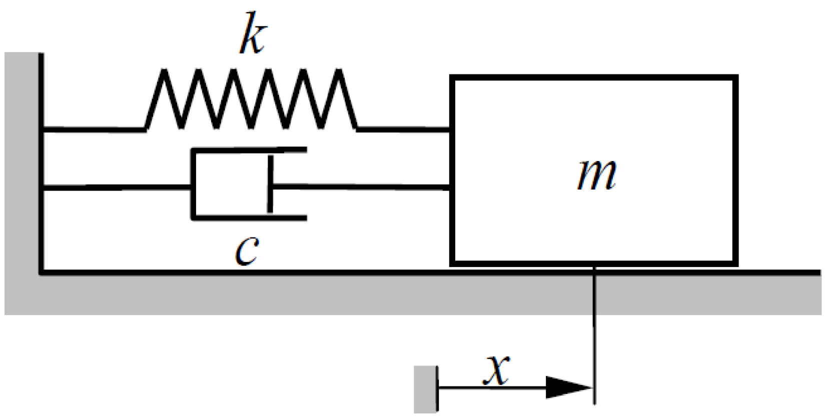


Parametri del sistema vibrante

$m := 5$ $k := 1500$ $c := 30$

Condizioni iniziali

$x_0 := 0.05$ $v_0 := 3$

Caso sottosmorzato

$$x(t) := e^{-\xi \cdot \omega \cdot t} \cdot \left(A \cdot cos\left(\omega_s \cdot t\right) + B \cdot sin\left(\omega_s \cdot t\right)\right)$$

$$x'(t) := \frac{d}{dt} x(t) \qquad x''(t) := \frac{d^2}{dt^2} x(t)$$

Derivate temporali (velocità ed accelerazione) simboliche

$$x'(t) \left| \begin{array}{l} simplify \\ collect, cos\left(\omega_s \cdot t\right), sin\left(\omega_s \cdot t\right) \end{array} \right. \rightarrow e^{-\xi \cdot \omega \cdot t} \cdot \left(B \cdot \omega_s - A \cdot \xi \cdot \omega\right) \cdot cos\left(t \cdot \omega_s\right) + \left[-e^{-\xi \cdot \omega \cdot t} \cdot \left(A \cdot \omega_s + B \cdot \xi \cdot \omega\right)\right] \cdot sin\left(t \cdot \omega_s\right)$$

$$x''(t) \left| \begin{array}{l} simplify \\ collect, cos\left(\omega_s \cdot t\right), sin\left(\omega_s \cdot t\right) \end{array} \right. \rightarrow \left[-e^{-\xi \cdot \omega \cdot t} \cdot \left(2 \cdot B \cdot \xi \cdot \omega \cdot \omega_s - A \cdot \xi^2 \cdot \omega^2 + A \cdot \omega_s^2\right)\right] \cdot cos\left(t \cdot \omega_s\right) + e^{-\xi \cdot \omega \cdot t} \cdot \left(B \cdot \xi^2 \cdot \omega^2 + 2 \cdot A \cdot \xi \cdot \omega \cdot \omega_s - B \cdot \omega_s^2\right) \cdot sin\left(t \cdot \omega_s\right)$$

$\omega := \sqrt{\frac{k}{m}} = 17.321$ **Pulsazione propria [rad/s]**

$f := \frac{\omega}{2 \cdot \pi} = 2.757$ **Frequenza propria [Hz]**

$T := \frac{1}{f} = 0.363$ **Periodo della vibrazione libera [s]**

$\xi := \frac{c}{2 \cdot m \cdot \omega} = 0.173$ $\xi = 17.321 \cdot \%$ **Fattore di smorzamento**

$\omega_s := \omega \cdot \sqrt{1 - \xi^2} = 17.059$ **Pulsazione propria smorzata [rad/s]**

$f_s := \frac{\omega_s}{2 \cdot \pi} = 2.715$ **Frequenza propria smorzata [Hz]**

