot \left(A \cdot cos\left(\omega_s \cdot t\right) + B \cdot sin\left(\omega_s \cdot t\right)\right)$$

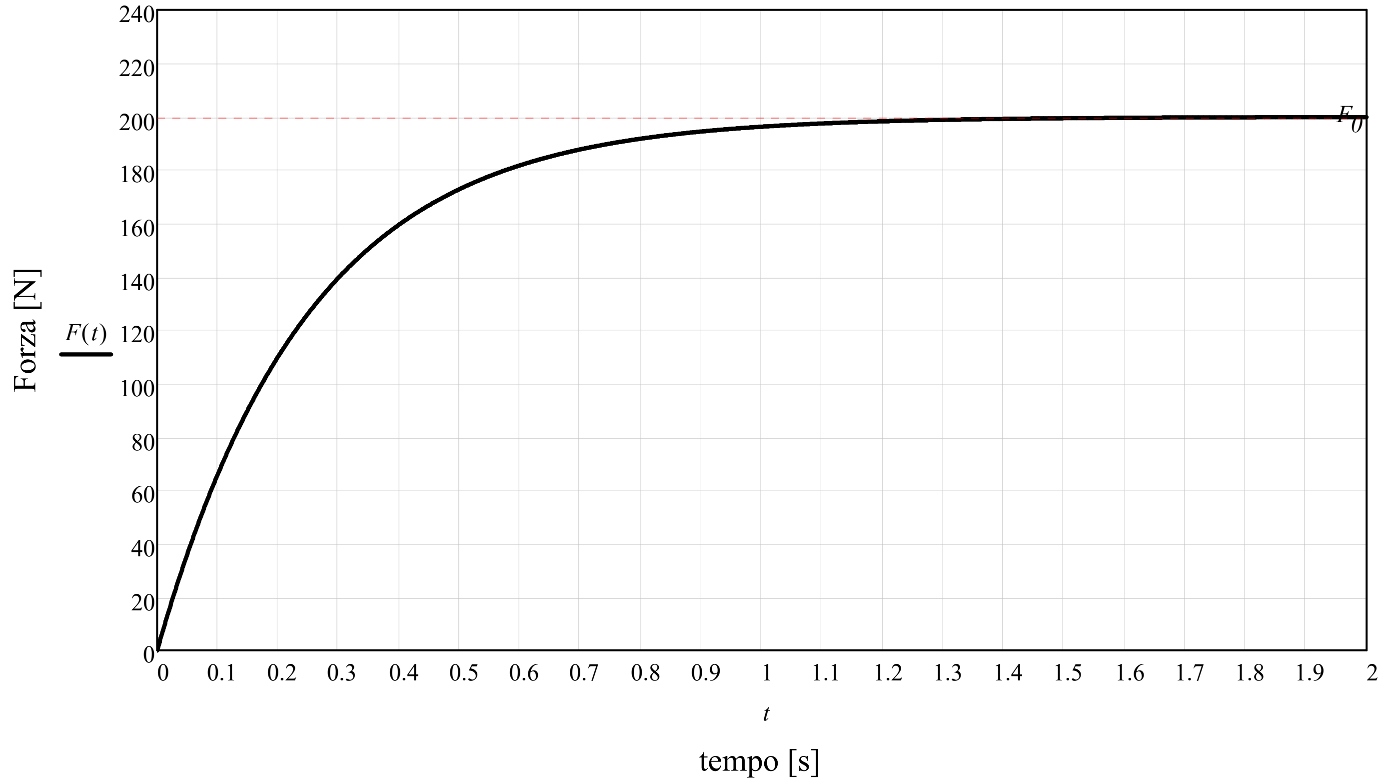
$$x(t) := x_{omo}(t) + x_{part}(t)$$

$$x'(t) := \frac{d}{dt}x(t) \qquad x''(t) := \frac{d}{dt}x'(t)$$









