Vibrazioni ad 1 GdL con forzante esponenziale

$x_{omo}(t) := e^{-\xi\cdot\omega\cdot t}\cdot\left(A\cdot cos\left(\omega_s\cdot t\right) + B\cdot sin\left(\omega_s\cdot t\right)\right)$

$x_{part}(t) := Q - P\cdot e^{\frac{-t}{\gamma}}$

$x(t) := x_{omo}(t) + x_{part}(t)$ $\qquad$ $x'(t) := \frac{d}{dt}x(t)$

$x(t) \rightarrow Q + e^{-\xi\cdot\omega\cdot t}\cdot\left(A\cdot cos\left(t\cdot\omega_s\right) + B\cdot sin\left(t\cdot\omega_s\right)\right) - P\cdot e^{-\frac{t}{\gamma}}$

$x'(t) \rightarrow e^{-\xi\cdot\omega\cdot t}\cdot\left(B\cdot\omega_s\cdot cos\left(t\cdot\omega_s\right) - A\cdot\omega_s\cdot sin\left(t\cdot\omega_s\right)\right) + \frac{P\cdot e^{-\frac{t}{\gamma}}}{\gamma} - \xi\cdot\omega\cdot e^{-\xi\cdot\omega\cdot t}\cdot\left(A\cdot cos\left(t\cdot\omega_s\right) + B\cdot sin\left(t\cdot\omega_s\right)\right)$

$x(0) \rightarrow A - P + Q$

$x'(0) \rightarrow B\cdot\omega_s + \frac{P}{\gamma} - A\cdot\xi\cdot\omega$

Given

$x(0) = 0$

$x'(0) = 0$

$$Find(A, B) \rightarrow \begin{pmatrix} P - Q \\ -\frac{P - \gamma\cdot P\cdot\xi\cdot\omega + \gamma\cdot Q\cdot\xi\cdot\omega}{\gamma\cdot\omega_s} \end{pmatrix}$$

Parametri della forzante

$p_0 := 100000$

$D := 120\cdot 10^{-3} = 0.12$ $\qquad$ $A_{pist} := \left(\frac{\pi\cdot D^2}{4}\right) = 0.011$

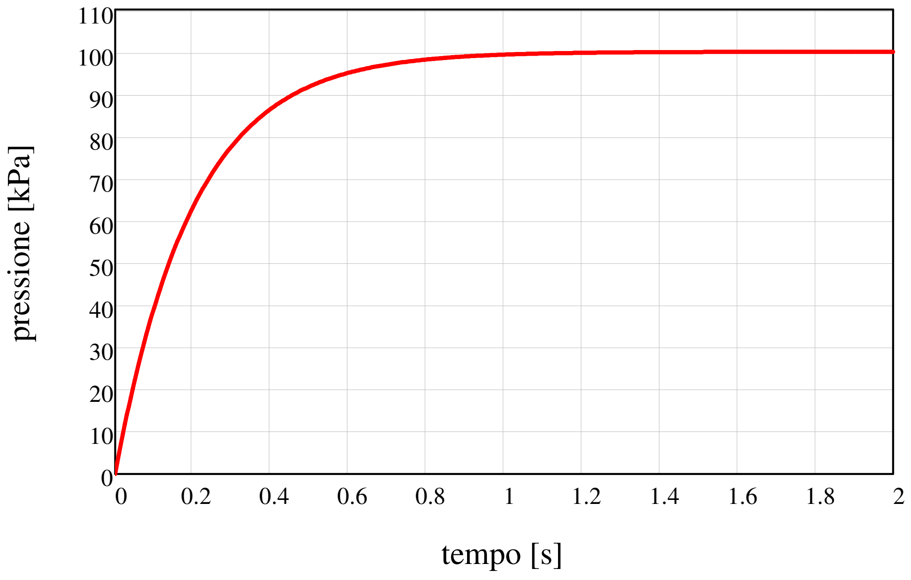
$F_0 := A_{pist}\cdot p_0 = 1130.973$ $\qquad$ Forza max.