$T_s := \frac{1}{f_s} = 0.368$ **Periodo della vibrazione libera smorzata [s]**

$A := x_0 = 0.05$

$B := \dfrac{\xi \cdot \omega \cdot x_0 + v_0}{\omega_s} = 0.185$

$C := \sqrt{A^2 + B^2} = 0.191$

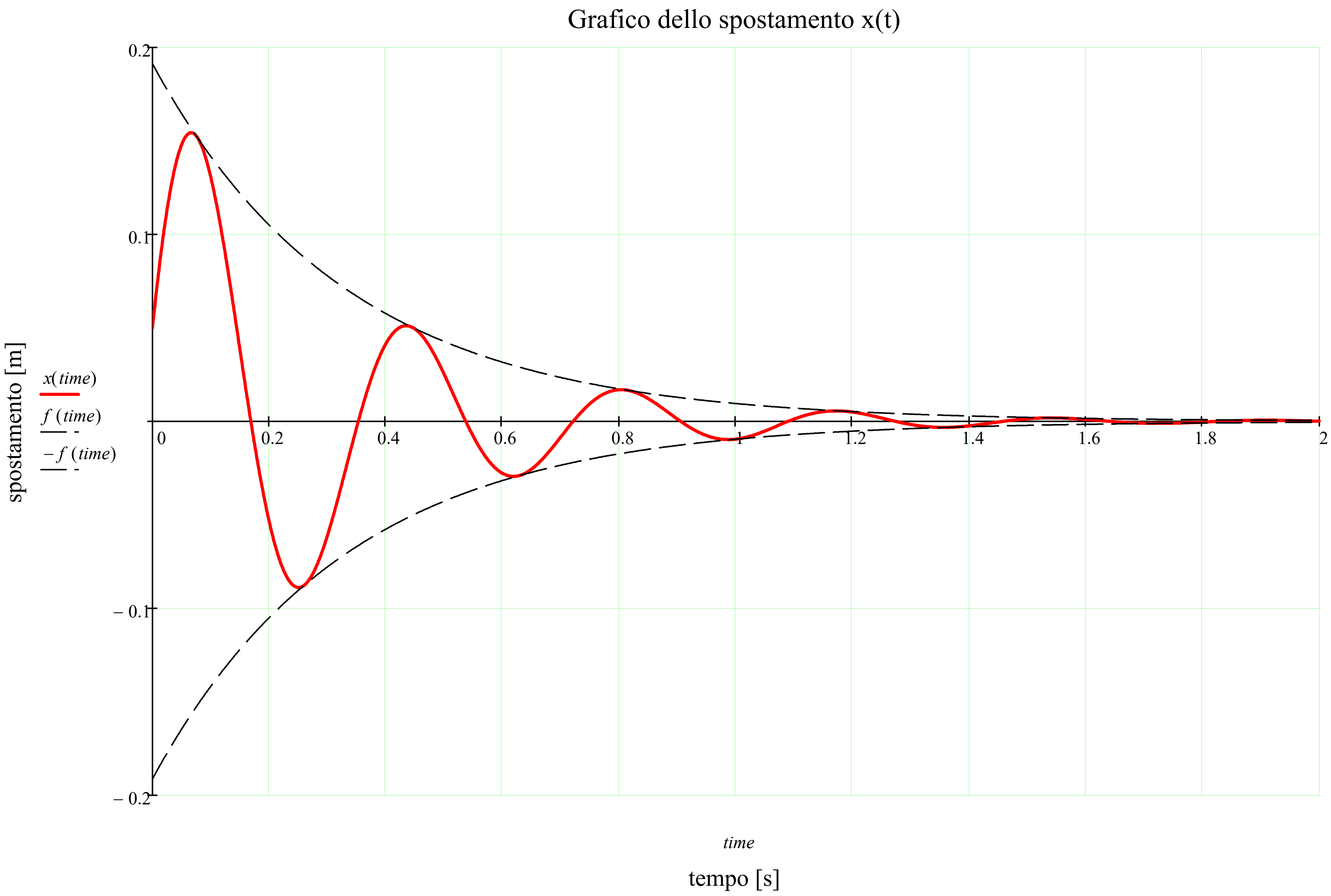
$f(t) := C \cdot e^{-\xi \cdot \omega \cdot t}$