Integrazione per via numerica

$$\mathbf{F}(x, x', t) := \frac{1}{m} \cdot (F(t) - c \cdot x' - k \cdot x)$$

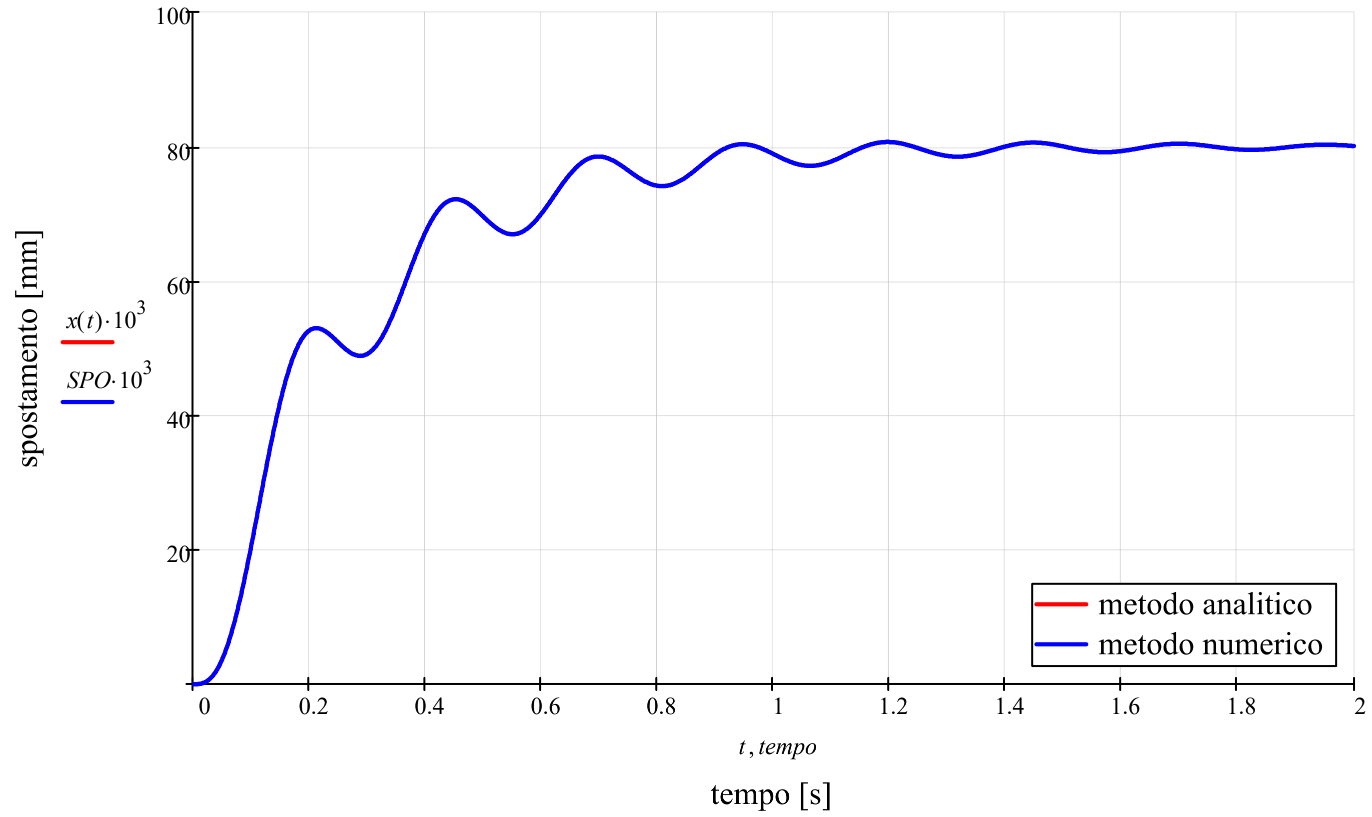
$$u := \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$N := \frac{T_{max}}{\Delta t} = 2000$$