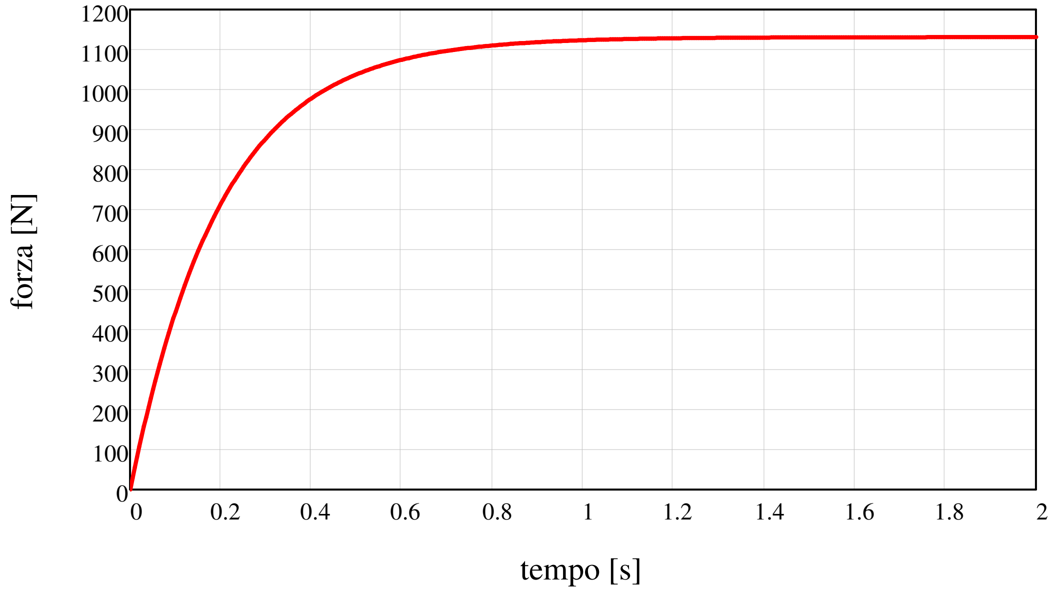
$\gamma := 0.2$ $\qquad$ Costante di tempo

$p(t) := p_0 \cdot \left(1 - e^{\frac{-t}{\gamma}}\right)$ $F(t) := F_0 \cdot \left(1 - e^{\frac{-t}{\gamma}}\right)$

$\Delta t := 5 \cdot 10^{-3} = 5 \times 10^{-3}$ $T_{max} := 2$ $N := \frac{T_{max}}{\Delta t} = 400$

$t := 0, \Delta t .. T_{max}$





Parametri del sistema vibrante

$m := 4$

$c := 10$

$k := 2500$

$\omega := \sqrt{\frac{k}{m}} = 25$

$c_{cr} := 2 \cdot m \cdot \omega = 200$

$\xi := \frac{c}{c_{cr}} = 0.05$ $\xi = 5 \cdot \%$

$\omega_s := \omega \cdot \sqrt{1 - \xi^2} = 24.969$

$$P := \frac{F_0}{\frac{m}{\gamma^2} - \frac{c}{\gamma} + k} = 0.444 \qquad Q := \frac{F_0}{k} = 0.452$$