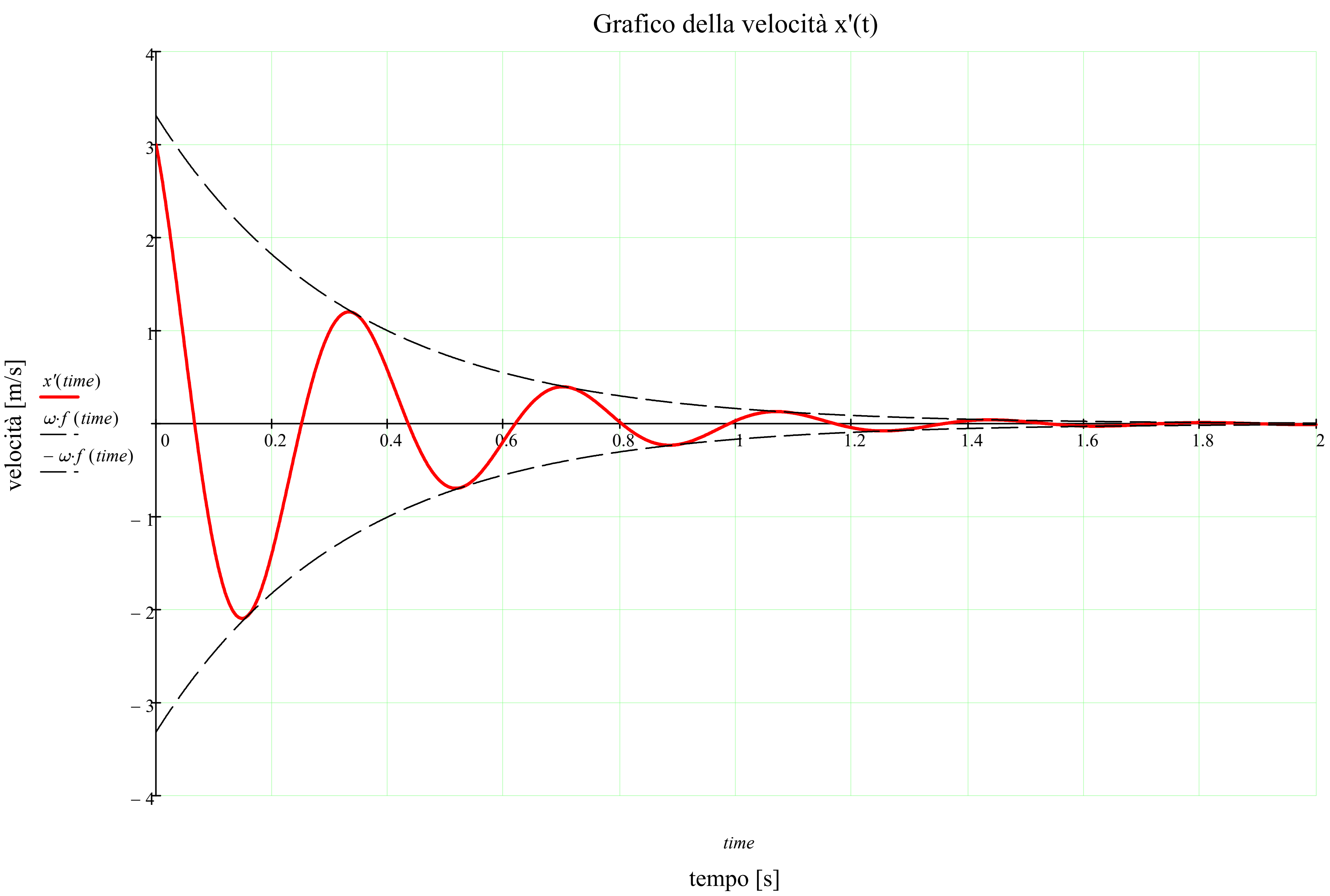
$T_{max} := 2$

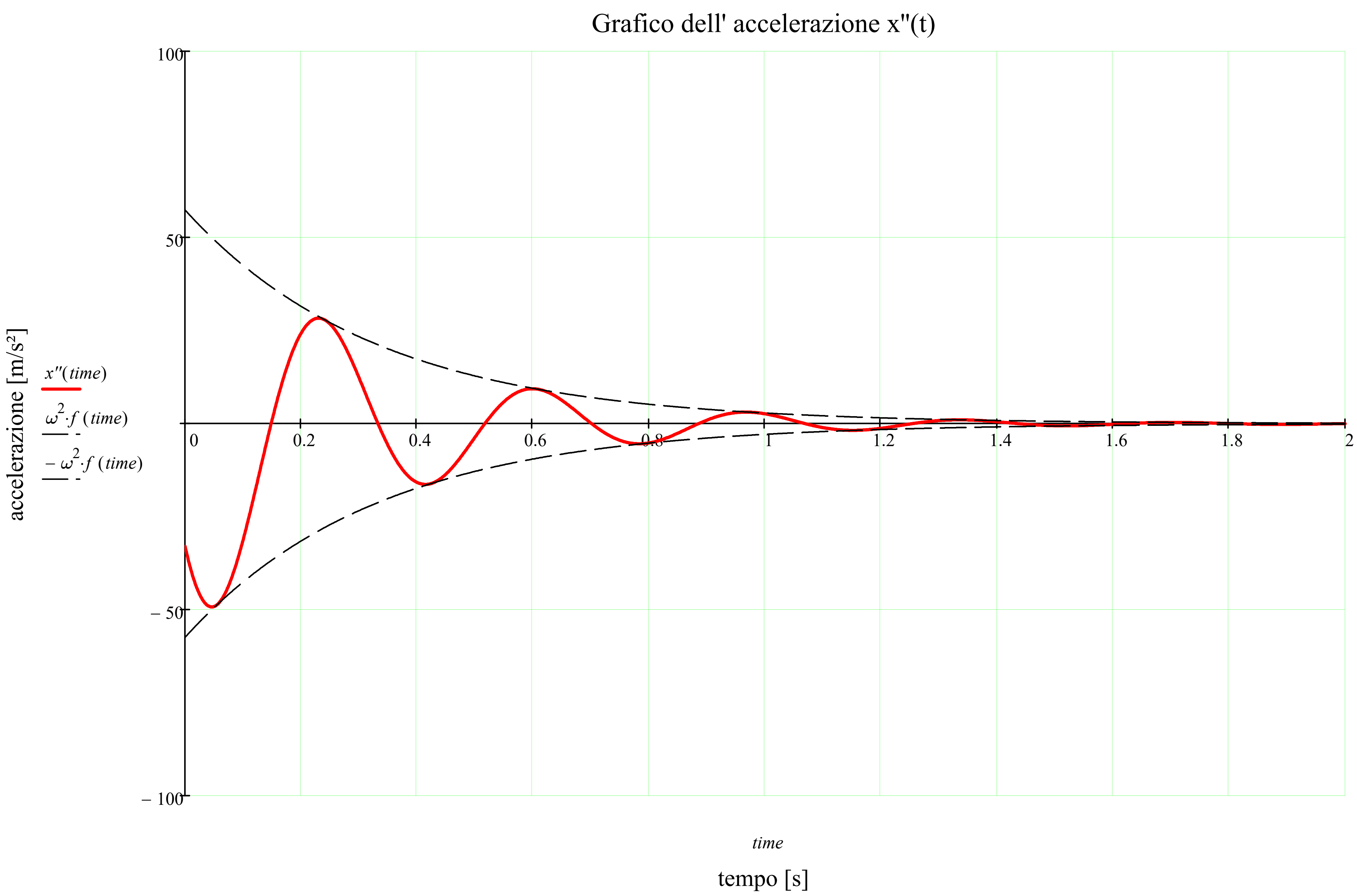
$\Delta t_{cons} := \dfrac{1}{100} \cdot T = 3.628 \times 10^{-3}$ $\Delta t := 1 \cdot 10^{-3}$