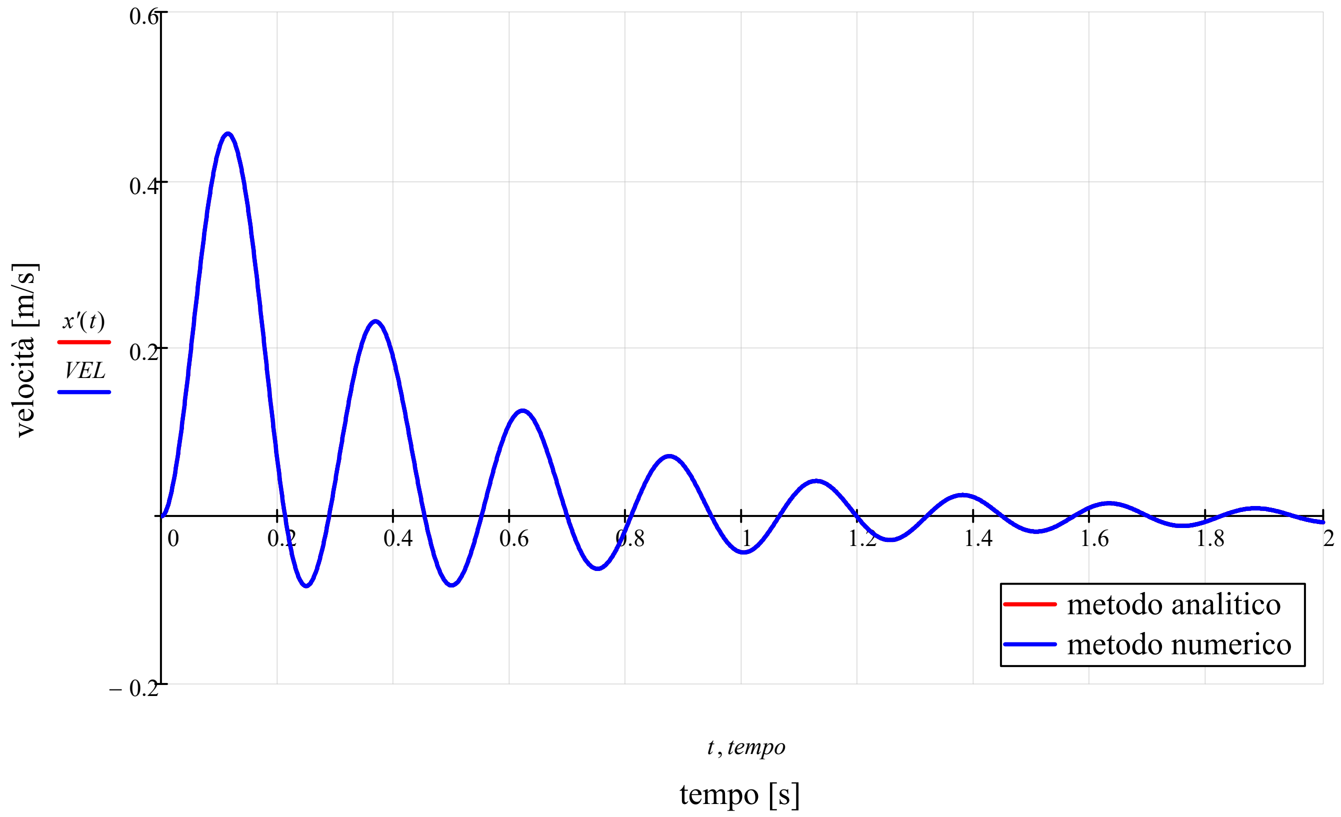
$$EQMOTO(t, u) := \begin{pmatrix} u_2 \\ \mathbf{F}(u_1, u_2, t) \end{pmatrix}$$