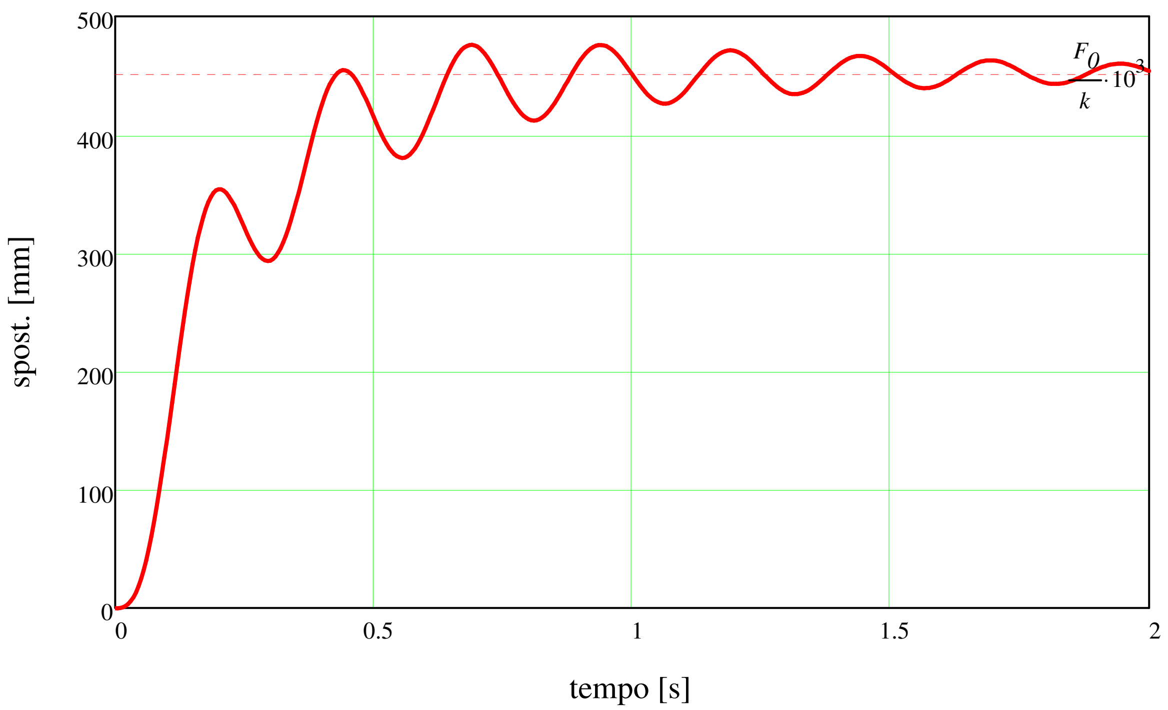
$$\begin{pmatrix} A \\ B \end{pmatrix} := \begin{pmatrix} P - Q \\ -\frac{P - \gamma \cdot P \cdot \xi \cdot \omega + \gamma \cdot Q \cdot \xi \cdot \omega}{\gamma \cdot \omega_s} \end{pmatrix} = \begin{pmatrix} -8.87 \times 10^{-3} \\ -0.089 \end{pmatrix}$$

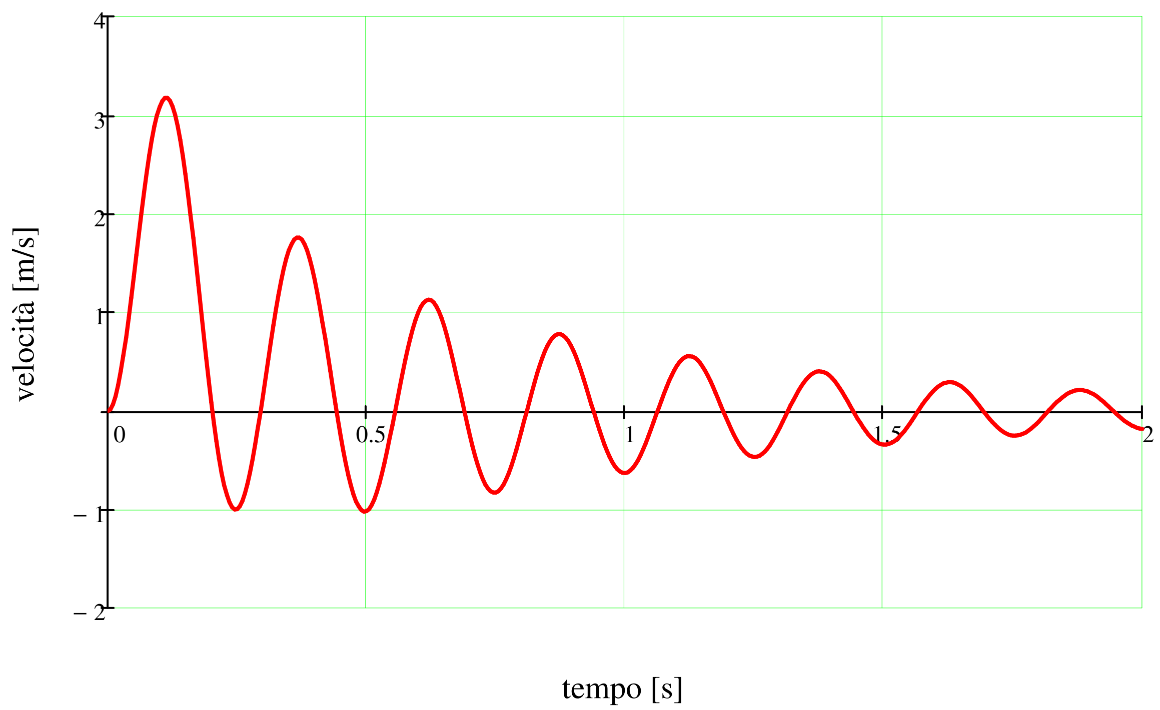
$$x_{omo}(t) := e^{-\xi \cdot \omega \cdot t} \cdot \left(A \cdot cos\left(\omega_s \cdot t\right) + B \cdot sin\left(\omega_s \cdot t\right)\right) \qquad x_{part}(t) := Q - P \cdot e^{\frac{-t}{\gamma}}$$