$N_{int} := \dfrac{T_{max}}{\Delta t} = 2 \times 10^3$

$time := 0, \Delta t .. T_{max}$





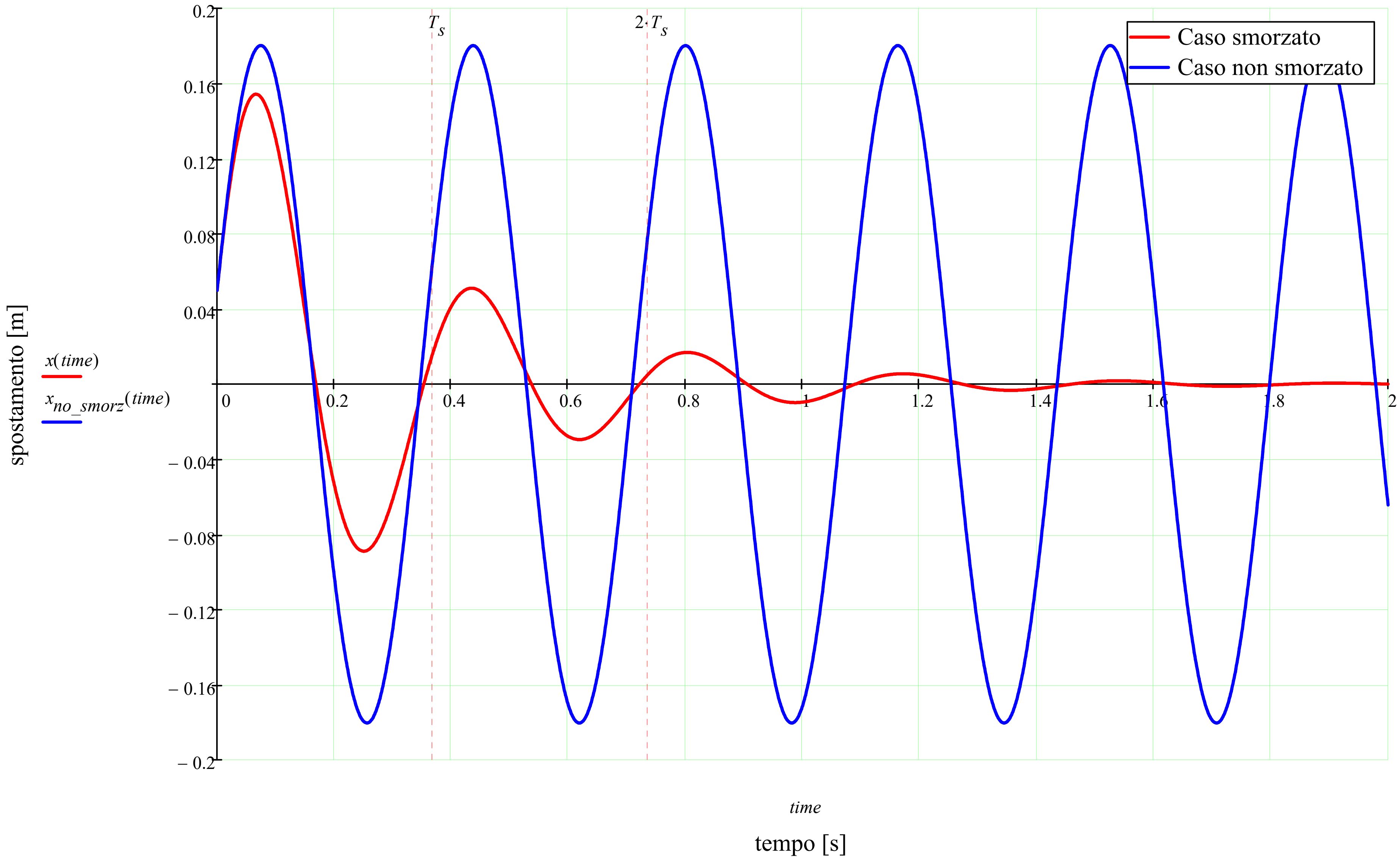


Caso non smorzato

$A_1 := x_0 = 0.05$ $\qquad B_1 := \frac{v_0}{\omega} = 0.173$

$x_0 = 0.05$ $\qquad v_0 = 3$

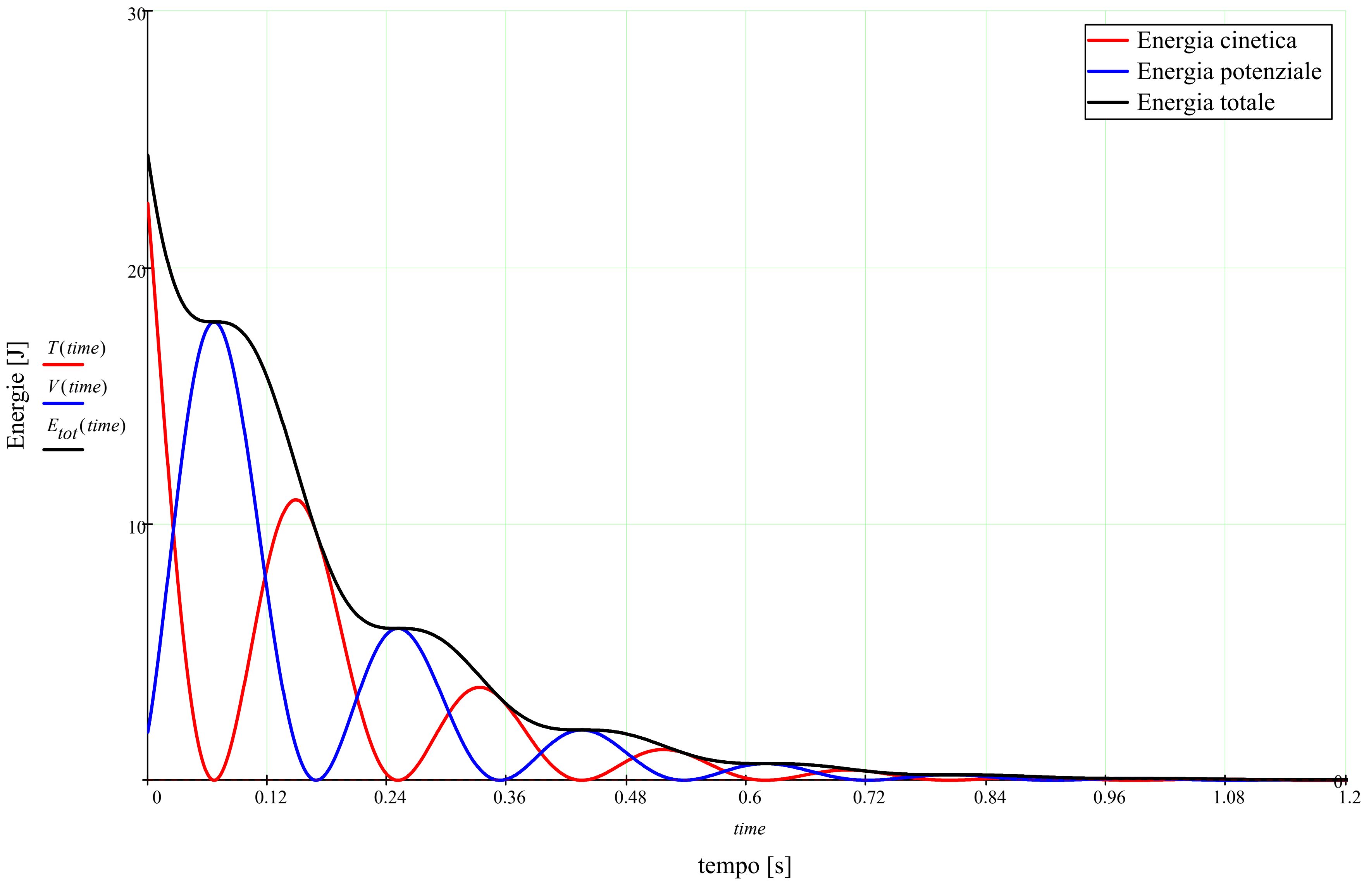
$x_{no\_smorz}(t) := A_1 \cdot cos(\omega \cdot t) + B_1 \cdot sin(\omega \cdot t)$