$$TAB := rkfixed(u, 0, T_{max}, N, EQMOTO)$$

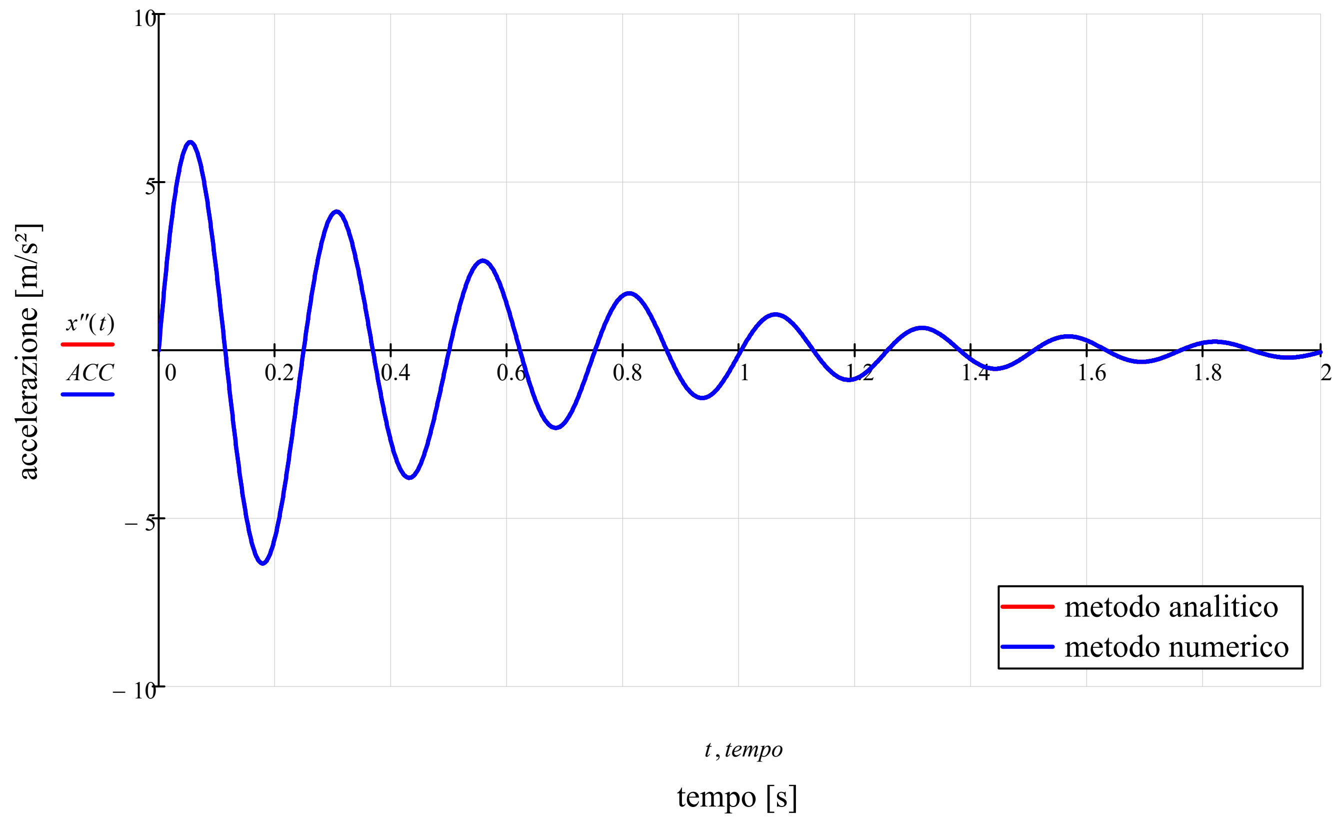
$tempo := TAB^{\langle 1 \rangle}$ $\quad SPO := TAB^{\langle 2 \rangle}$ $\quad VEL := TAB^{\langle 3 \rangle}$ $\quad ACC := \overrightarrow{\mathbf{F}(SPO, VEL, tempo)}$



spostamento [mm]
x(t)·10^3
SPO·10^3
100
80
60
40
20
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
metodo analitico
metodo numerico
t , tempo
tempo [s]



velocità [m/s]
x′(t)
VEL
0.6
0.4
0.2
− 0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
metodo analitico
metodo numerico
t , tempo
tempo [s]



accelerazione [m/s²]
x″(t)
ACC
10
5
− 5
− 10
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
metodo analitico
metodo numerico
t , tempo
tempo [s]