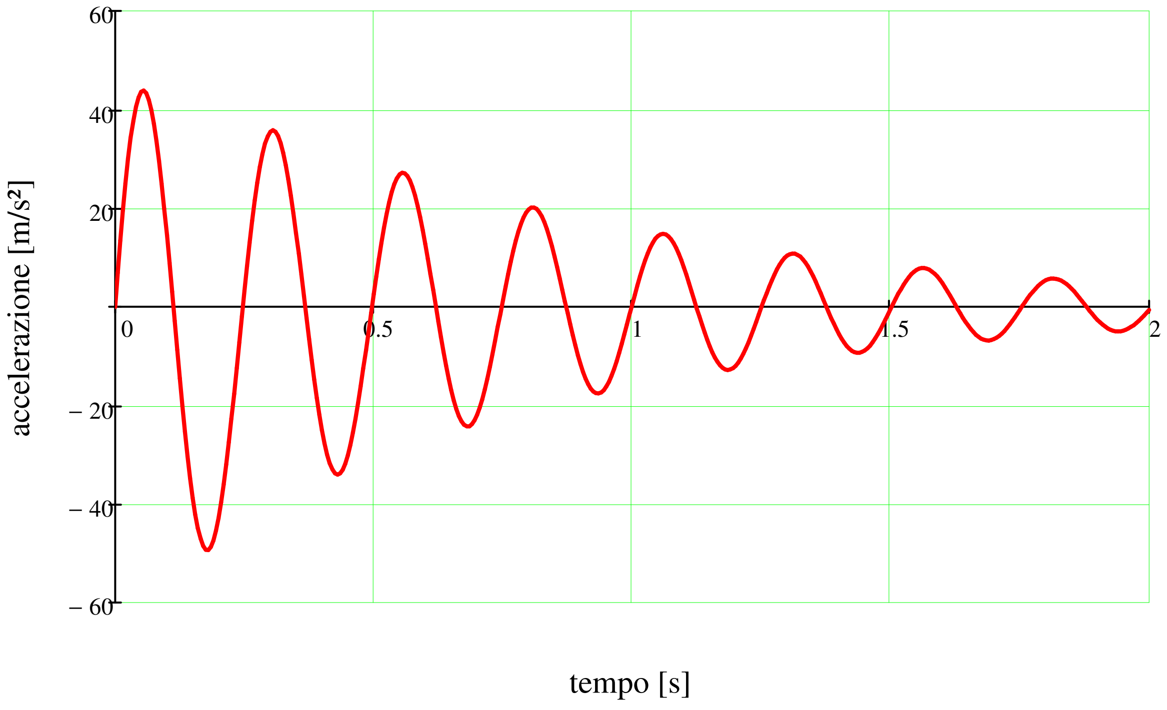
$$x(t) := x_{omo}(t) + x_{part}(t)$$

$$x'(t) := \frac{d}{dt} x(t)$$

$$x''(t) := \frac{d}{dt} x'(t)$$







Integrazione numerica dell'equazione di moto

$$\mathbf{F}(x, x', t) := \frac{1}{m} \cdot (F(t) - c \cdot x' - k \cdot x)$$

$$EQMOTO(t, u) := \begin{pmatrix} u_1 \\ \mathbf{F}(u_0, u_1, t) \end{pmatrix}$$

$\omega = 25$ Pulsazione propria [rad/s]