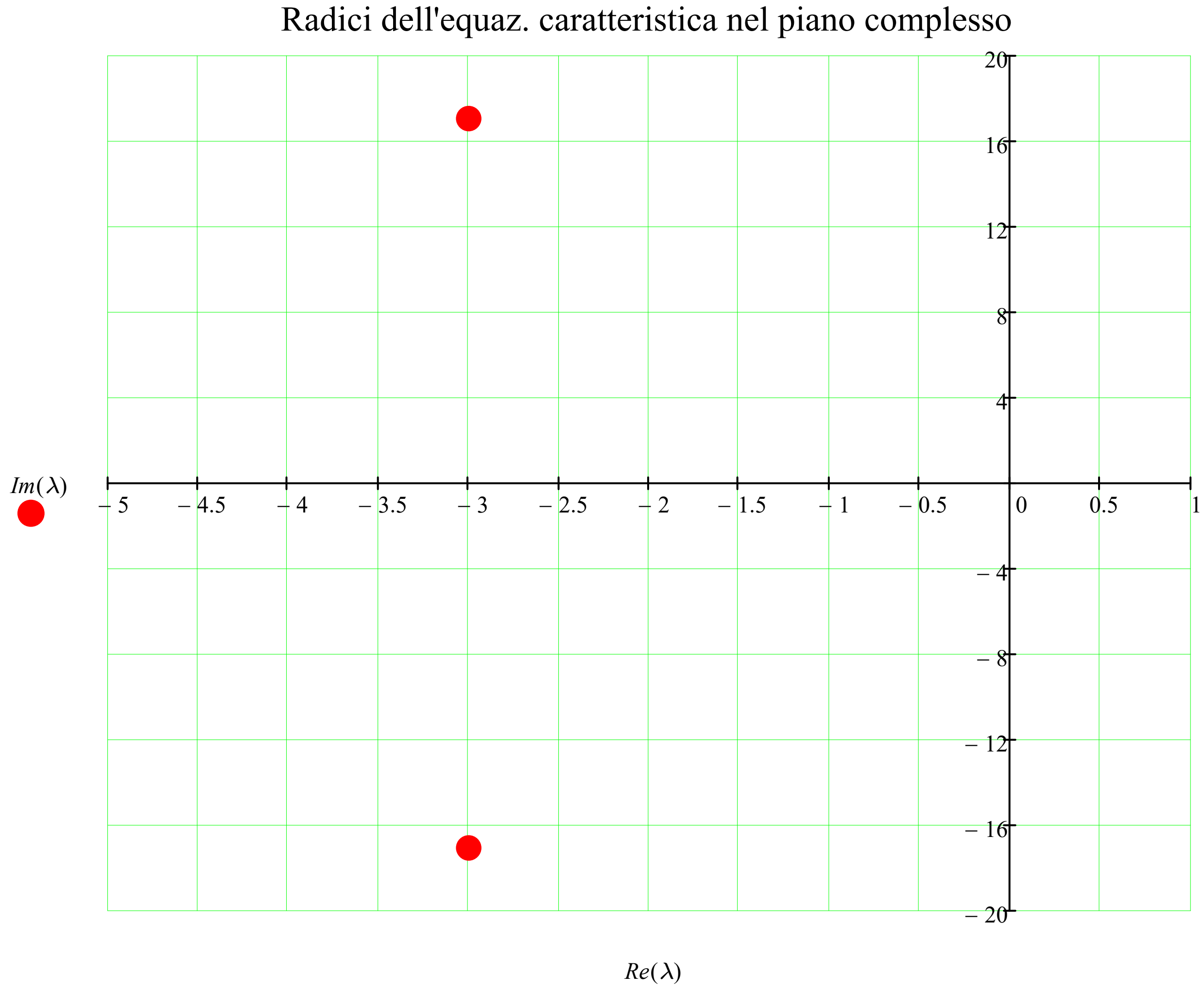
$T(t) := \frac{1}{2} \cdot m \cdot x'(t)^2$ **Energia cinetica**

$V(t) := \frac{1}{2} \cdot k \cdot x(t)^2$ **Energia potenziale**

$E_{tot}(t) := T(t) + V(t)$ **Energia totale**



$\lambda := polyroots\left(\begin{pmatrix} k \\ c \\ m \end{pmatrix}\right) = \begin{pmatrix} -3 - 17.059i \\ -3 + 17.059i \end{pmatrix}$