$T := \frac{2 \cdot \pi}{\omega} = 0.251$ Periodo proprio [s]

$f := \frac{1}{T} = 3.979$ Frequenza propria [Hz]

$\Delta t_{cons} := \frac{1}{20} \cdot T = 0.013$ $\Delta t = 5 \times 10^{-3}$

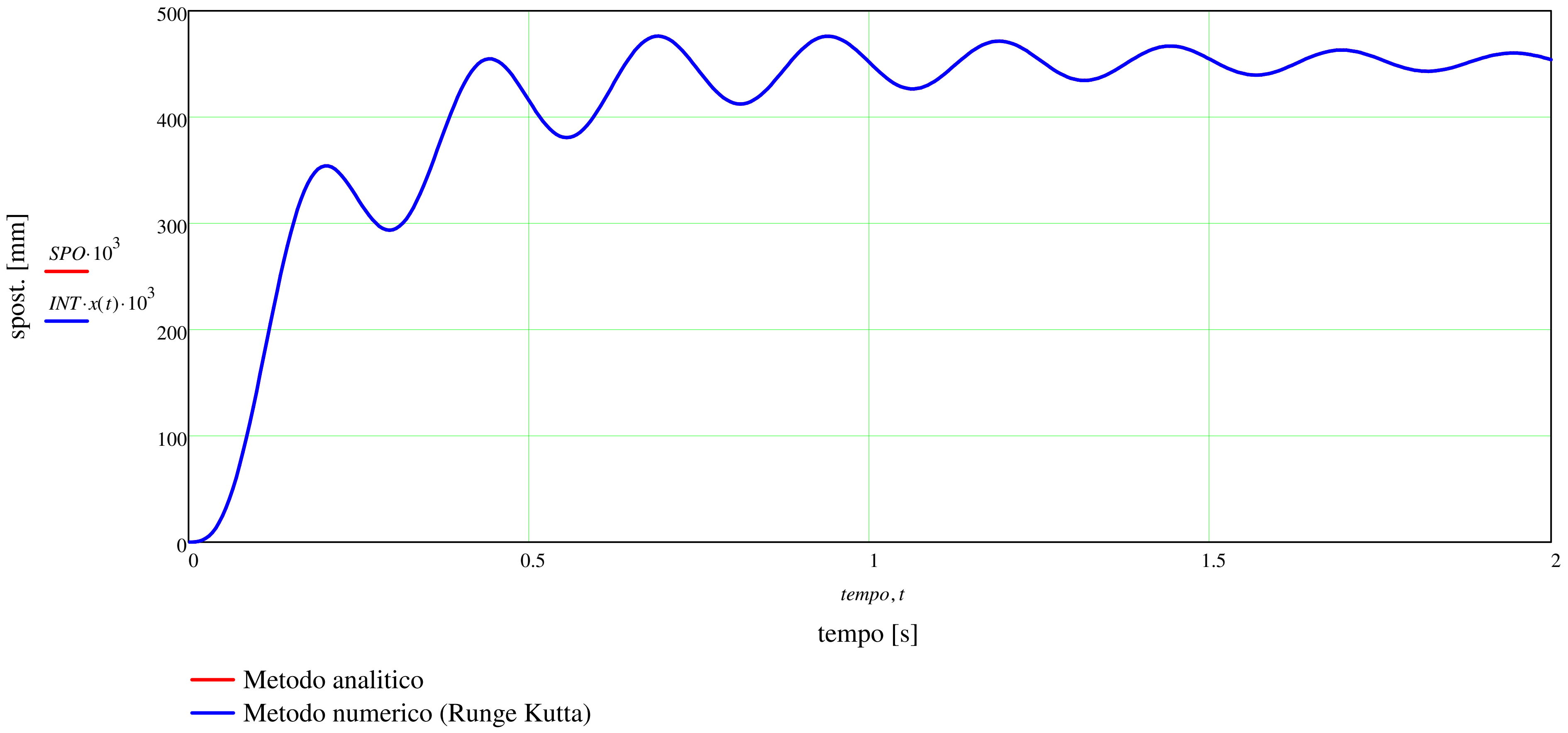
$u := \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ Condizioni iniziali

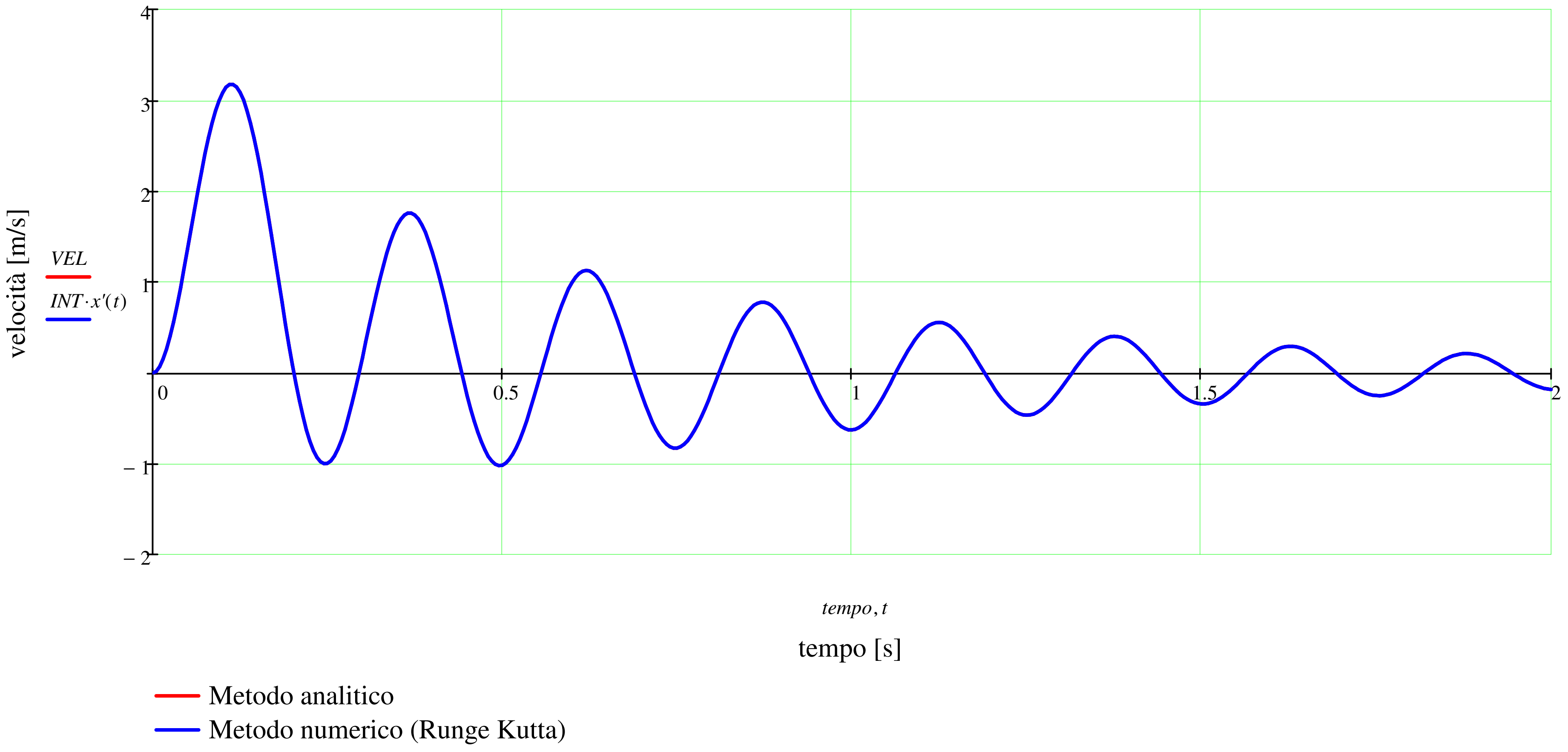
$N := \frac{T_{max}}{\Delta t} = 400$

$TAB := rkfixed(u, 0, T_{max}, N, EQMOTO)$

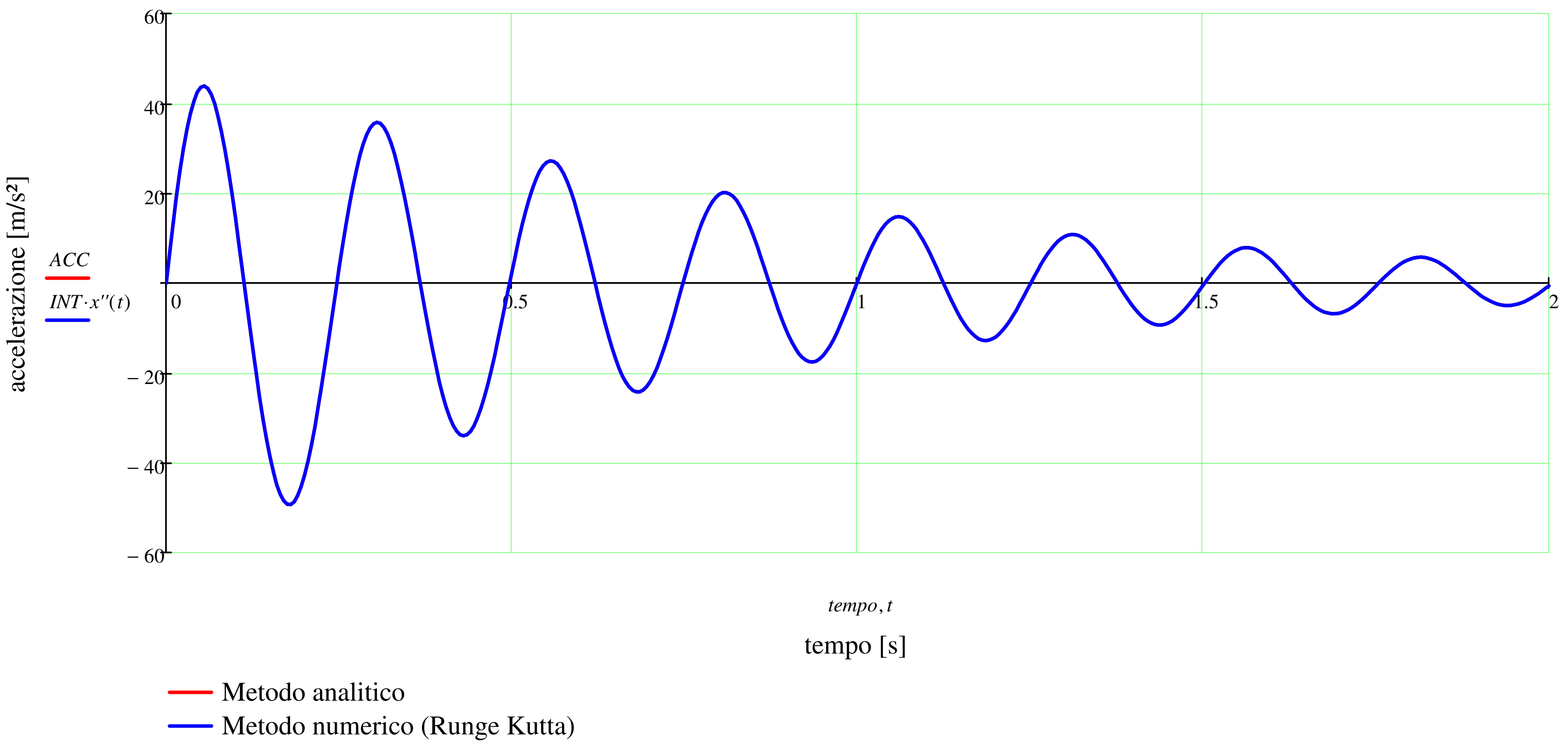
$tempo := TAB^{\langle 0 \rangle}$ $SPO := TAB^{\langle 1 \rangle}$ $VEL := TAB^{\langle 2 \rangle}$

$ACC := \mathbf{F}(SPO, VEL, tempo)$ $INT := 1$ Interruttore (ON/OFF) per abilitare/disabilitare la soluzione numerica





velocità [m/s]
VEL
INT·x'(t)
tempo, t
tempo [s]
Metodo analitico
Metodo numerico (Runge Kutta)



accelerazione [m/s²]
ACC
INT·x''(t)
tempo, t
tempo [s]
Metodo analitico
Metodo numerico (Runge Kutta)