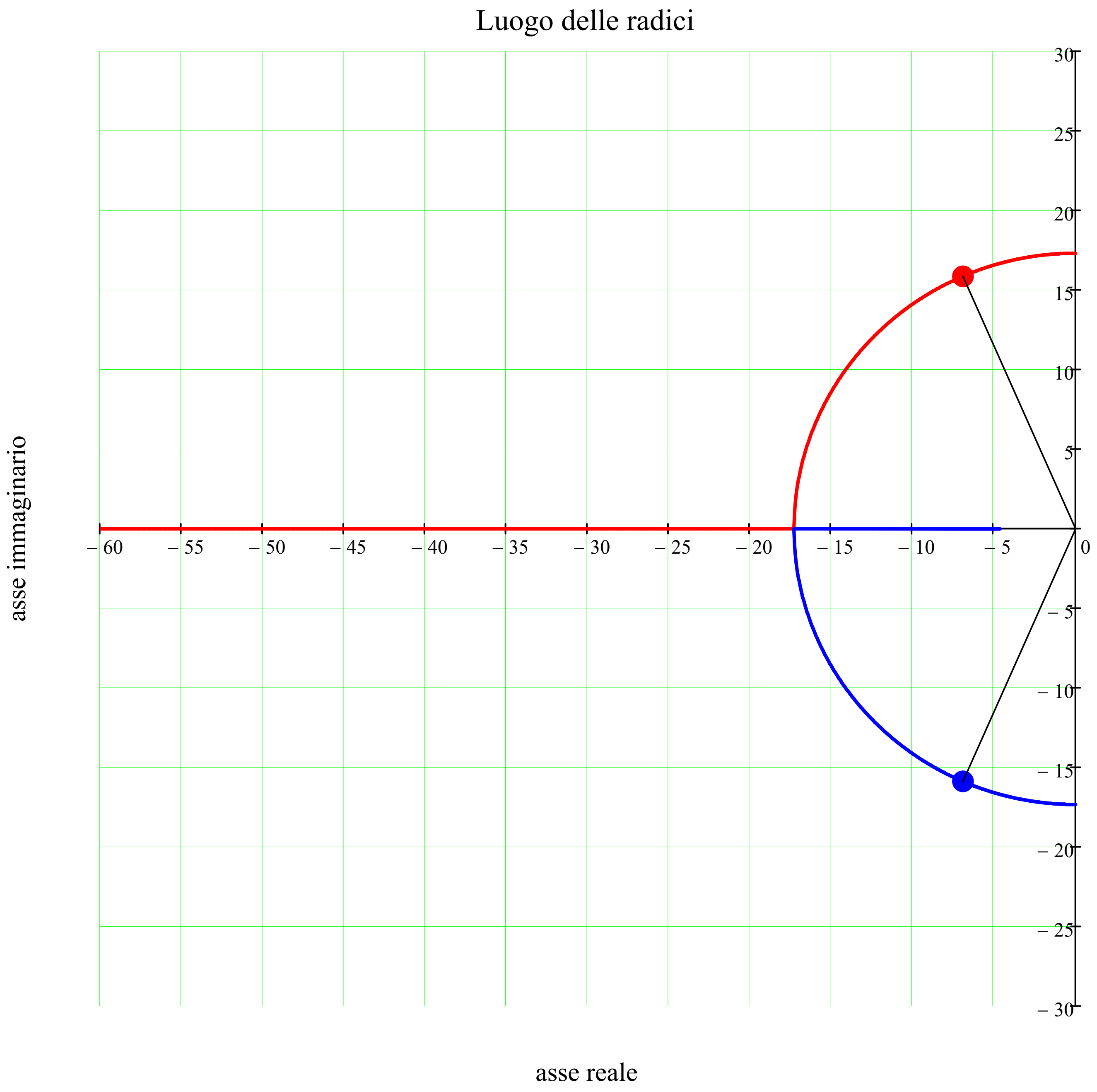
$\xi = 0.173$

$\lambda_1(\xi) := \omega\cdot\left(-\xi + i\cdot\sqrt{1 - \xi^2}\right)$ $\lambda_2(\xi) := \omega\cdot\left(-\xi - i\cdot\sqrt{1 - \xi^2}\right)$

$\xi_0 := 0.4$ **Modificare questo valore per modificare la posizione dei pallini sul grafico**

$\lambda_1(\xi_0) = -6.928 + 15.875i$ $\lambda_2(\xi_0) = -6.928 - 15.875i$

$ZOOM := 30$



Smorzamento critico

$m = 5$ $\qquad k = 1500$

$c_{cr} := 2\cdot\sqrt{k\cdot m} = 173.205$ $\qquad 2\cdot\sqrt{k\cdot m} = 173.205$ $\qquad \frac{2\cdot k}{\omega} = 173.205$

Formule alternative per il calcolo dello smorzamento critico

$c_{cr\_bis} := 2\cdot m\cdot\omega = 173.205$ $\qquad c_{cr\_ter} := \frac{2\cdot k}{\omega} = 173.205$

Caso critico

$x_{crit}(t) := \left(C_1 + C_2\cdot t\right)\cdot e^{-\omega\cdot t}$

$x'_{crit}(t) := \frac{d}{dt}x_{crit}(t)$ $\qquad x''_{crit}(t) := \frac{d^2}{dt^2}x_{crit}(t)$

Derivate temporali (velocità ed accelerazione) simboliche

$x'_{crit}(t)$ simplify $\rightarrow -e^{-\omega\cdot t}\cdot\left(C_1\cdot\omega - C_2 + C_2\cdot\omega\cdot t\right)$

$x''_{crit}(t)$ simplify $\rightarrow \omega\cdot e^{-\omega\cdot t}\cdot\left(C_1\cdot\omega - 2\cdot C_2 + C_2\cdot\omega\cdot t\right)$

$C_1 := x_0 = 0.05$

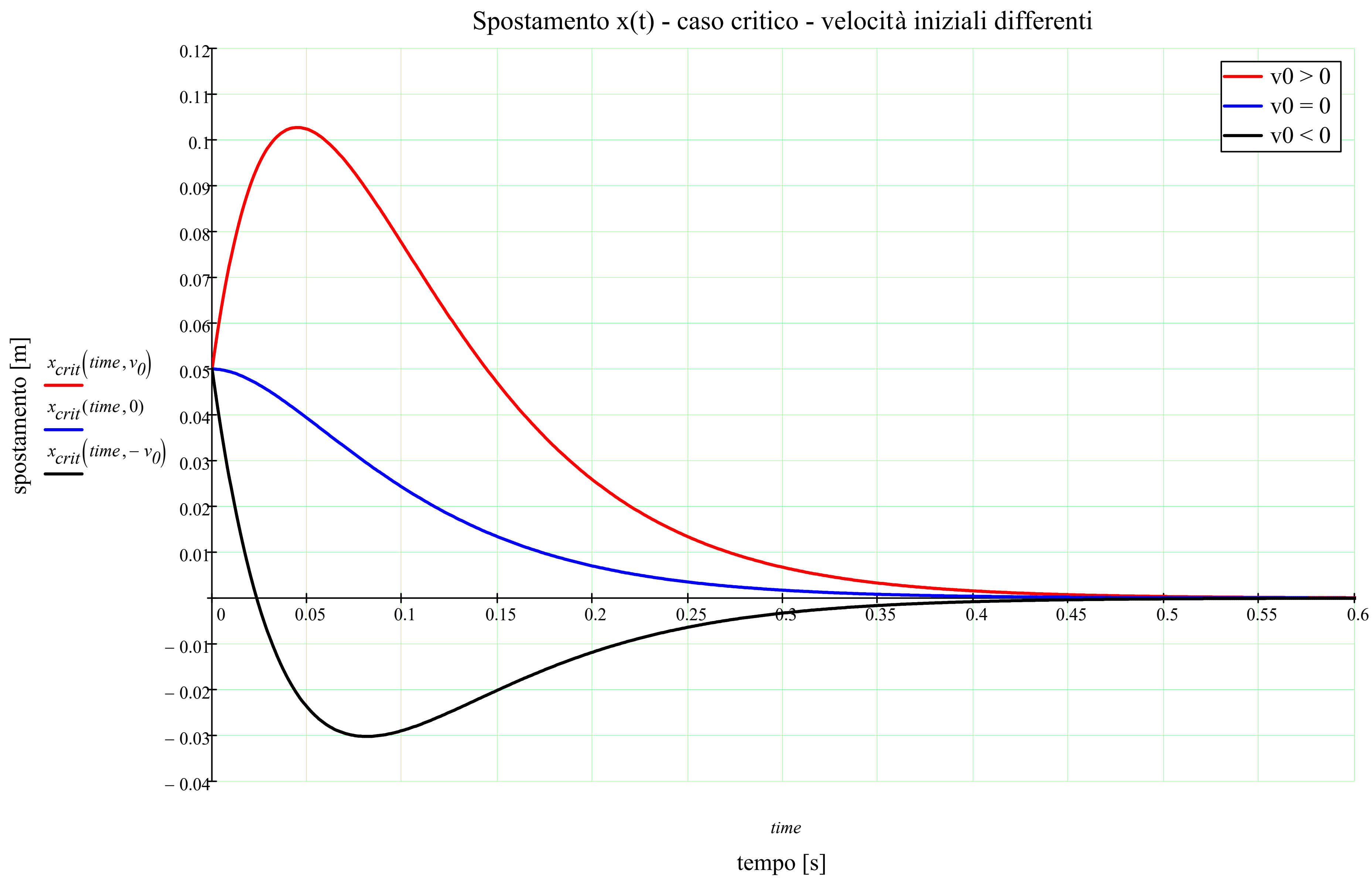
$C_2(v_0) := \omega\cdot x_0 + v_0$

$v_0 = 3$

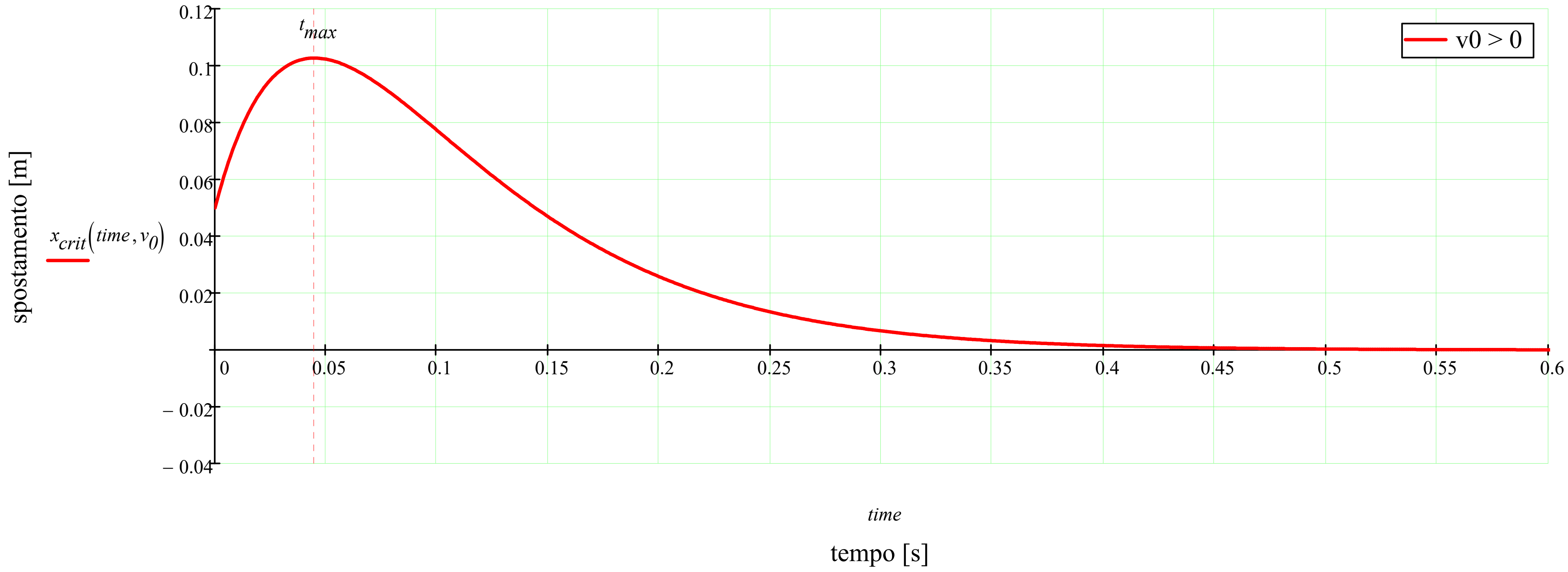
$x_{crit}(t, v_0) := (C_1 + C_2(v_0) \cdot t) \cdot e^{-\omega \cdot t}$ $\quad C_1 = 0.05 \quad C_2(v_0) = 3.866$

$C_2(0) = 0.866$

$C_2(-v_0) = -2.134$



$t_{max} := \frac{C_2(v_0) - C_1 \cdot \omega}{C_2(v_0) \cdot \omega} = 0.045$ **Calcolo del punto di max. (curva rossa - velocità iniziale positiva)**

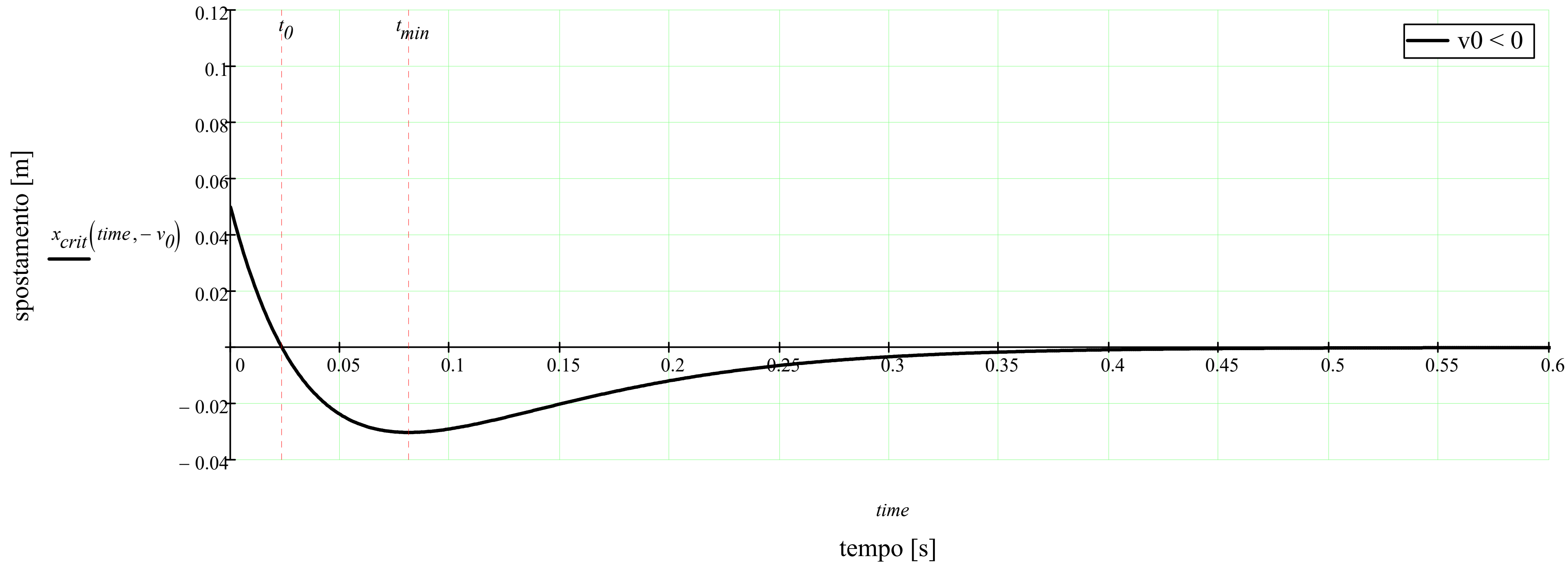


$t_0 := \frac{-C_1}{C_2(-v_0)} = 0.023$

Calcolo del punto di molla scarica (x=0) - (curva nera - velocità iniziale negativa)

$t_{min} := \frac{C_2(-v_0) - C_1 \cdot \omega}{C_2(-v_0) \cdot \omega} = 0.081$

Calcolo del punto di min. (curva nera - velocità iniziale negativa)



Caso sovrasmorzato

$x_0 = 0.05$

$v_0 = 3$

$x_{sovra}(t) := C_1 \cdot e^{\lambda_1 \cdot t} + C_2 \cdot e^{\lambda_2 \cdot t}$

$x'_{sovra}(t) := \frac{d}{dt} x_{sovra}(t)$ $\qquad$ $x''_{sovra}(t) := \frac{d^2}{dt^2} x_{sovra}(t)$

Derivate temporali (velocità ed accelerazione) simboliche

$x'_{sovra}(t) \text{ simplify } \rightarrow C_1 \cdot \lambda_1 \cdot e^{t \cdot \lambda_1} + C_2 \cdot \lambda_2 \cdot e^{t \cdot \lambda_2}$

$x''_{sovra}(t) \text{ simplify } \rightarrow C_1 \cdot \lambda_1^2 \cdot e^{t \cdot \lambda_1} + C_2 \cdot \lambda_2^2 \cdot e^{t \cdot \lambda_2}$

$c_{cr} = 173.205$

$c := 350$ $\qquad$ $\xi := \frac{c}{c_{cr}} = 2.021$

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} := \begin{pmatrix} \frac{-c - \sqrt{c^2 - 4 \cdot k \cdot m}}{2 \cdot m} \\ \frac{-c + \sqrt{c^2 - 4 \cdot k \cdot m}}{2 \cdot m} \end{pmatrix} = \begin{pmatrix} -65.414 \\ -4.586 \end{pmatrix}$$

$-\xi \cdot \omega - \omega \cdot \sqrt{\xi^2 - 1} = -65.414$

$-\xi \cdot \omega + \omega \cdot \sqrt{\xi^2 - 1} = -4.586$

$$\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} := \begin{pmatrix} \frac{-v_0 + x_0 \cdot \lambda_2}{\lambda_2 - \lambda_1} \\ \frac{v_0 - x_0 \cdot \lambda_1}{\lambda_2 - \lambda_1} \end{pmatrix} = \begin{pmatrix} -0.053 \\ 0.103 \end{pmatrix}$$

$x_0 = 0.05$ $v_0 = 3$

$C_1 := x_0 = 0.05$

$C_2 := \omega \cdot x_0 + v_0 = 3.866$

$x_{crit}(t) := \left(C_1 + C_2 \cdot t\right) \cdot e^{-\omega \cdot t}$

$x_{sovra}(t) := C_1 \cdot e^{\lambda_1 \cdot t} + C_2 \cdot e^{\lambda_2 \cdot t}$